Kinetic Energy and the Work–Energy Theorem

4.2

Imagine the energy that a jet, like the one in **Figure 1**, needs to climb into the air. The jet has a mass of hundreds of thousands of kilograms, yet during takeoff, it seems to rise easily above the ground.

How can the jet have so much energy? You read in the previous section that the work a force does on an object is proportional to the distance the object moves. The work is also proportional to the component of the force in the direction of the object's displacement. The jet engines cause a force that pushes the jet forward. As it moves along the runway, air rushing past the wings exerts an upward force on the wings. The engines help maintain this effect. The forces of the engines and the air do work on the jet as it takes off. In this section, you will learn about the relationship between work done on an object and energy transferred to the object. The jet engines and air are the sources of energy that the jet uses to fly.

Figure 1 Work done by jet engines and the surrounding air gives a jet the kinetic energy needed to take off.

Kinetic Energy

Kinetic energy, E_k, is the energy an object has due to its motion. An object's kinetic energy is directly related to its mass and the square of its speed, according to the following relationship:

kinetic energy (E_k) the energy an object has because of its motion

$$E_k = \frac{1}{2}mv^2$$

where m is the mass of the object and v is its speed.

Consider how this relationship affects the kinetic energy of the jet in Figure 1. The jet's kinetic energy increases during takeoff because its speed increases. If the speed doubles, for example, the kinetic energy increases by a factor of 4. The jet has a tremendous amount of kinetic energy because its mass is so great.

Notice that kinetic energy is a scalar quantity. The equation above defines the magnitude of kinetic energy, but kinetic energy does not have a direction associated with it. The mass and speed of the airplane, and not its direction, determine its kinetic energy.

You can use dimensional analysis of the above equation to identify the units of kinetic energy. Expressing the mass in kilograms (kg) and the speed in metres per second (m/s) shows that the units of kinetic energy are joules (J):

$$\begin{aligned}[E_k] &= [\text{kg}]\left[\frac{\text{m}}{\text{s}}\right]^2 \\ &= [\text{kg}]\frac{[\text{m}]^2}{[\text{s}]^2} \\ &= \frac{[\text{kg}][\text{m}]}{[\text{s}]^2}[\text{m}] \\ &= [\text{N}]\cdot[\text{m}] \\ [E_k] &= [\text{J}]\end{aligned}$$

Notice that the units of kinetic energy are the same as the units of work. We will explore this important result further after Tutorial 1, which shows how to determine the kinetic energy of a moving object when you know its mass and its speed.

Tutorial 1 Solving Problems Involving Kinetic Energy

Sample Problem 1: Calculating Kinetic Energy

A car has a mass of 1.50×10^3 kg and is travelling at a speed of 85.0 km/h. Calculate the car's kinetic energy.

Given: $m = 1.50 \times 10^3$ kg; $v = 85.0$ km/h

Required: E_k

Analysis: Use the equation for kinetic energy, $E_k = \frac{1}{2}mv^2$. First, convert speed into units of metres per second.

Solution: $v = 85.0 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}}$

$$v = 23.6\ \text{m/s}$$

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(1.50 \times 10^3\ \text{kg})(23.6\ \text{m/s})^2$$

$$E_k = 4.18 \times 10^5\ \text{J}$$

Statement: The car's kinetic energy is 4.18×10^5 J.

Sample Problem 2: Using Kinetic Energy to Determine the Speed of an Object

When fleeing a predator, a 1.4 kg rabbit has a kinetic energy of 96 J. Calculate the speed of the rabbit.

Given: $m = 1.4$ kg; $E_k = 96$ J

Required: v

Analysis: Rearrange the kinetic energy equation, $E_k = \frac{1}{2}mv^2$, to isolate the unknown variable, v.

Solution: $E_k = \frac{1}{2}mv^2$

Multiply both sides of the equation by 2. Then, divide both sides by m to isolate v on one side of the equation.

$$2E_k = \cancel{2}\left(\frac{1}{\cancel{2}}mv^2\right)$$

$$2E_k = mv^2$$

$$\frac{2E_k}{m} = \frac{\cancel{m}v^2}{\cancel{m}}$$

$$\frac{2E_k}{m} = v^2$$

Take the square root of both sides.

$$v = \sqrt{\frac{2E_k}{m}}$$

Substitute known values into the equation and solve.

$$v = \sqrt{\frac{2(96\ \text{J})}{1.4\ \text{kg}}}$$

$$v = 12\ \text{m/s}$$

Statement: The rabbit's speed is 12 m/s.

Practice

1. By what factor does a car's kinetic energy increase when the car's speed
   (a) doubles [ans: 4] (b) triples [ans: 9] (c) increases by 26 % T/I [ans: 1.6]
2. If a bowling ball with mass 8.0 kg travels down the lane at 2.0 m/s, what is its kinetic energy? T/I [ans: 16 J]
3. Calculate the mass of a blue jay moving at 15 km/h with 0.83 J of kinetic energy. T/I [ans: 0.095 kg]

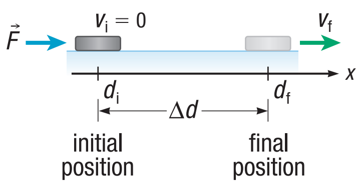

Figure 2 When a force, F, acts on this hockey puck, the puck accelerates. The force does work on the puck, and the puck's kinetic energy changes.

Kinetic Energy and the Work–Energy Theorem

Newton's second law of motion tells us that when an object is subject to a net external force, it accelerates in the same direction as the force. This motion results in work being done on the object. When the object's speed changes from this acceleration, then its kinetic energy also changes (**Figure 2**).

You have seen that for the simplest case of one-dimensional motion, with the force directed parallel to the displacement, the work done by a force on an object is equal to the magnitude of the force multiplied by the object's displacement: $W = F\Delta d$. Using Newton's second law ($F_T = ma$ for this one-dimensional case), the equation becomes

$$W = F_T \Delta d$$

$$W = ma\Delta d$$

Assume that the force is constant so that the acceleration is also constant. Recall the following kinematics equation for motion with constant acceleration:

$$v^2 = v_i^2 + 2a\Delta d$$

Notice that the subscript i indicates the *initial* velocity and position and the subscript f indicates the *final* velocity. The displacement is just $\Delta d = (d_f - d_i)$, so

$$v^2 = v_i^2 + 2a(d_f - d_i) \quad \text{or} \quad a(d_f - d_i) = \frac{v_f^2 - v_i^2}{2}$$

To calculate the work done on the object as it moves from the initial position d_i to the final position d_f, combine the previous equations:

$$W = ma(d_f - d_i)$$
$$= m\frac{v_f^2 - v_i^2}{2}$$
$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The equation shows that doing work on an object changes its kinetic energy. The following shows this relationship explicitly:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= E_{k\,f} - E_{k\,i}$$
$$W = \Delta E_k$$

This equation is the **work–energy theorem**: the total work done on an object by an external force equals the change in its kinetic energy. This relation tells us how work, force, and displacement connect to the kinetic energy of an object. Notice that this theorem is also consistent with the result discovered earlier, that work and kinetic energy are both measured in the same units, joules.

work–energy theorem the total work done on an object equals the change in its kinetic energy

Tutorial 2 Applying the Work–Energy Theorem

You can use the work–energy theorem to solve problems involving the kinetic energy transferred when a force does work on an object.

Sample Problem 1: Using the Work–Energy Theorem to Calculate Work Done

A blue whale with a mass of 1.5×10^5 kg is swimming with a speed of 6.1 m/s. A nearby boat startles the whale, and the whale increases its speed to 12.8 m/s. Calculate the work done on the whale by the water.

Given: $m = 1.5 \times 10^5$ kg; $v_i = 6.1$ m/s; $v_f = 12.8$ m/s

Required: W

Analysis: As the whale swims, it exerts a backward force on the water. By Newton's third law, the water exerts an equal and opposite forward force on the whale, causing it to accelerate and gain kinetic energy. Use the work–energy theorem, $W = \Delta E_k$, to calculate the positive work done by the water on the whale.

Solution: First, determine the initial and final kinetic energies.

$$E_{k\,i} = \frac{1}{2}mv_i^2$$
$$= \frac{1}{2}(1.5 \times 10^5 \text{ kg})(6.1 \text{ m/s})^2$$
$$E_{k\,i} = 2.79 \times 10^6 \text{ J}$$
$$E_{k\,f} = \frac{1}{2}mv_f^2$$
$$= \frac{1}{2}(1.5 \times 10^5 \text{ kg})(12.8 \text{ m/s})^2$$
$$E_{k\,f} = 1.23 \times 10^7 \text{ J}$$

Apply the work–energy theorem.

$$W = \Delta E_k$$
$$= E_{k\,f} - E_{k\,i}$$
$$= 1.23 \times 10^7 \text{ J} - 2.79 \times 10^6 \text{ J}$$
$$W = 9.5 \times 10^6 \text{ J}$$

Statement: The water does 9.5×10^6 J of work on the whale.

Sample Problem 2: Applying the Work–Energy Theorem in the Presence of Friction

A shuffleboard player wants to slide a 430 g disc a distance of precisely 12 m. If the coefficient of kinetic friction between the disc and the playing surface is 0.62, calculate the initial speed at which the player must release the disc.

Given: $m = 430\text{ g} = 0.43\text{ kg}$; $\mu_K = 0.62$; $\Delta d = 12\text{ m}$

Required: v

Analysis: The force due to friction is $\vec{F}_f = \mu_K \vec{F}_N$, where $\vec{F}_N$ is the normal force. Calculate the force due to friction, then the work done by friction. The work done by friction is $W = F_f \Delta d \cos\theta$.

The work–energy theorem is $W = \Delta E_k$, and $E_k = \frac{1}{2}mv^2$.

Solution: The force due to friction is

$$\begin{aligned}\vec{F}_f &= \mu_K \vec{F}_N \\ &= \mu_K mg \\ &= (0.62)(0.43\text{ kg})(9.8\text{ m/s}^2) \\ \vec{F}_f &= 2.61\text{ N}\end{aligned}$$

Friction opposes the motion of the disc, so θ is 180°, and $\cos\theta$ is −1.

The work done by friction is

$$\begin{aligned}W &= F\Delta d\cos\theta \\ &= (2.61\text{ N})(12\text{ m})(\cos 180°) \\ W &= -31.3\text{ J}\end{aligned}$$

The work–energy theorem tells us that the change in kinetic energy will equal the work done, or −31.3 J. The final velocity is zero, so the final kinetic energy is zero, and the initial kinetic energy is the negative of the work done.

$$\begin{aligned}E_k &= E_f - E_i \\ &= 0 - (-31.3\text{ J}) \\ E_k &= 31.3\text{ J}\end{aligned}$$

We can solve for the initial speed:

$$\begin{aligned}E_k &= \frac{1}{2}mv^2 \\ \frac{2E_k}{m} &= v^2 \\ v &= \sqrt{\left(\frac{2E_k}{m}\right)} \\ &= \sqrt{\left(\frac{2(31.3\text{ J})}{0.43\text{ kg}}\right)} \\ v &= 12\text{ m/s}\end{aligned}$$

Statement: The initial speed of the disc must be 12 m/s.

Sample Problem 3: Applying the Work–Energy Theorem to Calculate Initial Speed

A police car of mass 2.4×10^3 kg is travelling on the highway when the officers receive an emergency call. They increase the speed of the car to 33 m/s. The increase in speed results in 3.1×10^5 J of work done on the car. Determine the initial speed of the police car in kilometres per hour.

Given: $m = 2.4 \times 10^3\text{ kg}$; $v_f = 33\text{ m/s}$; $W = 3.1 \times 10^5\text{ J}$

Required: v_i

Analysis: Rearrange the work–energy equation to solve for the initial speed of the car. First, calculate the final kinetic energy of the car; then, subtract the work done to determine the initial kinetic energy.

Solution: First, determine the final and initial kinetic energies.

$$\begin{aligned}E_{k\,f} &= \frac{1}{2}mv_f^2 \\ &= \frac{1}{2}(2.4 \times 10^3\text{ kg})(33\text{ m/s})^2 \\ E_{k\,f} &= 1.31 \times 10^6\text{ N}\end{aligned}$$

$$\begin{aligned}W &= \Delta E_k \\ &= E_{k\,f} - E_{k\,i} \\ E_{k\,i} &= E_{k\,f} - W \\ &= 1.31 \times 10^6 - 3.1 \times 10^5\text{ J} \\ E_{k\,i} &= 1.00 \times 10^6\text{ J}\end{aligned}$$

Then, solve for v_i.

$$\begin{aligned}E_{k\,i} &= \frac{1}{2}mv_i^2 \\ \left(\frac{2}{m}\right)E_{k\,i} &= \left(\frac{2}{\cancel{m}}\right)\frac{1}{\cancel{2}}\cancel{m}v_i^2 \\ v_i^2 &= \left(\frac{2}{m}\right)E_{k\,i} \\ v_i &= \sqrt{\left(\frac{2}{m}\right)E_{k\,i}} \\ &= \sqrt{\left(\frac{2}{2.4 \times 10^3\text{ kg}}\right)1.00 \times 10^6\text{ J}} \\ v_i &= 28.9\text{ m/s}\end{aligned}$$

The initial speed is 29 m/s. To determine the speed in kilometres per hour,

$$\begin{aligned}v_i &= 29\,\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\,\frac{1\text{ km}}{1000\,\cancel{\text{m}}}\,\frac{3600\,\cancel{\text{s}}}{1\text{ h}} \\ v_i &= 1.0 \times 10^2\text{ km/h}\end{aligned}$$

Statement: The initial speed of the police car was 1.0×10^2 km/h.

Practice

1. An archer pulls back her bowstring (**Figure 3**) loaded with a 22 g arrow and then releases the string. The arrow's speed as it leaves the bowstring is 220 km/h. Calculate the work done on the arrow by the bowstring. T/I [ans: 41 J]

Figure 3

2. A space probe travels far out in the galaxy to a point where the force of gravity is very weak. The probe has a mass of 3.8×10^4 kg and an initial speed of 1.5×10^4 m/s. The probe's engines exert a force of 2.2×10^5 N in the original direction of motion as the probe travels a distance of 2.8×10^6 m. Calculate the final speed of the probe. K/U T/I [ans: 1.6×10^4 m/s]
3. A skater moves across the ice a distance of 12 m before a constant frictional force of 15 N causes him to stop. His initial speed is 2.2 m/s. Calculate the skater's mass. K/U T/I [ans: 74 kg]

Underlying Assumptions Related to the Work–Energy Theorem

You can use the work–energy theorem to solve several types of physics problems. However, you cannot control all of the variables in the real world as easily as you can in a physics experiment. The work–energy theorem is only true if no energy losses occur. In many real-world situations, energy will seem to disappear in the form of light, sound, heat, or changes in the shape of an object. For instance, in a car collision, energy goes into the sounds of the crash and the bending of materials in the car. The work done on a given car does not equal the change in the kinetic energy of that car.

This discussion assumes that the applied force is constant. The derivation of the work–energy theorem for a varying force requires calculus, but the result is the same. The work–energy theorem holds true, even when the applied force is not constant.

Investigation 4.2.1

The Work–Energy Theorem **(page 209)**

Now that you have learned about the work–energy theorem, you are ready to complete an investigation to test the theorem. This Controlled Experiment will give you an opportunity to calculate the force of friction.

4.2 Review

Summary

- Kinetic energy, E_k, is the energy an object has due to its motion. It is a scalar quantity because no direction is associated with it. The units of kinetic energy are joules (J).
- An object's kinetic energy is related to its mass, m, and its speed, v, by the equation $E_k = \frac{1}{2}mv^2$.
- According to the work–energy theorem, the total work done on an object is equal to the change in the object's kinetic energy: $W = \Delta E_k$.

Questions

1. Could an elephant walking slowly across a field have more kinetic energy than a cheetah chasing its prey? Explain your answer. K/U T/I
2. A cat with a mass of 5.0 kg is chasing a mouse with a mass of 35 g. The mouse is running away from the cat at a constant speed in a straight line. K/U T/I
   (a) The cat's kinetic energy is 100 times the mouse's kinetic energy. Will the cat be able to catch up with the mouse? Explain your answer.
   (b) What is the minimum kinetic energy that the cat must have to keep up with the mouse?
3. A car of mass 1.5×10^3 kg is initially travelling at a speed of 11 m/s. The driver then accelerates to a speed of 25 m/s over a distance of 0.20 km. Calculate the work done on the car. K/U T/I
4. A truck of mass 9.1×10^3 kg is travelling along a level road at an initial speed of 98 km/h and then slows to a final speed of 27 km/h. Determine the total work done on the truck. T/I
5. Two objects have the same kinetic energy. One has a speed that is 2.5 times the speed of the other. Determine the ratio of their masses. K/U T/I
6. Consider a small car of mass 1.2×10^3 kg and a large sport utility vehicle (SUV) of mass 4.1×10^3 kg. The car is travelling at 99 km/h. The car and the SUV have the same kinetic energy. Calculate the speed of the SUV. K/U T/I
7. An archer is able to shoot an arrow with a mass of 0.020 kg at a speed of 250 km/h. If a baseball of mass 0.14 kg is given the same kinetic energy, determine its speed. T/I
8. A hockey player shoots a puck at a speed of 150 km/h. The mass of the puck is 0.16 kg, and the player's stick is in contact with it over a distance of 0.25 m. Calculate the average force exerted on the puck by the player. T/I
9. At room temperature, an oxygen molecule with mass 5.31×10^{-26} kg has kinetic energy of about 6.25×10^{-21} J. Determine the speed of the molecule. T/I
10. A horizontal force of 15 N pulls a block of mass 3.9 kg across a level floor. The coefficient of kinetic friction between the block and the floor is $\mu_K = 0.25$. If the block begins with a speed of 0.0 m/s and is pulled for a distance of 12 m, determine the final speed of the block. T/I
11. Centripetal forces do non-zero work on objects in non-circular orbits. Satellites in non-circular orbits around Earth have different speeds at different positions in their orbit. The change in kinetic energy comes from work done on the satellites by gravity. A satellite of mass 5.55×10^3 kg has a speed of 2.81 km/s at one point in its orbit and a speed of 3.24 km/s at a second point. Calculate the work done on the satellite by gravity as the satellite moves from the first point to the second point. T/I A
12. **Figure 4** shows the kinetic energy of a robot as a function of its speed. K/U T/I

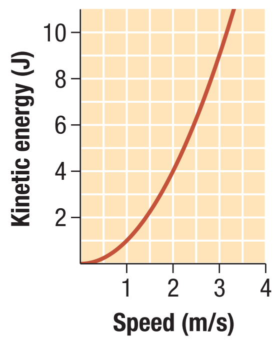

Figure 4

   (a) What type of function is this?
   (b) Why does the graph pass through the origin?
   (c) Determine the mass of the robot.
   (d) Write an equation that relates kinetic energy as a function of speed for the robot.