Gravitational Potential Energy

4.3

The extreme sport of heli-skiing starts with a helicopter ride to the top of a remote mountain. A skier like the one in **Figure 1** then jumps out of the helicopter onto the steep slopes. How do you know that the skier in Figure 1 has energy before jumping from the helicopter? How can this energy be transformed into another form? In this section, you will learn the answers to these questions.

Figure 1 The skier's kinetic energy will increase as she falls. Her height gives her potential energy.

Potential Energy

You can sense that the skier in Figure 1 has energy because of her height above the ground. The force of gravity will do work on the skier as soon as she steps out of the helicopter. As she falls, and then skis down the mountain, her kinetic energy will increase. Her height above the ground means that she has the potential to pick up kinetic energy from the force of gravity. This potential to increase kinetic energy is a form of stored energy. The stored energy that an object has that can be released into another form of energy is **potential energy**.

potential energy the stored energy an object has that can be converted into another form of energy

Many forms of potential energy exist. A long-jumper poised to jump has potential energy stored in his muscles. The flexed muscles have stored biomechanical energy that is released as the jumper springs into the air. A stretched elastic band also has elastic potential energy. When you let an elastic band go, the potential energy becomes kinetic energy.

Kinetic energy and potential energy have a close relationship, and one form can transform into the other as work is done on or by an object. The sum of the kinetic energy and potential energy of an object is called the **mechanical energy** of the object. You will read more about properties of mechanical energy in Section 4.5. This section will focus on the potential energy an object has due to gravity.

mechanical energy the sum of an object's kinetic and potential energies

Gravitational Potential Energy

An object near Earth's surface has a potential energy that depends on the object's mass, m, and height, h. The **gravitational potential energy**, E_g, is stored energy as a result of the gravitational force between the object and Earth. Although we will often speak of the gravitational potential energy of an object, note that E_g is actually a property of the object and Earth together. Potential energy is always a property of a *system* of objects, as we will discuss in Section 4.5.

gravitational potential energy (E_g) stored energy an object has because of its position and the applied gravitational force

Analyzing gravitational potential energy mathematically can help clarify how to apply it when solving problems. Suppose a worker lifts a crate onto the back of a pickup truck (**Figure 2**).

Figure 2 A worker applies a force F_a to a crate, doing work as he lifts the crate from the ground up to the truck bed. The change in height of the crate is Δy.

Let Δy be the change in the elevation of the crate. The force the worker applies to the crate to raise it is in the same direction as the crate's displacement. The work done on the crate by the force is

$$W = F\Delta d\cos\theta$$
$$= F\Delta y\cos\theta$$
$$= mg\Delta y\,(\cos 0°)$$
$$W = mg\Delta y$$

Note that we replaced Δd with Δy because we often use y to represent vertical displacements. When the bottom of the crate reaches the height of the truck's bed, the crate has gravitational potential energy relative to the ground. Suppose the worker decides instead to place crates on top of each other. You could then describe a crate's gravitational potential energy relative to the ground, or describe its gravitational potential energy relative to the truck bed. Since gravitational potential energy depends on position *relative* to an object, it is a relative quantity. Its value depends on the height of the object above some point of reference that you choose. You can choose the level that is most convenient for solving any given problem. The work done to increase the elevation of an object relates to the *change* in the elevation instead of a specific height:

$$\Delta E_g = mg\Delta y$$

where ΔE_g is the change in gravitational potential energy, m is the mass, g is the gravitational acceleration, and Δy is the vertical component of the displacement.

The equation for gravitational potential energy presents a few important points to consider. First, the equation determines the *change* in gravitational potential energy for a given change in elevation. It does not assign a fixed value of potential energy to a given elevation, but only the difference in potential energy between different elevations, Δy. You can freely choose a reference point to act as a zero gravitational potential elevation. When solving a problem, choose the elevation that is most convenient for that problem. Earth's surface is often a convenient choice, but it is not the only choice, or necessarily the best choice. Choose a reference point that will result in the easiest calculations. Often, choosing the surface of Earth for this point is most convenient because the gravitational potential energy will then be either zero or positive.

Second, Δy does not depend on any changes in the horizontal position or on the path the object took to reach its new height. When the object increases its height, Δy is positive; when the object falls to a lower height, Δy is negative.

Finally, you will read in Chapter 6 that the acceleration due to gravity, g, actually varies slightly by height above Earth's surface. The equation above is only accurate when the change in height is small enough that you can ignore the change in g.

An object thrown upward will begin with kinetic energy, but the amount of kinetic energy decreases as the object slows down because of the gravitational force. At the same time, the object gains potential energy as its height increases. Whenever an object falls, the force of gravity will do work on the object, giving it kinetic energy according to the work–energy theorem. At the same time, it loses gravitational potential energy.

Suppose that you hold a physics book above your desk. If you drop the book, its initial velocity is zero, but it has gravitational potential energy relative to the desktop. As it falls, the force of gravity does work on the book and converts its gravitational potential energy to kinetic energy. Before it hits the desktop, all of the initial potential energy relative to the desktop has converted into kinetic energy. As the book strikes the desktop, kinetic energy is converted into sound energy, thermal energy, and other forms of energy.

UNIT TASK **BOOKMARK**

You can apply what you have learned about work, kinetic energy, and gravitational potential energy to the Unit Task on page 270.

Units of Gravitational Potential Energy

Gravitational potential energy and kinetic energy are different manifestations of the same quantity—energy. They both have the same units—joules—and both are scalar quantities. In the equations below, you can see how to calculate the units of gravitational potential energy, $\Delta E_g = mg\Delta y$. The units of m are kilograms. The units of g are metres per second squared. The units of Δy are metres. Putting these facts together gives

$$\begin{aligned} [E_g] &= [\text{kg}]\frac{[\text{m}]}{[\text{s}^2]}[\text{m}] \\ &= [\text{N}]\cdot[\text{m}] \\ [E_g] &= [\text{J}] \end{aligned}$$

Similar to kinetic energy, no direction is associated with gravitational potential energy. The following Tutorial examines how to use gravitational potential energy in problems involving rising and falling objects.

Tutorial 1 Applications Involving Gravitational Potential Energy

Sample Problem 1: Calculating Gravitational Potential Energy

A hiker stands near the edge of a cliff and accidentally drops a rock of mass 1.2 kg to the ground at the base of the cliff 28 m below (**Figure 3**). Calculate the potential energy of the rock relative to the ground just before the hiker drops it.

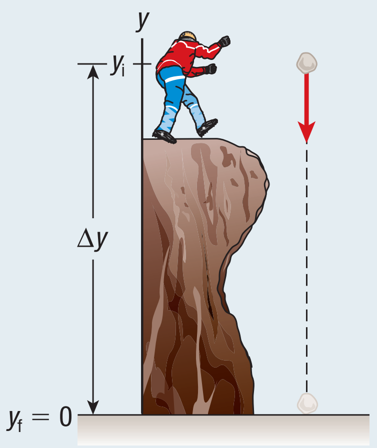

Figure 3

Given: $m = 1.2$ kg; $g = 9.8$ m/s^2; $\Delta y = 28$ m

Required: ΔE_g

Analysis: Use the gravitational potential energy equation, $\Delta E_g = mg\Delta y$. Let the positive y-direction be upward. Let the reference point, $y_f = 0$, be the base of the cliff.

Solution:
$$\begin{aligned} \Delta E_g &= mg\Delta y \\ &= (1.2\text{ kg})(9.8\text{ m/s}^2)(28\text{ m}) \\ \Delta E_g &= 3.3 \times 10^2\text{ J} \end{aligned}$$

Statement: The rock's potential energy just before the hiker drops it is 3.3×10^2 J.

Sample Problem 2: Applying Gravitational Potential Energy to Determine Mass

A weightlifter raises a loaded barbell 2.2 m. The lift increases the gravitational potential energy of the barbell by 490 J. Determine the mass of the loaded barbell.

Given: $\Delta y = 2.2$ m; $g = 9.8$ m/s^2; $\Delta E_g = 490$ J

Required: m

Analysis: Rearrange the gravitational potential energy equation, $\Delta E_g = mg\Delta y$, to solve for m.

Solution: $\Delta E_g = mg\Delta y$

Divide both sides of the equation by $g\Delta y$.

$$m = \frac{\Delta E_g}{g\Delta y}$$

$$= \frac{490\ \text{J}}{(9.8\ \text{m/s}^2)(2.2\ \text{m})}$$

$$m = 23\ \text{kg}$$

Statement: The mass of the loaded barbell is 23 kg.

Sample Problem 3: Gravitational Potential Energy and the Work–Energy Theorem

A physics textbook is 3.6 cm thick and has a mass of 1.6 kg. A student stacks 10 of the books in a single pile on a table. Each book starts from table level before the student lifts it to the top of the stack.

(a) Calculate the gravitational potential energy of the stack of books with respect to the table.

(b) Determine the total work done by the student to make the stack of books.

(c) Calculate the work done by the student to move the books from the stack and set each on the desk.

Given: height of each book, $h = 3.6$ cm $= 0.036$ m; number of books $= 10$

Required: W

Analysis: $\Delta E_g = mg\Delta y$

(a) Each book has a separate gravitational potential energy. Since the first book does not move, no energy is expended. The second book is lifted 3.6 cm, the third is lifted 2×3.6 cm, and so on. The gravitational potential energy of the stack is the sum of the gravitational potential energies for the individual books.

(b) Since work and energy use the same units, W is equal to the gravitational potential energy of the stack.

(c) The work done to move the books from the stack is equal in magnitude to W but opposite in sign.

Solution: $\Delta E_g = mg\Delta y$

$$= (1.6\ \text{kg})(9.8\ \text{m/s}^2)[0(0.036\ \text{m}) + 1(0.036\ \text{m}) + 2(0.036\ \text{m}) + \cdots + 9(0.036\ \text{m})]$$

$$= (1.6\ \text{kg})(9.8\ \text{m/s}^2)(0.036\ \text{m})(0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)$$

$$= (0.56\ \text{J})(45)$$

$$\Delta E_g = 25\ \text{J}$$

$$W = 25\ \text{J}$$

Statement:

(a) The potential energy of the stack relative to the table is 25 J.

(b) The work done by the student to create the stack is 25 J.

(c) The work done to move the books from the stack is −25 J.

Practice

1. A grey squirrel drops a 0.02 kg walnut from a branch that is 8.0 m high. Determine the change in potential energy of the walnut between the branch and the ground. T/I [ans: 1.6 J]
2. The weightlifter in Sample Problem 2 increases the mass of the weights on the barbell and lifts it one more time. The new lift increases the potential energy of the bar by 660 J. Calculate the new mass of the barbell. T/I [ans: 31 kg]
3. Calculate the work required by the student in Sample Problem 3 to stack 2 more physics books on the original stack of 10. T/I [ans: 12 J]

4.3 Review

Summary

- Potential energy is the stored energy an object has that can be released into another form of energy.
- Mechanical energy is the sum of the kinetic and potential energies.
- Gravitational potential energy is the energy that an object has due to its height above a reference point. It is a scalar quantity and is measured in joules (J).
- When solving problems related to gravitational potential energy, choose a reference point, $y = 0$, from which to measure the gravitational potential energy.
- The gravitational potential energy of an object near Earth's surface depends on the object's mass, m; the acceleration due to gravity, g; and the object's change in height as measured from a reference height, Δy: $\Delta E_g = mg\Delta y$.

Questions

1. A 2.5 kg piece of wood falls onto a carpenter's table from a height of 2.0 m above the table. T/I
   (a) Calculate the kinetic energy of the wood as it hits the table.
   (b) Calculate the speed of the wood as it hits the table.
2. Calculate the gravitational potential energy relative to the ground of a 5.0 kg Canada goose flying at a height of 553 m above the ground. T/I
3. A hockey referee drops a 175 g hockey puck from rest vertically downward from a height of 1.05 m above the ice surface. K/U T/I
   (a) Determine the gravitational potential energy of the puck relative to the ice before the referee drops it.
   (b) Calculate the change in gravitational potential energy of the puck as it drops from the referee's hand to the ice surface.
   (c) Calculate the work done on the puck by gravity as the puck travels from the referee's hand to the ice surface.
4. You lift your pet cat vertically by 2.0 m, and then you lower it vertically by 2.0 m. During this exercise, is the total work done by gravity positive, negative, or zero? Explain your answer. K/U C
5. A pole vaulter clears the bar at a height of 5.4 m, and then falls to the safety mat. The change in the pole vaulter's gravitational potential energy from the bar to the mat is -3.1×10^3 J. Calculate the pole vaulter's mass. T/I
6. A 0.46 kg golf ball on a tee is struck by a golf club. The golf ball reaches a maximum height where its gravitational potential energy has increased by 155 J from the tee. Determine the ball's maximum height above the tee. T/I
7. A 59 kg snowboarder descends a 1.3 km ski hill from the top of a mountain to the base (**Figure 4**). The slope is at an angle of 14° to the horizontal. Determine the snowboarder's gravitational potential energy relative to the mountain base when she is at the top. T/I

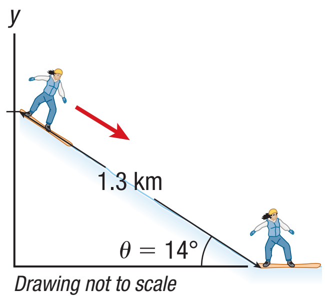

Figure 4

8. Suppose that you have N identical boxes in your room, each with mass m and height Δy. You stack the boxes in a vertical pile. K/U T/I
   (a) Determine the work done to raise the last box to the top of the pile. Express your answer in terms of the variables m, g, N, and Δy.
   (b) Determine the gravitational potential energy, in terms of m, g, N, and Δy, that is stored in the entire pile.
9. A gallon of gas contains about 1.3×10^8 J of chemical potential energy. Determine how many joules of chemical potential energy are stored in each litre of gas (3.79 L = 1 gallon). Calculate the height that the amount of chemical potential energy in 1 L of gas could raise all the students in your class if it was all converted to gravitational potential energy. You will have to make assumptions about the mass of the students. State your assumptions and show your calculations. T/I A