4.5 The Law of Conservation of Energy

Imagine the thrill of the riders as the roller coaster in **Figure 1** moves up and around the loops, over and over again. What do riders feel as they near the top of each loop? What do they feel as they move down toward the ground again? What makes them feel these sensations?

In this section, you will explore some of the physics related to roller coasters, sports activities, and other movements you experience every day. You will read about the exchange between gravitational potential energy and kinetic energy that occurs when objects move.

Figure 1 Roller coasters offer both thrills and a chance to learn about physics.

Energy Transformations

As a diver climbs the steps to the top of a diving platform, her gravitational potential energy increases relative to the water surface. As she dives toward the water, her gravitational potential energy decreases. At the same time, her kinetic energy increases as her downward speed builds. When she hits the water, her gravitational potential energy relative to the water surface is zero, and her kinetic energy is at a maximum.

Although her gravitational potential energy decreases, that energy does not just disappear. As her kinetic energy increases, it too does not just appear from nowhere. The potential energy transforms into kinetic energy as the diver falls. Energy is neither created nor destroyed; it simply changes form. In fact, the total mechanical energy—the sum of kinetic and potential energy—remains constant. This important law of nature is called the **law of conservation of energy:**

> **Law of Conservation of Energy**
>
> Energy is neither created nor destroyed. It can only change form.

The law of conservation of energy is one of the fundamental principles of physics. To take into account apparent energy losses due to friction and other effects, the statement above will be refined in the next section. You will use the law of conservation of energy as a tool for solving many problems.

Mini Investigation

Various Energies of a Roller Coaster

Skills: Predicting, Analyzing, Evaluating, Communicating

SKILLS HANDBOOK A5.5

In this activity, you will analyze differences in energy at various heights of a roller coaster car. A roller coaster car starts from rest at point A, moves down the hill, and up and around the loop, to point F (**Figure 2**). Assume that energy losses due to friction are negligible and can be ignored. In this scenario, the height of the roller coaster is the independent variable, and the different types of energy are the dependent variables.

1. Create a table with the headings Height, Gravitational Potential Energy, Kinetic Energy, and Total Energy.
2. Record the height values for the six labelled points in Figure 2.
3. Calculate the potential, kinetic, and total mechanical energy values for each of the six points, and record the values in your table. Assume the mass of the roller coaster car is 875 kg.

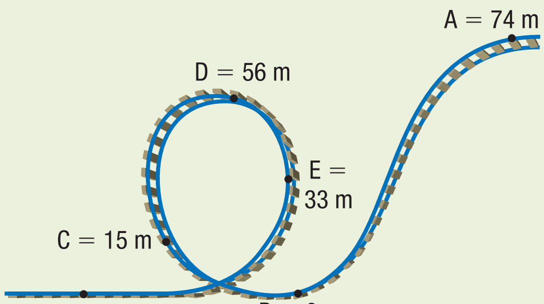

Figure 2

4. Sketch a graph of energy versus height for the roller coaster. Show each of the three energies (gravitational potential energy, kinetic energy, and total energy) on the same graph, but use different colours or line styles for each type.

A. Describe and explain the shape of the total energy graph. K/U T/I A

B. Compare the shapes of the graphs for gravitational potential energy and kinetic energy. How do they relate to the total energy graph? T/I A

C. Explain why it was necessary to know the height of point A. How would the graph change if the height of point A were greater? T/I A

D. Discuss how your graph would change if the mass of the roller coaster car were greater. K/U T/I A

WEB LINK

The total mechanical energy of a moving roller coaster car is the same at every point. That is, the sum of the gravitational energy and the kinetic energy remains constant. The total initial energy, $E_{\text{T i}}$, equals the total final energy, $E_{\text{T f}}$:

$$E_{\text{T i}} = E_{\text{T f}}$$

For situations involving only gravitational potential energy and kinetic energy, the equation can be written as

$$E_{\text{g i}} + E_{\text{k i}} = E_{\text{g f}} + E_{\text{k f}}$$

Knowing the energy of a roller coaster car at the start of the ride enables you to determine the gravitational potential energy and kinetic energy at other points. The following Tutorial examines how to use conservation of energy to solve problems involving motion. WEB LINK

Tutorial 1 Applying the Law of Conservation of Energy

Sample Problem 1: Making Connections between Gravitational Potential Energy and Kinetic Energy

A 67 kg snowboarder starts at the top of an icy (frictionless) hill of vertical height 22 m with an initial speed of 15 m/s (**Figure 3**).

(a) Calculate the snowboarder's mechanical energy at the top of the hill.

(b) Calculate the snowboarder's speed at the midway point and at the bottom of the hill.

(c) Describe the energy transformation that occurs as the snowboarder moves down the hill.

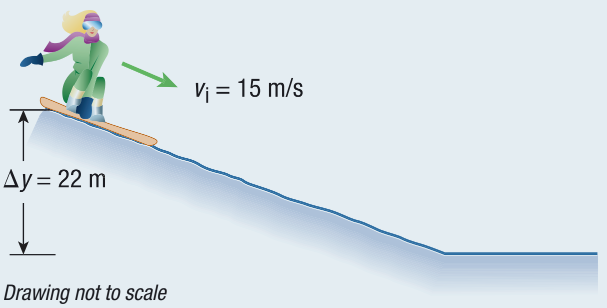

Figure 3

Solution

(a) **Given:** $m = 67$ kg; $\Delta y = 22$ m; $v_i = 15$ m/s

Required: E_T

Analysis: Choose the bottom of the hill as the $h = 0$ reference point. Then, set the gravitational potential energy at the top of the hill equal to this amount.

$$\Delta E_g = mg\Delta y;\ E_k = \frac{1}{2}mv^2;\ E_T = E_g + E_k$$

Solution: The change in gravitational potential energy from the bottom of the hill to the top of the hill is

$$\begin{aligned}\Delta E_g &= mg\Delta y \\ &= (67\text{ kg})(9.8\text{ m/s}^2)(22\text{ m}) \\ \Delta E_g &= 1.444 \times 10^4\text{ J (two extra digits carried)}\end{aligned}$$

The kinetic energy at the top of the hill is

$$\begin{aligned}E_k &= \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(67\text{ kg})(15\text{ m/s})^2 \\ E_k &= 7.537 \times 10^3\text{ J (two extra digits carried)}\end{aligned}$$

The total mechanical energy at the top of the hill is then

$$\begin{aligned} E_T &= E_g + E_k \\ &= 1.444 \times 10^4\ \text{J} + 7.537 \times 10^3\ \text{J} \\ E_T &= 2.198 \times 10^4\ \text{J (two extra digits carried)} \end{aligned}$$

Statement: The snowboarder's mechanical energy at the top of the hill, relative to the bottom of the hill, is 2.2×10^4 J.

(b) **Given:** $m = 67$ kg; $\Delta y_{mid} = 11$ m and $\Delta y_{bottom} = 0$ m; $E_{T\,i} = 2.198 \times 10^4$ J

Required: v_{mid}; v_{bottom}

Analysis: Use the law of conservation of energy to relate the initial and final energies:

$$E_{g\,i} + E_{k\,i} = E_{g\,f} + E_{k\,f}$$

In each case,

$$\Delta E_g = mg\Delta y;\ E_k = \frac{1}{2}mv^2$$

Solution: The total mechanical energy at the midpoint and the bottom of the hill will equal the total energy at the top of the hill, 2.2×10^4 J. Since the hill bottom is the $h = 0$ reference point, the gravitational potential energy at the midpoint is

$$\begin{aligned} E_g &= mg\Delta y \\ &= (67\ \text{kg})(9.8\ \text{m/s}^2)(11\ \text{m}) \\ E_g &= 7.223 \times 10^3\ \text{J (two extra digits carried)} \end{aligned}$$

The kinetic energy is

$$\begin{aligned} E_k &= E_T - E_g \\ &= 2.198 \times 10^4\ \text{J} - 7.223 \times 10^3\ \text{J} \\ E_k &= 1.476 \times 10^4\ \text{J} \end{aligned}$$

Then, calculate the speed at the midpoint.

$$\begin{aligned} E_{k\,mid} &= \frac{1}{2}mv^2 \\ \frac{2E_{k\,mid}}{m} &= v^2 \\ \sqrt{\frac{2E_{k\,mid}}{m}} &= v \\ v &= \sqrt{\frac{2(1.476 \times 10^4\ \text{J})}{67\ \text{kg}}} \\ v &= 21\ \text{m/s} \end{aligned}$$

Next, perform the same calculations for the bottom of the hill. Note that at the bottom of the hill the mechanical energy is all kinetic energy.

$$\begin{aligned} E_{g\,bottom} &= mg\Delta y \\ &= (67\ \text{kg})(9.8\ \text{m/s}^2)(0\ \text{m}) \\ E_{g\,bottom} &= 0\ \text{J} \\ E_{k\,bottom} &= E_T - E_{g\,bottom} \\ &= 2.198 \times 10^4\ \text{J} - 0\ \text{J} \\ E_{k\,bottom} &= 2.198 \times 10^4\ \text{J (two extra digits carried)} \end{aligned}$$

Now calculate her speed at the bottom of the hill:

$$\begin{aligned} v_{bottom} &= \sqrt{\frac{2E_{k\,bottom}}{m}} \\ &= \sqrt{\frac{2(2.198 \times 10^4\ \text{J})}{67\ \text{kg}}} \\ v_{bottom} &= 26\ \text{m/s} \end{aligned}$$

Statement: Halfway down the hill, the snowboarder's speed is 21 m/s. Her speed at the bottom of the hill is 26 m/s.

(c) As the snowboarder moves down the hill, her height above the reference point decreases, reducing her gravitational potential energy. Her speed, however, increases. The gravitational potential energy continuously transforms into kinetic energy until she reaches the bottom.

Sample Problem 2: Determining Maximum Height and Speed Using the Law of Conservation of Energy

Figure 4 shows a 0.45 kg bullfrog jumping with an initial speed of 6.2 m/s at an angle of 49° above the horizontal. Assume that energy losses due to air resistance are negligible and can be ignored.

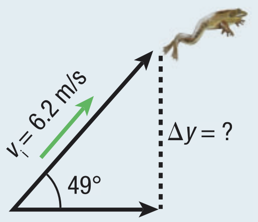

Figure 4

(a) Calculate the maximum height of the bullfrog's jump.

(b) Calculate the components of the bullfrog's velocity when it first reaches a height of 0.82 m.

Solution

(a) **Given:** $m = 0.45$ kg; $h_i = 0$; $v_i = 6.2$ m/s; $\theta = 49°$

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$; $E_k = \frac{1}{2}mv^2$; $E_T = E_g + E_k$;

$v_x = v_i \cos\theta$

The bullfrog's velocity in the horizontal direction does not change, because no horizontal force acts on the frog. Since the change in the horizontal velocity is zero, it has no effect on the change in kinetic energy of the bullfrog. The kinetic energy from the bullfrog's vertical component of velocity decreases and transforms into gravitational potential energy. You can solve the problem by comparing just the vertical parts of the kinetic energy and the gravitational potential energy.

Solution: The initial speed is 6.2 m/s. The initial horizontal velocity is

$$v_x = v_i \cos\theta$$
$$= (6.2 \text{ m/s}) \cos 49°$$
$$v_x = 4.07 \text{ m/s}$$

At the high point of the jump, the vertical component of the velocity is zero, but the horizontal component is still 4.07 m/s. The speed at the high point is then 4.07 m/s. Using conservation of energy,

$$E_{k\,i} + E_{g\,i} = E_{k\,f} + E_{g\,f}$$
$$E_{g\,i} - E_{g\,f} = E_{k\,f} - E_{k\,i}$$
$$-\Delta E_g = \Delta E_k$$

Use this equation to solve for the change in height.

$$-\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}(v_f^2 - v_i^2)$$
$$\Delta y = \left(\frac{v_i^2 - v_f^2}{2g}\right)$$
$$= \left(\frac{(6.2 \text{ m/s})^2 - (4.07 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)}\right)$$
$$\Delta y = 1.1 \text{ m}$$

Statement: The bullfrog reaches a maximum height of 1.1 m above the ground.

(b) **Given:** $y_i = 0$; $m = 0.45$ kg; $v_i = 6.2$ m/s; $y_f = 0.82$ m

Required: $v_{x\,f}$; $v_{y\,f}$

Analysis: Use the law of conservation of energy to relate the initial and final energies.

Solution: The change in the gravitational potential energy is

$$\Delta E_g = mg\Delta y$$
$$= (0.45 \text{ kg})(9.8 \text{ m/s}^2)(0.82 \text{ m})$$
$$\Delta E_g = 3.6 \text{ J}$$

As in part (a), the horizontal speed of the bullfrog does not change, so the only change to its kinetic energy comes from a change in its vertical speed.

$$\Delta E_k = \frac{1}{2}m(v_{x\,f}^2 + v_{y\,f}^2) - \frac{1}{2}m(v_{x\,i}^2 + v_{y\,i}^2)$$
$$= \frac{1}{2}m(\cancel{v_{x\,f}^2} + v_{y\,f}^2 - \cancel{v_{x\,i}^2} - v_{y\,i}^2)$$
$$\Delta E_k = \frac{1}{2}m(v_{y\,f}^2 - v_{y\,i}^2)$$

The initial vertical speed is

$$v_{y\,i} = v_i \sin\theta$$
$$= (6.2 \text{ m/s}) \sin 49°$$
$$v_{y\,i} = 4.68 \text{ m/s}$$

Now use conservation of energy:

$$\Delta E_k = -\Delta E_g$$
$$\frac{1}{2}mv_{y\,f}^2 = \frac{1}{2}mv_{y\,i}^2 - \Delta E_g$$

Multiply both sides of the equation by 2, and divide both sides by m.

$$v_{y\,f}^2 = v_{y\,i}^2 - \frac{2\Delta E_g}{m}$$
$$v_{y\,f} = \sqrt{v_{y\,i}^2 - \frac{2\Delta E_g}{m}}$$
$$= \sqrt{(4.68 \text{ m/s})^2 - \frac{2(3.6 \text{ J})}{0.45 \text{ kg}}}$$
$$v_{y\,f} = 2.4 \text{ m/s}$$

Statement: The bullfrog's speed at a height of 0.82 m is 2.4 m/s.

Practice

1. A soccer player kicks a 0.43 kg soccer ball down a smooth (frictionless) hill 18 m high with an initial speed of 7.4 m/s (**Figure 5**). T/I
   (a) Calculate the ball's speed as it reaches the bottom of the hill. [ans: 2.0×10^1 m/s]
   (b) The soccer player stands at the same point on the hill and gives the ball a kick up the hill at 4.2 m/s. The ball moves up the hill, comes to rest, and rolls back down the hill. Determine the ball's speed as it reaches the bottom of the hill. T/I [ans: 19 m/s]
2. A tennis player begins a serve by tossing a 57 g tennis ball straight up. T/I
   (a) After leaving the player's hand, the ball rises another 1.8 m. Calculate the speed of the ball as it leaves the player's hand. [ans: 5.9 m/s]
   (b) On the next serve, the tennis player tosses the ball with $\frac{1}{4}$ the speed in (a). Determine the ratio of the maximum rise of the ball after leaving the player's hand after this toss to the maximum rise in (a). [ans: 1:16]

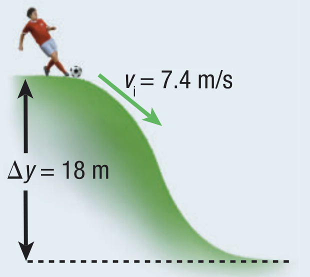

Figure 5

Investigation 4.5.1

Energy and Pulleys (page 210)
The law of conservation of energy applies to real-world mechanical systems. In this controlled experiment you will investigate this law.

Isolated and Open Systems

In the previous section, you learned about the conservation of mechanical energy. Mechanical energy includes gravitational potential energy and kinetic energy. However, the conservation of energy includes other forms of energy, such as thermal, elastic, electrical, chemical, light, and sound. To understand transformations between these forms of energy, it is important to know the distinction between an isolated and an open system.

isolated system a system that cannot interact or exchange energy with external systems; also called a closed system

An **isolated system** is a system that cannot interact with any other system or exchange energy with its surroundings. An object in an isolated system might exchange energy with other objects within the system, but energy never moves into or out of the system. The parts of the system are isolated from any influences outside the system.

In reality, the universe itself is the only completely isolated system, but you can define a system that has minor influence from its surroundings as an isolated system. For example, you may consider a diver to be an isolated system if you ignore influences such as air friction and the energy of vibrations in the diving board.

open system a system that can interact with another external system

An **open system** is a system that can interact with another external system. An open system can exchange energy with its surroundings. In reality, a diver is an open system that exchanges energy with the air, the diving board, the water, and other real-world systems. Physicists sometimes refer to an isolated system as a *closed system* to contrast it with an open system. Many systems analyzed in this book can be modelled as isolated systems. The concepts of isolated and open systems allow us to state the **law of conservation of energy** more formally:

Law of Conservation of Energy

Energy is neither created nor destroyed in an *isolated system*. It can only change form.

Biological Energy Transformations

biochemical energy a type of chemical potential energy stored in the cells and other basic structures of biological organisms

It is difficult to imagine a second of the day without observing some type of energy transformation. Each time you turn on a light, walk to class, or listen to your favourite song, energy changes from one form to another. Energy conservation involves transformations between many types of energy. Many biologically important energy transformations involve chemical energy. **Biochemical energy** is the energy stored in the cells of organisms and is used to perform all life processes. Green plants produce biochemical energy during photosynthesis. Every time you blink your eyes, raise your arm, or move muscles, you transform biochemical energy stored in your body to mechanical energy that enables you to move.

Biochemical energy can also change to forms other than mechanical energy. The bioluminescent jellyfish in **Figure 6** uses the transformation of biochemical energy to light energy as a defence mechanism against predators. Electric eels transform biochemical energy to electrical energy to stun their prey. During intense exercise, the change of biochemical energy to thermal energy in your body causes you to feel warm. In each of these transformations, total energy is conserved.

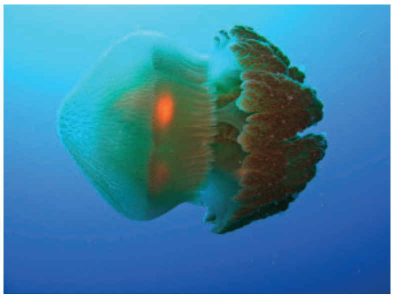

Figure 6 Some jellyfish transform biochemical energy into light energy.

Power

You learned that hydroelectric stations generate millions of kilowatts of power. The terms *power* and *kilowatt* are probably familiar to you, but what do they actually mean? Time enters into work–energy ideas through the concept of **power**, which relates the rate of change in energy of a system over time. A man using a rope to lift a crate does work on the crate as it moves from the floor to a height h (**Figure 7**). The displacement changes the potential energy of the crate by an amount $mg\Delta y$. This energy comes from the man as he pulls on the other end of the rope and does an amount of work $W = mg\Delta y$ on the rope.

power the rate of work done by a force over time, or the rate at which the energy of an open system changes

If this work is expended during a time t, then the power P exerted by the man is defined as

$$P = \frac{W}{t}$$

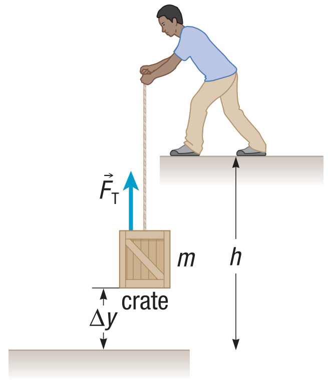

Figure 7 The man produces power as he lifts the crate.

The SI unit of power is the watt (W), named after James Watt (1736–1819), a developer of the steam engine. One watt is equal to one joule per second. Note that the symbol for watt is not italicized (W), but the variable used for work is an italic W.

Notice that power is the *rate* at which work is done. Work is a way to transfer energy from one system to another over time. As a result, power is also equal to the energy output of a device per unit time.

In addition to mechanical energy, you can consider how other types of energy, including chemical energy and electrical energy, are involved in conservation of energy situations. The concept of power also applies to chemical and electrical processes and devices. There is a power output or input associated with many of the electrical devices in your home. For example, a typical compact fluorescent lamp is rated at 23 W, or 23 J/s. This rating means that the lamp consumes 23 J of electrical energy for every second it is turned on. Likewise, electronic devices such as computers and DVD players also have a power rating. **Table 1** lists the power output and consumption of a number of common devices.

Table 1 Typical Values for the Power Output or Consumption of Some Common Appliances and Devices

Device	Power output or consumption (W)
portable DVD player	20
laptop computer	40
desktop computer	125
elite bicycle racer	400
automobile engine (small car)	7.5×10^4
automobile engine (race car)	5.2×10^5

The distinction between power consumption and power output is important. For example, a fluorescent lamp consumes a certain amount of electrical energy (for which you pay the utility company), and it outputs a certain amount of energy in the form of visible light along with a certain amount of thermal energy.

The following Tutorial illustrates the definition of power as change in energy over time.

Tutorial 2 Calculating Power

Sample Problem 1: Power as a Rate of Change in Kinetic Energy

A car accelerates from rest to a speed of 27.8 m/s in 7.7 s. The mass of the car is 1.1×10^3 kg. Ignoring friction, determine how much power the car requires.

Given: $m = 1.1 \times 10^3$ kg; $v_i = 0$ m/s; $v_f = 27.8$ m/s; $t = 7.7$ s

Required: P

Analysis: Use the power equation, $P = \frac{W}{t}$. Use the work–energy theorem to relate W to the change in kinetic energy ΔE_k.

Solution: The work done on the car equals its change in kinetic energy. The car starts from rest, so the change in kinetic energy equals the final kinetic energy:

$$\begin{aligned} W &= \Delta E_k \\ &= E_{k\,f} \\ &= \frac{1}{2}mv_f^2 \\ &= \frac{1}{2}(1.1 \times 10^3 \text{ kg})(27.8 \text{ m/s})^2 \\ W &= 4.251 \times 10^5 \text{ J (two extra digits carried)} \end{aligned}$$

Therefore,

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{4.251 \times 10^5 \text{ J}}{7.7 \text{ s}} \\ P &= 5.5 \times 10^4 \text{ W} \end{aligned}$$

Statement: The car requires 55 kW of power.

Practice

1. A firefighter climbs a ladder at a speed of 1.4 m/s. The ladder is 5.0 m long, and the firefighter weighs 65 kg. T/I A
   (a) Determine the firefighter's power output while climbing the ladder. [ans: 890 W]
   (b) How long does it take her to climb the ladder? [ans: 3.6 s]
2. A Grand Prix race car accelerates to twice the speed of the car in Sample Problem 1, in the same amount of time. Calculate the ratio of the power needed by the Grand Prix car to the power needed by the car in Sample Problem 1. K/U T/I A [ans: 4:1]
3. Every year, the Calgary Tower hosts a foot race to the top of the tower. The vertical distance travelled up the 802 steps is about 190 m, and a champion racer can make the climb in less than 5.0 min. If a 62 kg racer completes the climb in 4 min 50 s, determine his average power output during the race. T/I A [ans: 0.40 kW]

UNIT TASK BOOKMARK

You can apply what you have learned about energy transformations and the conservation of energy to the Unit Task on page 270.

4.5 Review

Summary

- The law of conservation of energy states that energy can neither be created nor destroyed in an isolated system; it can only change form. This law is useful for solving many physics problems involving motion.
- An isolated, or closed, system cannot exchange energy with its surroundings. However, an open system can exchange energy with its surroundings.
- Power is the rate of work done during a time interval, or the rate at which the energy of a system changes.

Questions

1. A child tosses a tennis ball straight up into the air with an initial speed of 11 m/s. Ignore the height of the child, and assume that air resistance is negligible. K/U T/I C
   (a) Determine the maximum height that the ball will reach.
   (b) Sketch a graph of potential energy versus time for the ball during its time in the air. Explain why the graph has the shape that it does. Label the minimum and maximum potential energy values.
   (c) On the same set of axes, sketch a graph of total energy versus time and a graph of kinetic energy versus time. Explain why they are shaped the way that they are. Identify any maximum or minimum values.
2. An apple falls from a branch to the ground below. K/U
   (a) At what moment is the kinetic energy of the apple greatest?
   (b) At what moment is the gravitational potential energy greatest?
3. A hockey puck slides along a level surface, eventually coming to rest. K/U T/I
   (a) Is the energy of the hockey puck conserved? Explain your answer.
   (b) Discuss what happens to the initial kinetic energy of the puck.
4. (a) A skier of mass 110 kg travels down a frictionless ski trail with a top elevation of 210 m. Calculate the work done on the skier by gravity as the skier travels from the top of the trail to the bottom.
   (b) Calculate the speed of the skier when he reaches the bottom of the ski trail. Assume he starts from rest. T/I A
5. A 62 kg snowboarder is moving across a horizontal ledge at 8.1 m/s when she encounters a drop-off and becomes airborne. Ignore air resistance. The snowboarder lands 3.7 m below the drop-off. Calculate her speed at the moment she hits the ground. T/I A
6. A dolphin is trying to jump through a hoop that is fixed at a height of 3.5 m above the surface of her pool. The dolphin leaves the water at an angle of inclination of 40°. Determine the minimum speed the dolphin will need when leaving the water in order to reach the height of the hoop. T/I
7. A roller coaster car with mass 640 kg moves along the track shown in **Figure 8**. Assume all friction is negligible. K/U T/I A

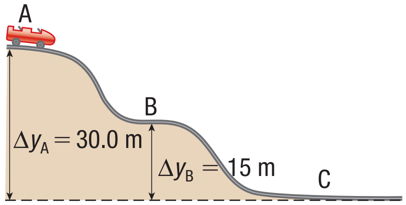

Figure 8

   (a) Is the mechanical energy of the roller coaster conserved? Explain your answer.
   (b) If the roller coaster starts from rest at point A, what is its total mechanical energy at point A?
   (c) What is the total mechanical energy at point B?
   (d) Calculate the speed of the roller coaster when it reaches points B and C.
   (e) If the car starts with a speed of 12 m/s at point A, calculate the speed of the roller coaster when it reaches points B and C.
8. A 52 kg woman jogs up a hill in 24 s. Calculate the power the woman exerts if the hill is 18 m high. T/I A