# 3.3 Centripetal Force

Think of a time when you were a passenger in a car going around a sharp curve at high speed (**Figure 1**). If the car were going fast enough, you might feel the side of the car door pushing on your side. If you look closely at Figure 1, you can see that the road is banked. In this case, the bank assists the force of friction to help make the car and passengers move in a circle more safely for a given speed. A force is also required on objects when you play crack the whip when ice skating, or when you swing a yo-yo on a string around your head. If you were to let go of the crack-the-whip line, you would no longer feel the force and would continue in a straight line in the direction you were moving. The implications of the forces causing circular motion is one reason highway exit ramps often have banked curves like the one in Figure 1. CAREER LINK

**Figure 1** Passengers in a car that is going around a sharp curve at high speed will experience a strong force pushing them toward the centre of the curve.

## Forces That Cause Centripetal Acceleration

As you learned in Section 3.2, any object moving with uniform circular motion has a centripetal acceleration of magnitude

$$a_c = \frac{v^2}{r}$$

From Newton's second law, we know that forces cause accelerations. So, for an object moving with uniform circular motion, we have

$$\Sigma F = ma_c$$

$$F_c = \frac{mv^2}{r}$$

where $F_c$ is the magnitude of the net force required to make an object of mass $m$ travel with a constant speed $v$ in a circle of radius $r$. Centripetal acceleration is directed toward the centre of the circle, so the centripetal force must also be directed toward the centre of the circle according to Newton's second law.

To further analyze the forces involved in uniform circular motion, suppose a person is twirling a yo-yo on a string so that the yo-yo moves in a circle. To keep the situation simple, assume this demonstration is being performed by an astronaut in deep space, where gravitational forces are negligible (**Figure 2(a)**).

Since gravitational forces are negligible, the only force on the yo-yo comes from the string. According to Newton's second law and the equation

$$F_c = \frac{mv^2}{r}$$

a force of magnitude $\frac{mv^2}{r}$ causes this acceleration. And since the force comes from the tension $F_T$ in the string,

$$F_T = \frac{mv^2}{r}$$

**Investigation 3.3.1**

**Simulating Uniform Circular Motion (page 135)**

You have learned about the principles related to objects in uniform circular motion. In this investigation, you will use a simulation program to verify the relationship between frequency and variables such as force, mass, speed, radius, and period.

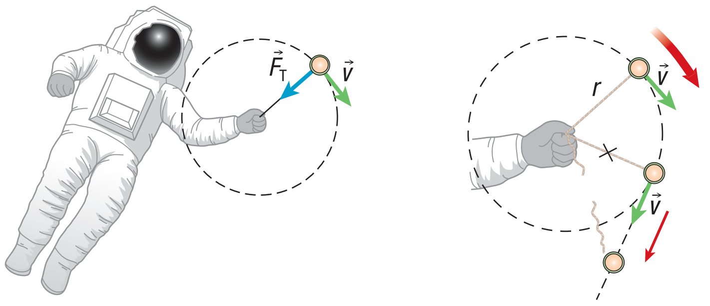



**Figure 2** (a) An astronaut in deep space twirls a yo-yo on a string. In deep space all gravitational forces are negligible, so the only force on the yo-yo is due to the tension in the string. (b) When $F_T = \frac{mv^2}{r}$, the yo-yo will move with uniform circular motion. If the string breaks, the yo-yo will move along a straight line, obeying Newton's first law.

**Investigation 3.3.2**

**Analyzing Uniform Circular Motion (page 136)**

An object in circular motion at a constant speed is constantly undergoing centripetal acceleration directed toward the centre of the circle. This investigation will give you an opportunity to observe an object in uniform circular motion and collect data to describe relationships between the object, its mass, and the radius of its path.

For the yo-yo to travel in a circle, the tension must have this value. In other situations, the force might be due to gravity, friction, or some other source. The net force that causes centripetal acceleration is called the **centripetal force**, $F_c$. Without such a force, the object cannot move in uniform circular motion.

**centripetal force ($F_c$)** the net force that causes centripetal acceleration

What happens if the string in Figure 2 suddenly breaks? After the string breaks (**Figure 2(b)**), the force on the yo-yo is zero. According to Newton's first law, the yo-yo will then move away in a straight-line path with a constant velocity. The yo-yo does not move radially outward, nor does it "remember" its circular trajectory. The only way the yo-yo can move in a circle is when there is a force that makes it do so. Before the string broke, the string provided that force. The following Tutorial models how to solve problems that involve different centripetal forces.

## Tutorial 1 Solving Problems Related to Centripetal Force

In this Tutorial, you will solve for different variables in situations in which an object is moving with uniform circular motion.

### Sample Problem 1: Determining Centripetal Acceleration and Identifying the Centripetal Force

Suppose a bug is sitting on the edge of a horizontal DVD. The bug has a mass of 5.0 g, and the DVD has a radius of 6.0 cm. The DVD is spinning such that the bug travels around its circular path three times per second. Calculate the centripetal acceleration of the bug and the net force on the bug. Also, identify the force or forces responsible for the centripetal force.

**Given:** $m = 5.0$ g $= 0.0050$ kg; $r = 6.0$ cm $= 0.060$ m; $t = 1.0$ s

**Required:** $a_c$; $F_c$; origin of $F_c$

**Analysis:** Draw an FBD to determine the force or forces responsible for the centripetal force. Calculate the speed of the DVD using $v = \frac{\Delta d}{\Delta t}$. Then use the speed in the equation for centripetal acceleration, $a_c = \frac{v^2}{r}$, and calculate the centripetal force, $\Sigma F_c = ma_c$; circumference of a circle $= 2\pi r$.

**Solution:** The FBD is shown in **Figure 3**. The bug's acceleration is in the horizontal plane, so the bug's acceleration in the vertical plane is zero. Therefore, the normal force, $F_N$, and the force of gravity, $mg$, must cancel: $F_N = mg$. The bug is moving with uniform circular motion, so we know a third force must provide the force required to produce the centripetal acceleration, $a_c$. This force keeps the bug from slipping relative to the DVD, so the force is the force of static friction. To make the bug move with uniform circular motion, the force of static friction must be directed toward the centre of the circle.

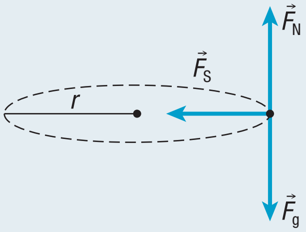



**Figure 3**

The bug travels around the circle three times in 1.0 s, so it travels a distance equal to three times the circumference each second:

$$\begin{aligned} v &= \frac{\Delta d}{\Delta t} \\ &= \frac{(3)(2\pi r)}{\Delta t} \\ &= \frac{(6\pi)(0.060\text{ m})}{1.0\text{ s}} \\ v &= 1.13\text{ m/s (one extra digit carried)} \\ a_c &= \frac{v^2}{r} \\ &= \frac{(1.13\ \cancel{\text{m}}\text{/s})^2}{0.060\ \cancel{\text{m}}} \\ a_c &= 21.3\text{ m/s}^2\text{ (one extra digit carried)} \end{aligned}$$

The force is

$$\begin{aligned} \Sigma F_c &= ma_c \\ &= (0.0050\text{ kg})(21.3\text{ m/s}^2) \\ \Sigma F_c &= 0.11\text{ N} \end{aligned}$$

**Statement:** The centripetal acceleration of the bug is 21 m/s$^2$, and the total force on the bug is 0.11 N. The centripetal force on the bug is static friction.

## **Sample Problem 2:** Calculating Speed Using Apparent Weight

A roller coaster car is at the lowest point on its circular track. The radius of curvature is 22 m. The apparent weight of one of the passengers in the roller coaster car is 3.0 times her true weight. Determine the speed of the roller coaster.

**Given:** $r = 22$ m; $F_N = 3.0mg$

**Required:** $v$

**Analysis:** Draw an FBD for the scenario. The uniform circular motion is in the vertical plane in this problem. At the lowest point on the circular track, the forces on the person are gravity and the normal force. The normal force is the apparent weight. Since the roller coaster is at the low point of the track, the normal force is directed toward the centre of the circular arc defined by the track (up), and gravity is downward, away from the centre. Apply Newton's second law to relate the normal force to the speed of the roller coaster. Then apply the equation for circular motion, $F_c = \frac{mv^2}{r}$.

**Solution: Figure 4** shows the FBD.

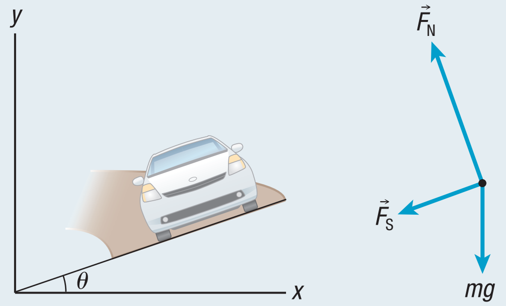
$$\begin{aligned} \Sigma F &= +F_N + (-mg) \\ F_c &= F_N - mg \\ \frac{mv^2}{r} &= 3.0mg - mg \\ \frac{mv^2}{r} &= 2.0mg \\ \frac{\cancel{m}v^2}{r} &= 2.0\cancel{m}g \\ v &= \sqrt{2.0rg} \\ &= \sqrt{(2.0)(22\text{ m})(9.8\text{ m/s}^2)} \\ v &= 21\text{ m/s} \end{aligned}$$


**Figure 4**

**Statement:** The speed of the roller coaster is 21 m/s.

## **Sample Problem 3:** Calculating Speed on a Banked Turn

A car making a turn on a dry, banked highway ramp is experiencing friction (**Figure 5**). The coefficient of static friction between the tires and the road is 0.60. Determine the maximum speed at which the car can safely negotiate a turn of radius $2.0 \times 10^2$ m with a banking angle of 20.0°.




(a) (b)

**Figure 5**

**Given:** $\mu_S = 0.60$; $r = 2.0 \times 10^2$ m; $\theta = 20.0°$

**Required:** $v$

**Analysis:** In two separate diagrams, draw the vector components of the frictional force and the vector components of the normal force. The car is moving with uniform circular motion, so the centripetal force on the car is $F_c = \frac{mv^2}{r}$ directed toward the centre of the circular path. In Figure 5, the centre is $2.0 \times 10^2$ m to the left of the car. Use the direction toward the centre as positive for the $x$-direction, and use up as positive for the $y$-direction. We have two unknown quantities: the speed of the car and the normal force. We can get two equations by applying Newton's second law along the vertical and horizontal directions. We can then solve for $F_N$ and $v$; $F_S = \mu_S F_N$

**Solution:** The vector component diagrams are shown in **Figure 6**.

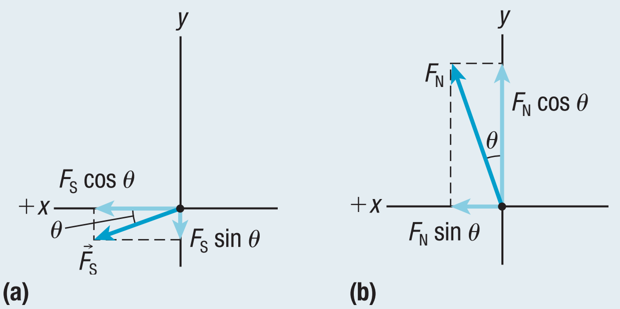



**Figure 6** (a) The vector components for the force of static friction (b) The vector components for the normal force

Vertical components of force:

The car is not slipping up or down the incline, so the acceleration along *y* is zero. The total force along *y* must be zero. From Figure 6(a) and 6(b),

$$\Sigma F_y = 0$$
$$\Sigma F_y = +F_N \cos\theta - F_S \sin\theta - mg$$
$$+F_N \cos\theta - F_S \sin\theta - mg = 0$$
$$F_S = \mu_S F_N$$
$$F_N \cos\theta - \mu_S F_N \sin\theta - mg = 0$$
$$F_N(\cos\theta - \mu_S \sin\theta) = mg$$
$$F_N = \frac{mg}{\cos\theta - \mu_S \sin\theta}$$

Horizontal components of force:

The total force along the horizontal direction provides the centripetal acceleration. From Figure 6(a) and 6(b),

$$\Sigma F_x = F_N \sin\theta + F_S \cos\theta$$
$$F_S = \mu_S F_N$$
$$\Sigma F_x = F_N \sin\theta + \mu_S F_N \cos\theta$$
$$ma_c = F_N \sin\theta + \mu_S F_N \cos\theta$$
$$\frac{mv^2}{r} = F_N(\sin\theta + \mu_S \cos\theta)$$

Solving for *v* then gives

$$v = \sqrt{\frac{F_N r(\sin\theta + \mu_S \cos\theta)}{m}}$$

Insert the result for the normal force:

$$v = \sqrt{\left(\frac{\cancel{m}g}{\cos\theta - \mu_S \sin\theta}\right)\left(\frac{r(\sin\theta + \mu_S \cos\theta)}{\cancel{m}}\right)}$$
$$= \sqrt{gr\left(\frac{\sin\theta + \mu_S \cos\theta}{\cos\theta - \mu_S \sin\theta}\right)}$$
$$= \sqrt{(9.8\ \text{m/s}^2)(2.0 \times 10^2\ \text{m})\left(\frac{\sin 20.0^\circ + (0.60)\cos 20.0^\circ}{\cos 20.0^\circ - (0.60)\sin 20.0^\circ}\right)}$$
$$v = 49\ \text{m/s}$$

**Statement:** The maximum speed at which the car can safely negotiate a turn with a radius of $2.0 \times 10^2$ m and with a banking angle of 20.0° is 49 m/s. It is really the horizontal components of the normal force and the force of friction that contribute to the net force and the acceleration. (Note: This is the maximum speed the car can go but not the speed the car should go.)

## Practice

1. A model airplane with a mass of 0.211 kg pulls out of a dive. The bottom of the dive is a circular arc with a radius of 25.6 m. At the bottom of the arc, the plane's speed is a constant 21.7 m/s. Determine the magnitude of the upward lift on the plane's wings at the bottom of the arc. K/U T/I A [ans: 5.9 N]
2. A curved road with a radius of 450 m in the horizontal plane is banked so that the cars can safely navigate the curve. Calculate the banking angle for the road that will allow a car travelling at 97 km/h to make it safely around the curve when the road is covered with black ice. (Assume no friction.) K/U T/I A [ans: 9.3°]
3. A 2.00 kg mass is spinning horizontally in a circle on a virtually frictionless surface. It completes 5.00 revolutions in 2.00 s. The mass is attached to a string 4.00 m long. Calculate the magnitude of the tension in the string. Air resistance is negligible. K/U T/I A [ans: $2.0 \times 10^3$ N]
4. A barn swallow chasing a moth is flying in a vertical loop of radius 150 m. At the top of the loop, the vertical force exerted by the air on the bird is zero. At what speed is the swallow flying at this point? K/U T/I A [ans: 38 m/s]
5. The highway ramp in Sample Problem 3 was dry. Now suppose the highway is wet or covered in ice. Predict how the maximum speed will change. Test your prediction by using the value 0.25 for the coefficient of static friction and determining the maximum speed. K/U T/I A [ans: 36 m/s]

# 3.3 Review

## Summary

- An object moving with uniform circular motion experiences a net force directed toward the centre of the object's circular path.
- The net force that causes uniform circular motion is the centripetal force, which may comprise one or more other forces such as gravity, the normal force, or tension.
- Combine the equation for Newton's second law with the equations for centripetal acceleration to calculate the magnitude of the net force: $\Sigma F = ma_c$; $F_c = \dfrac{mv^2}{r}$.

## Questions

1. The track near the top of a roller coaster has a circular shape with a diameter of 24 m forming a hill. When you are at the top, you feel as if your weight is only one-third your true weight. Calculate the speed of the roller coaster as it rolls over the top of the hill. T/I A
2. A car with a mass of 1000.0 kg is travelling over the top of a hill, as shown in **Figure 7**. The hill's curvature has a radius of 40.0 m, and the car is travelling at 15 m/s. T/I C A

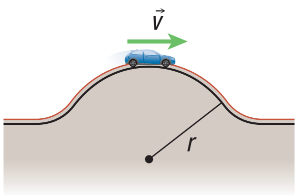



**Figure 7**

   (a) Draw an FBD.
   (b) Determine the magnitude of the normal force between the hill and the car at the top of the hill.
   (c) Determine the speed required to make the driver feel weightless at the top of the hill.
3. A civil engineer is designing a banked curve on a highway. The banked curve is designed to allow the cars to move safely in a horizontal circle. What will happen to the maximum speed of a car on the curve when the following changes are made? Explain your reasoning, considering each change separately. K/U T/I A
   (a) The banking angle between the road and the horizontal is increased.
   (b) The coefficient of friction between the tires and the road is larger.
   (c) A heavier car is used.
4. A car moves in a horizontal circle on a test track with a radius of $1.2 \times 10^2$ m. The coefficient of static friction between the tires and the road is 0.72. Draw an FBD, and calculate the maximum speed of the car. T/I C A
5. Consider a banked curve on an exit ramp for a highway in the middle of winter when the road surface is covered with very slippery ice. K/U T/I A
   (a) How does the banking angle of the road help drivers make it safely around the curve? What force (or component of force) is responsible? Explain your reasoning.
   (b) Explain why drivers must go much more slowly under these circumstances.
   (c) A student claims, "If banking angles help drivers safely navigate curved sections of road, why not make the banking angles significantly larger?" Identify one problem that might occur if this suggestion were used. Justify your answer.
6. An air puck with a mass of 0.26 kg is tied to a string and moves at a constant speed in a circle of radius 1.2 m. The other end of the string goes through a hole in the air table and straight down to a suspended mass of 0.68 kg, which hangs at rest (**Figure 8**). Calculate the speed of the air puck. K/U T/I A

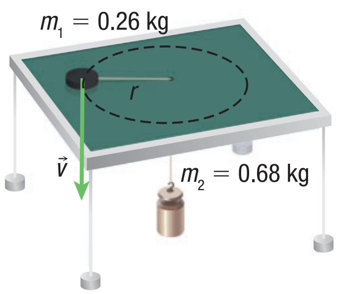



**Figure 8**