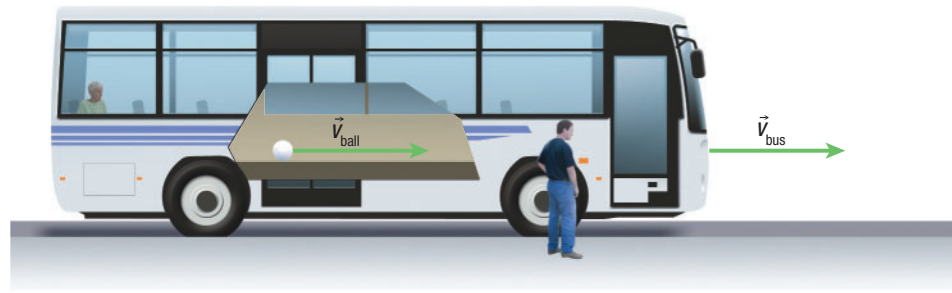


# Inertial and Non-inertial Frames of Reference

Think about riding on a bus. When the bus moves at a steady speed without changing direction, you could close your eyes and possibly not even be aware that the bus is moving. If you place a ball in the aisle of the bus, the ball does not move relative to the bus (**Figure 1**). When the bus slows down, however, you will see the ball roll forward. When the bus speeds up again, the ball will roll backward. When the bus turns, the ball will roll to the side. What forces cause the ball to accelerate? The fact is, no forces act on the ball to accelerate it. [WEB LINK](#)



**Figure 1** The bus and the ball placed in the aisle of the bus are moving at the same velocity with respect to the ground, although the ball is not moving relative to the bus.

## Defining Frames of Reference

**frame of reference** a coordinate system relative to which motion is described or observed

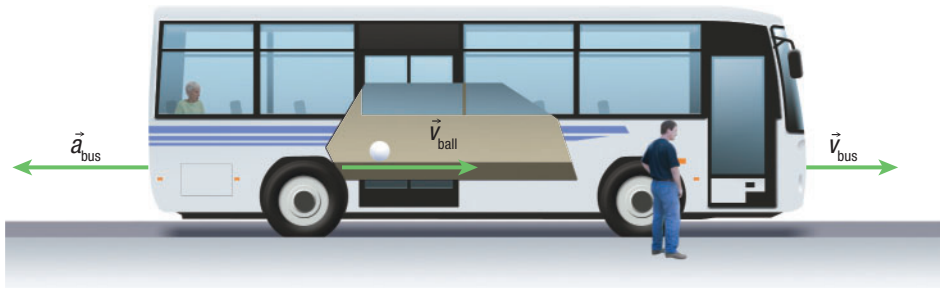
**inertial frame of reference** a frame of reference that moves at a zero or constant velocity; a frame in which the law of inertia holds

**non-inertial frame of reference** a frame of reference that accelerates with respect to an inertial frame; the law of inertia does not hold

A person on the bus will view all motion from the point of view of the bus, so to her, the moving bus in Figure 1 is a moving frame of reference. A **frame of reference** is an observer's choice of coordinate system, including an origin, for describing motion. A person standing on the sidewalk is in the stationary frame of reference of the ground. In many cases, people use the ground as a frame of reference, especially when they are not moving with respect to the ground. When you are at rest in a frame of reference that has a constant velocity, whether that frame is itself at rest ( $\vec{v} = 0$ ) or moving with a non-zero velocity, your velocity relative to that frame is zero. When the bus is moving at a constant velocity, no net force acts on it. Therefore, no net force acts on you inside it. Newton's first law of motion (the law of inertia) states that any object moving at a constant velocity (including  $\vec{v} = 0$ ) remains at that velocity if no net force acts on it. The bus in this case is an **inertial frame of reference**, which is a frame of reference in which the law of inertia is valid.

Now think about what happens when the bus slows down and the ball begins to roll forward inside the bus. What causes the ball to move forward? As the bus slows, it accelerates (that is, changes velocity) in a direction that is opposite to the direction of motion. Therefore, the bus is no longer an inertial frame of reference. It becomes a **non-inertial frame of reference**, which is a frame of reference in which the law of inertia is no longer valid. The reason there appears to be a net force on the ball is that the ball continues to move with the same velocity it had before. Obeying the law of inertia, the ball does not immediately slow down with the bus (**Figure 2**). Within the accelerating bus, or non-inertial frame of reference, this looks as if a force is pushing the ball forward.

Similarly, when the bus starts from rest and speeds up, the ball tends to stay at rest. To you and other passengers in the non-inertial frame, it appears as if there was a push on the ball toward the back of the bus. There also appears to be a force on the ball when the bus turns a corner. The ball continues moving in the same direction as the bus before the turn. From your point of view in the bus, this would be like a force pushing the ball in the direction opposite to that in which the bus was turning.



**Figure 2** As the bus slows, the ball continues to move forward. In the bus, it appears as if a force has been applied to the ball.

To explain these various motions, we invent the idea of fictitious forces. **Fictitious forces** are apparent but non-existent forces that explain the motion in accelerating (non-inertial) frames of reference. In the bus example, a fictitious force pushed the ball in the direction opposite the acceleration of the bus. As you continue learning about mechanics, you will encounter other fictitious forces. Fictitious forces simply explain non-accelerated motion in accelerated frames of reference. Tutorial 1 shows how to solve problems involving an object placed in a non-inertial frame of reference.

**fictitious force** an apparent but non-existent force invented to explain the motion of objects within an accelerating (non-inertial) frame of reference

## Tutorial 1 Solving Problems Related to Objects in a Non-inertial Frame of Reference

This Tutorial models how to solve problems involving objects placed in a non-inertial frame of reference.

### Sample Problem 1: Calculating the Acceleration of a Non-inertial Frame of Reference

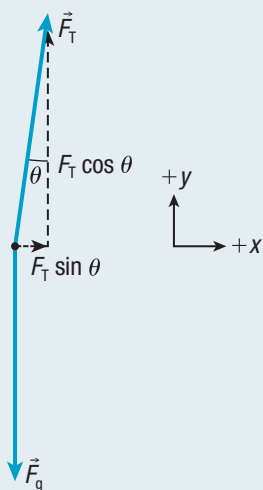
A teacher suspends a small cork ball from the ceiling of a bus. When the bus accelerates at a constant rate forward, the string suspending the ball makes an angle of  $10.0^\circ$  with the vertical. Calculate the magnitude of the acceleration of the bus.

**Given:**  $\theta = 10.0^\circ$ ;  $g = 9.8 \text{ m/s}^2$

**Required:**  $a_x$

**Analysis:** Look at the situation from an Earth (inertial) frame of reference. Draw an FBD to show the forces acting on the cork ball. The horizontal component of the tension  $\vec{F}_T$  balances the acceleration, so express the components of the tension in terms of the horizontal and vertical applied forces. Then calculate the magnitude of the acceleration.

**Solution:**



Vertical components of force:

$$\begin{aligned}\Sigma F_y &= 0 \\ F_T \cos \theta - mg &= 0 \\ F_T &= \frac{mg}{\cos \theta}\end{aligned}$$

Horizontal components of force:

$$\begin{aligned}\Sigma F_x &= ma \\ F_T \sin \theta &= ma \\ \text{Substitute } F_T &= \frac{mg}{\cos \theta} \\ \left(\frac{mg}{\cos \theta}\right) \sin \theta &= ma\end{aligned}$$

$$g \left(\frac{\sin \theta}{\cos \theta}\right) = a$$

$$\begin{aligned}\text{Substitute } \frac{\sin \theta}{\cos \theta} &= \tan \theta \\ a &= g \tan \theta \\ &= (9.8 \text{ m/s}^2)(\tan 10.0^\circ) \\ a &= 1.7 \text{ m/s}^2\end{aligned}$$

**Statement:** The magnitude of the bus's acceleration is  $1.7 \text{ m/s}^2$ .

## Practice

- You are in a car moving with a constant velocity of 14 m/s [E]. A baseball lies on the floor at your feet. **K/U T/I C**
  - Describe the motion of the ball from your point of view. How is it different from when the car is at rest?
  - How would an observer on the sidewalk describe the motion of the ball?
  - Now the car accelerates forward. Describe the ball's motion from your point of view.
  - Draw two FBDs showing the ball's motion in (c) from the frame of reference of the car and the frame of reference of the sidewalk. Which frame of reference is non-inertial? In which frame do you observe a fictitious force?
- You use a string to suspend a cork ball with a mass of 22.0 g from the ceiling of a moving speedboat. The ball and string hang at an angle of  $32.5^\circ$  from the vertical. **K/U T/I C A**
  - Calculate the magnitude of the speedboat's acceleration. Do you need to know the mass of the ball to make this calculation? Why or why not? [ans:  $6.2 \text{ m/s}^2$ ]
  - Determine the magnitude of the tension in the string. Do you need to know the mass of the ball to make this calculation? Why or why not? [ans: 0.26 N]
- A person is standing in a subway train holding a strap that is attached to a piece of luggage on wheels (**Figure 3**). The mass of the luggage is 14 kg. The strap makes an angle of  $35^\circ$  to the vertical. Assume there is no friction between the luggage wheels and the floor. **K/U T/I A**



**Figure 3**

- Determine the tension in the strap when the subway is moving at a constant velocity. [ans: 0 N]
  - Determine the tension in the strap when the subway is accelerating forward at  $1.4 \text{ m/s}^2$ . [ans: 34 N]
- A passenger stands in a train that is accelerating forward. The passenger is able to stay in place because of the force of static friction between his shoes and the floor. The coefficient of static friction between the shoes and the floor is 0.42. Determine the maximum amount the train, relative to the track, can accelerate before the passenger begins to slip along the floor. **K/U T/I A** [ans:  $4.1 \text{ m/s}^2$ ]

## Apparent Weight

The study of vertical acceleration can also introduce fictitious forces. To understand this, consider what happens when you stand on a bathroom scale (**Figure 4(a)**). As in all cases when you stand up, you feel a force pushing upward against the soles of your feet. This is the normal force, and it is equal and opposite to the weight ( $mg$ ) of your body when you stand on level ground.

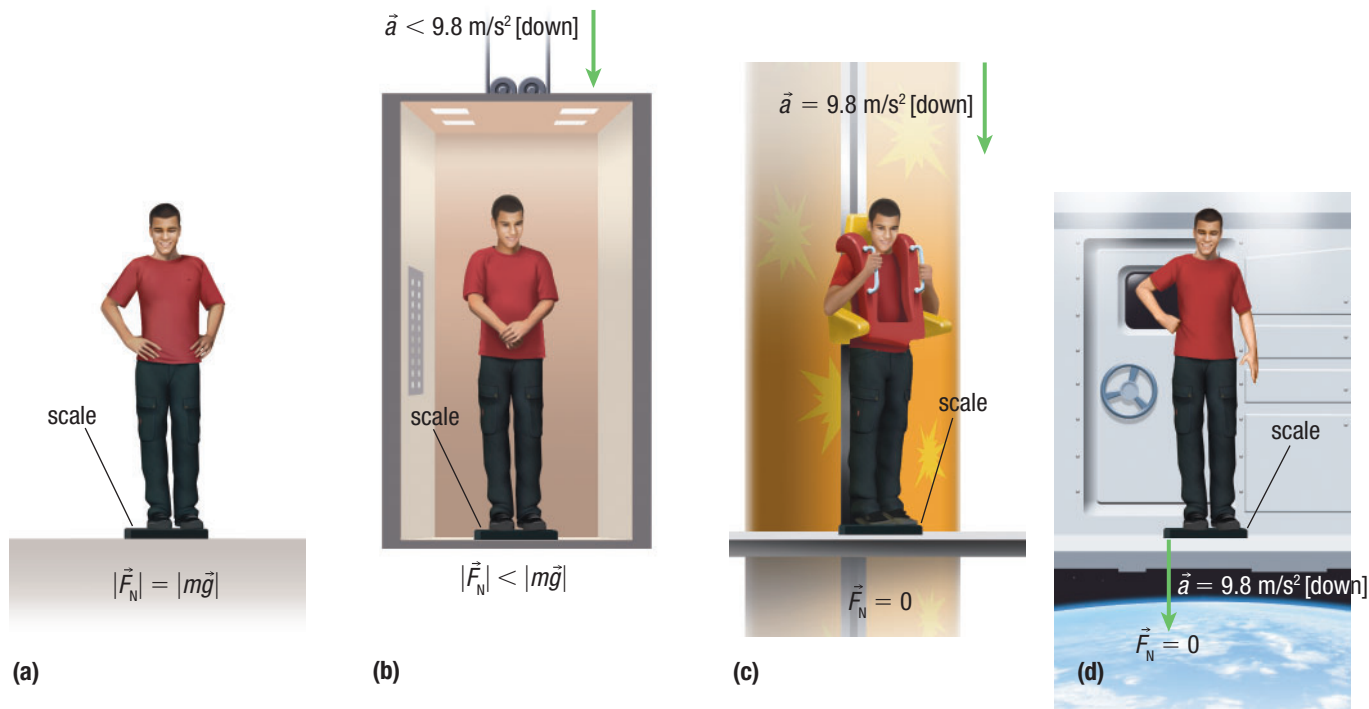
Now suppose that you stand on the same scale inside an elevator. When the elevator is at rest, the normal force is again the same as your weight. This is also true when the elevator is moving at a constant non-zero speed upward or downward. However, what happens when the elevator accelerates? When the elevator accelerates downward, the normal force decreases, so that the magnitude of the reading on the

scale is less than your weight,  $mg$  (**Figure 4(b)**). Similarly, when the elevator accelerates upward, the normal force increases, resulting in a greater reading on the scale.

As with the bus at the beginning of this section, the elevator can be an inertial frame of reference when it has a constant velocity going up or down. It becomes a non-inertial frame of reference when it accelerates, resulting in a normal force that is either greater or less than your weight. The magnitude of this normal force in a non-inertial frame of reference is called the **apparent weight**.

Other non-inertial frames of reference produce other values for apparent weight. On a free-fall ride at an amusement park, the acceleration of the ride is equal to  $g$ , and the normal force is zero (**Figure 4(c)**). Thus, the scale will read an apparent weight of zero. Similarly, an astronaut aboard the International Space Station is constantly in free fall. Therefore, the normal force acting on the astronaut is zero (**Figure 4(d)**).

**apparent weight** the magnitude of the normal force acting on an object in an accelerated (non-inertial) frame of reference



**Figure 4** The readings on the scale will be (a) equal to  $mg$  when standing still, (b) less than  $mg$  when the elevator accelerates downward, (c) equal to zero in vertical free fall on an amusement park ride, and (d) equal to zero in free fall during orbit.

In the following Tutorial, you will learn how to solve problems that involve the apparent weight of an object in a non-inertial frame of reference.

## Tutorial 2

### Solving Problems Related to Apparent Weight in a Non-inertial Frame of Reference

#### Sample Problem 1: Apparent Weight in an Accelerating Elevator

An elevator accelerates upward with an acceleration of magnitude  $1.5 \text{ m/s}^2$ , after which it moves with a constant velocity. As the elevator approaches its stopping point, it undergoes a downward acceleration of magnitude  $0.9 \text{ m/s}^2$ . Calculate the apparent weight of a passenger with a mass of  $75 \text{ kg}$  when

- the elevator undergoes positive acceleration
- the elevator moves at constant velocity
- the elevator undergoes negative acceleration

#### Solution

(a) **Given:**  $a = 1.5 \text{ m/s}^2 \text{ [up]}$ ;  $m = 75 \text{ kg}$ ;  $g = 9.8 \text{ m/s}^2 \text{ [down]}$

**Required:**  $F_N$

**Analysis:** Draw an FBD of the passenger, and solve for the normal force. Use up as positive.

**Solution:**



$$\begin{aligned}
 +F_N + (-mg) &= ma \\
 F_N &= mg + ma \\
 &= m(g + a) \\
 &= (75 \text{ kg})(9.8 \text{ m/s}^2 + 1.5 \text{ m/s}^2) \\
 F_N &= 8.5 \times 10^2 \text{ N}
 \end{aligned}$$

**Statement:** The apparent weight of the passenger when the elevator undergoes positive acceleration of  $1.5 \text{ m/s}^2$  is  $8.5 \times 10^2 \text{ N}$ .

(b) **Given:**  $a = 0 \text{ m/s}^2$ ;  $m = 75 \text{ kg}$ ;  $g = 9.8 \text{ m/s}^2$  [down]

**Required:**  $F_N$

**Analysis:** In an inertial frame, there is no acceleration, so the apparent weight of the passenger is  $mg$ ;  $F_N = mg + 0 = mg$ .

**Solution:**  $F_N = mg$   
 $= (75 \text{ kg})(9.8 \text{ m/s}^2)$   
 $F_N = 7.4 \times 10^2 \text{ N}$

**Statement:** The apparent weight of the passenger when the elevator moves at constant velocity is  $7.4 \times 10^2 \text{ N}$ .

(c) **Given:**  $a = 0.9 \text{ m/s}^2$  [down];  $m = 75 \text{ kg}$ ;  
 $g = 9.8 \text{ m/s}^2$  [down]

**Required:**  $F_N$

**Analysis:** Draw an FBD of the passenger, and solve for the normal force. Use down as positive.

**Solution:**

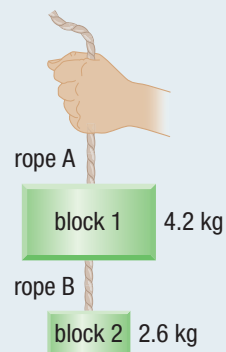


$$\begin{aligned}
 -F_N + (+mg) &= ma \\
 F_N &= mg - ma \\
 &= m(g - a) \\
 &= (75 \text{ kg})(9.8 \text{ m/s}^2 - 0.9 \text{ m/s}^2) \\
 F_N &= 6.7 \times 10^2 \text{ N}
 \end{aligned}$$

**Statement:** The apparent weight of the passenger when the elevator undergoes negative acceleration of  $0.9 \text{ m/s}^2$  is  $6.7 \times 10^2 \text{ N}$ .

## Practice

- A student with a mass of  $55 \text{ kg}$  stands in an elevator that accelerates (a) upward at  $2.9 \text{ m/s}^2$  and then (b) downward at  $2.9 \text{ m/s}^2$ . Determine the student's apparent weight during each acceleration. K/U T/I A [ans: (a)  $7.0 \times 10^2 \text{ N}$ ; (b)  $3.8 \times 10^2 \text{ N}$ ]
- Two boxes of books are stacked and placed on the floor of an elevator. The masses of the bottom and top boxes are  $9.5 \text{ kg}$  and  $2.5 \text{ kg}$ , respectively. The normal force between the floor and the bottom box is  $70.0 \text{ N}$ . K/U T/I A
  - Determine the magnitude and direction of the elevator's acceleration. [ans:  $4.0 \text{ m/s}^2$  [down]]
  - Determine the force that the larger box exerts on the smaller box. [ans:  $15 \text{ N}$  [up]]
- Rope A is tied to block 1, and rope B is attached to both block 1 and block 2 (**Figure 5**). The mass of block 1 is  $4.2 \text{ kg}$ , and the mass of block 2 is  $2.6 \text{ kg}$ . You lift both blocks straight up. Calculate the tension in the ropes when the blocks (a) move at a constant velocity of  $1.5 \text{ m/s}$  [up] and (b) accelerate at  $1.2 \text{ m/s}^2$  [up]. K/U T/I A  
[ans: (a) rope A:  $67 \text{ N}$ ; rope B:  $25 \text{ N}$ ; (b) rope A:  $75 \text{ N}$ ; rope B:  $29 \text{ N}$ ]
- The Taipei 101 Tower, in Taipei, has 101 floors. Although the elevators in the Taipei 101 Tower are the fastest in the world, they have an acceleration of only  $0.98 \text{ m/s}^2$ . Calculate the apparent weight of a passenger with a mass of  $61 \text{ kg}$  when one of these elevators is accelerating downward. K/U T/I A [ans:  $5.4 \times 10^2 \text{ N}$ ]



**Figure 5**

## 3.1 Review

### Summary

- A frame of reference is a coordinate system relative to which motion is described or observed.
- An inertial frame of reference is one that moves at a constant velocity or is at rest. The law of inertia holds.
- A non-inertial frame of reference is one that undergoes acceleration because of an external force. The law of inertia does not hold.
- Fictitious forces help explain motion in a non-inertial frame of reference.
- Apparent weight is the magnitude of the normal force acting on an object in a non-inertial frame of reference.

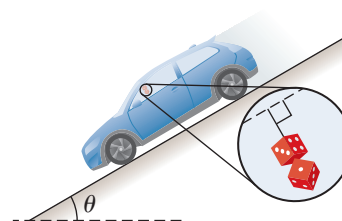
### Questions

- Suppose you are on a train moving with a constant velocity. Another train on parallel tracks is moving with the same velocity. A passenger in the other train is tossing a ball vertically in the air. **T/I C A**
  - Describe how the path of the ball would look to you.
  - Describe how the path of the ball would look if the trains moved in opposite directions.

- A mass on a string is suspended from the ceiling of an airplane. Calculate the angle that the mass makes when the airplane has a horizontal acceleration of magnitude  $1.5 \text{ m/s}^2$ . **K/U T/I A**
- A jet reaches a takeoff speed of  $255 \text{ km/h}$  in  $10.0 \text{ s}$ . This jet has a ball-on-a-string accelerometer hanging from the ceiling of the cabin. Assume the jet accelerates uniformly during takeoff. Calculate the angle of the string during takeoff. **K/U T/I A**
- A student constructs an accelerometer by attaching cork balls to strings anchored to the bottom of a fish tank. When the student fills the tank with water, the balls float to the surface. When the tank is at rest, the strings align in the vertical direction. While riding in a car and holding the tank level, the student notices that the strings make an angle of  $16^\circ$  with respect to the vertical. Calculate the magnitude of the car's acceleration. **K/U T/I A**

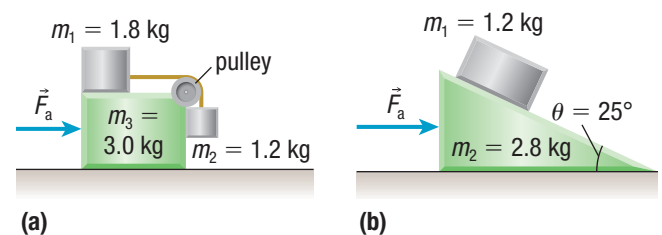
- The passenger elevators at the Brookfield Place towers in Toronto reach a top speed of about  $6.0 \text{ m/s}$  upward. Suppose one of the elevators reaches this speed in  $10.0 \text{ s}$ . Calculate the apparent weight of a passenger whose mass is  $64 \text{ kg}$ . **T/I**
- A student on a free-fall ride at an amusement park brings a scale to check her apparent weight during the ride. At one point, she notices a reading of  $255 \text{ N}$ . The student's mass is  $52 \text{ kg}$ . Calculate the acceleration of the ride at the time of the reading. **T/I A**

- A vintage sports car accelerates down a hill at an angle of  $17^\circ$  to the ground, as shown in **Figure 6**. The driver notices that the string of ornamental fuzzy dice hanging from his rear-view mirror is perpendicular to the roof of the car. **K/U T/I C A**



**Figure 6**

- Draw an FBD of the dice from the frame of reference of the level ground, both when the car is at rest on the hill and when it is accelerating. Explain how the two FBDs differ.
  - Calculate the car's acceleration.
- In **Figure 7**, mass 1 does not slide with respect to the surface when the horizontal force shown is applied. Determine the magnitude of the horizontal force in both **Figure 7(a)** and **Figure 7(b)**. Assume there is no friction. **K/U T/I A**



**Figure 7**