2.4 Forces of Friction

Friction may seem like it always makes movement more difficult because it always opposes motion. However, friction is actually essential for much of the motion that we rely on. Have you ever tried getting around on ice? The reason that walking, driving, or riding a bicycle on ice is difficult is the lack of friction.

When you take a step, you push backward on the ground. The static friction of the ground opposes the attempted motion of your foot and exerts a simultaneous and opposite force that propels you forward. A sprinter, such as the one in **Figure 1**, tries to maximize the forward force of the ground. This means wearing shoes that have a large force of static friction with the running surface. It also means pushing on the ground with a force that has a large component parallel to the ground. However, if the sprinter pushes on the ground at an angle that is too shallow, his feet will overcome the static friction and slip on the track.

Figure 1 The sprinter is using the static friction between his shoes, $\vec{F}_s$, and the running surface to accelerate.

Types of Friction: Kinetic and Static

Figure 2(a) shows a hockey puck sliding across the ice. Although this surface is quite slippery, there is still a small amount of friction. Eventually, the force of friction stops the puck. **Figure 2(b)** is an FBD of the hockey puck. The puck moves horizontally to the right. Only one force acts along the horizontal, the force of kinetic friction. The puck's velocity is to the right, so the force of friction opposes this motion to the left. Two forces act in the vertical direction: the weight of the puck (the force of gravity) and the normal force exerted by the icy surface acting on the puck. (You read about these forces in Section 2.2.)

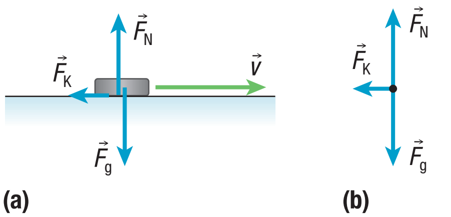


Figure 2 (a) The hockey puck experiences only one force in the horizontal direction, a small force of friction, which slows and eventually stops the puck. (b) The FBD shows all the forces acting on the puck.

Kinetic Friction

The force of gravity and the normal force cancel each other in the *y*-direction, and the puck's motion is entirely in the *x*-direction. So why do you think you need to know about the forces along *y* if they cancel each other? Actually, the normal force is closely connected to the force of friction. For a sliding object, the magnitudes of these two forces are related by $F_K = \mu_K F_N$. The number μ_K is the **coefficient of kinetic friction**, the number that relates the force of kinetic friction between two surfaces in contact with the normal force where they meet. The frictional force occurs when two surfaces are in motion (slipping) with respect to each other. As a coefficient, μ_K is a number without dimensions or units, and its value depends on the surface properties.

coefficient of kinetic friction (μ_K) the ratio of kinetic friction to the normal force

For a hockey puck on an icy surface, μ_K is relatively small, so the frictional force is similarly small. The value of μ_K depends on the smoothness of both the ice and the hockey puck, and might typically be 0.005. The coefficient of kinetic friction for two rough surfaces is larger. For example, the surface of wood is much rougher than the surface of ice, and the surface of wet snow is rougher than that of ice. For wood slipping on wet snow, the coefficient of kinetic friction is 0.10.

Table 1 lists coefficients of kinetic and static friction (discussed below) for some common materials. These values of μ_K show that the frictional force depends on the properties of the surfaces that are in contact. Note that the coefficient of kinetic friction is less than or equal to the coefficient of static friction. Note also the value for synovial joints in humans. Biomedical research into friction in joints is an advancing field. CAREER LINK

Table 1 Typical Values for the Coefficients of Kinetic Friction and Static Friction for Some Common Materials

Surface	μ_K	μ_S	Surface	μ_K	μ_S
rubber on dry concrete	0.6–0.85		steel on ice	0.01	0.1
rubber on wet concrete	0.45–0.75		rubber on ice	0.005	
rubber on dry asphalt	0.5–0.80		wood on dry snow	0.18	0.22
rubber on wet asphalt	0.25–0.75		wood on wet snow	0.10	0.14
steel on dry steel	0.42	0.78	Teflon on Teflon	0.04	0.04
steel on greasy steel	0.029–0.12	0.05–0.11	near-frictionless carbon	0.001	
leather on oak	0.52	0.61	synovial joints in humans	0.003	0.01
ice on ice	0.03	0.1			

Note: The values for μ_S for rubber and concrete are not normally provided because there are no reliable methods to determine them. In addition, the range depends on a variety of conditions.

Investigation **2.4.1**

Inclined Plane and Friction (page 96)

The coefficients of friction can be determined experimentally by exerting a horizontal force and using measuring equipment. In this investigation, you will estimate these coefficients using objects on an inclined plane.

Remember, too, that the magnitude of the frictional force depends on the normal force. If you increase the normal force—perhaps by adding an additional mass to the top of the hockey puck—the frictional force increases. However, this relationship is not a "law" of nature. It is simply an approximation that works well in a wide variety of cases. To derive a fundamental law or theory of friction, we need to consider in detail the atomic interactions that occur when two surfaces are in contact. This problem is quite complicated and is currently a topic of much research.

Static Friction

The example of the sliding hockey puck illustrates surfaces that are moving (slipping) against each other. What about when two surfaces are in contact but not slipping? Such cases involve static friction.

Investigation 2.4.2

Motion and Pulleys (page 97)

You will design your own investigation in which you will predict the acceleration of a mass and then measure the acceleration. Then you will evaluate your results by comparing the measured value with the calculated value.

In **Figure 3(a)**, a worker is trying to push a refrigerator, but the refrigerator is not moving. This example demonstrates static friction. Unlike the hockey puck example, here, the person pushing on the refrigerator adds an additional force. The FBD in **Figure 3(b)** shows the additional force of the push working against the force of static friction. Intuitively, you know that when the force exerted by the person is small, the refrigerator will not move. Since the acceleration is then zero, the force of static friction and the force of the push cancel each other:

$$ma = \Sigma F_x$$
$$= F_a + (-F_S)$$
$$ma = 0$$

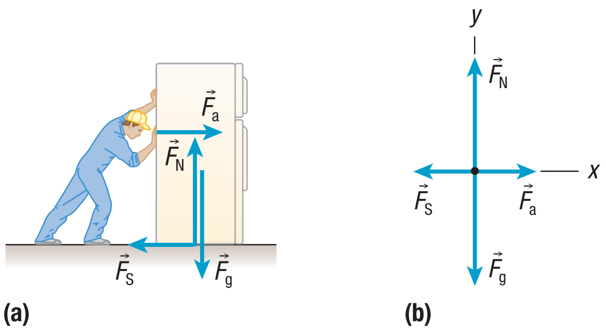


Figure 3 (a) Static friction opposes the attempted motion of the refrigerator. (b) FBD for the refrigerator.

The force of static friction is sufficiently strong that no relative motion (no slipping) occurs. In terms of magnitudes, $|\vec{F}_a| = |\vec{F}_S|$. However, the value of the applied force, F_a, can vary, and the refrigerator will still remain at rest. That is, the worker can push harder, a little, or not at all without the refrigerator moving. In all these cases, the frictional force exactly cancels F_a. The only way this can happen is when the magnitude of the frictional force varies depending on the value of F_a, as described by $|F_a| \leq \mu_S F_N$, where μ_S is the coefficient of static friction. The **coefficient of static friction** is the number that relates the force of static friction between two surfaces to the normal force where they meet.

coefficient of static friction (μ_S) the ratio of the maximum force of static friction to the normal force

The term *static* means that the two surfaces—the floor and the bottom of the refrigerator—are not moving relative to each other. The magnitude of the force of static friction can take any value up to a maximum of $\mu_S F_N$. If F_a in Figure 3 is small, the force of static friction is small and will cancel F_a so that the total horizontal force is zero. If F_a is increased, the force of static friction increases but again cancels F_a. However, the magnitude of the force of static friction has an upper limit of $\mu_S F_N$. If F_a is greater than this upper limit, the worker will succeed in moving the refrigerator.

UNIT TASK **BOOKMARK**

You can apply what you have learned about friction to the Unit Task on page 146.

Mini **Investigation**

Light from Friction

Skills: Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this investigation, you will observe the production of light from friction. This is called triboluminescence (from the Greek *tribein*, meaning "to rub") and means that light is generated from the friction of materials rubbing together.

Equipment and Materials: eye protection; pliers; wintergreen mints

1. Extinguish the room lights and close the blinds to make the room as dark as possible. Alternatively, enter a dark closet for the investigation. Wait until your eyes adapt to the dark.
2. Put on your eye protection. Place the mints in the pliers and crush them. Observe the result. If nothing happens, try repeating the process.

Use caution when working in the dark. Wear eye protection when crushing the candy. Never eat or taste anything while in the science laboratory.

A. What happened when you crushed the mints? T/I

B. What materials rubbed together to create the friction that produced the light? K/U T/I

In the following Tutorial, you will explore friction problems and calculate acceleration, angle, and mass.

Tutorial 1 Solving Friction Problems

The Sample Problems in this Tutorial model how to use the coefficients of kinetic and static friction to calculate other unknowns, such as acceleration, angle, and mass.

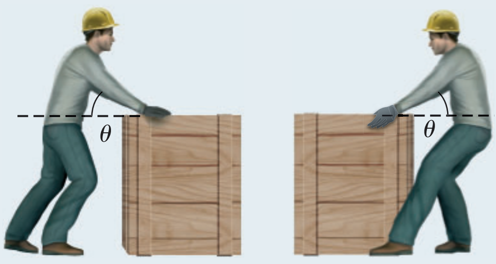
Sample Problem 1: Comparing Pushing and Pulling

A worker must move a crate that can be either pushed or pulled, as shown in **Figure 4**. The worker can exert a force of 3.6×10^2 N. The crate has a mass of 45 kg. The worker can push or pull the crate at an angle of 25°, and the coefficient of kinetic friction between the floor and the crate is 0.36. The worker wants to move the crate as quickly as possible, but he does not know whether it is better to push or pull.

(a) Calculate the acceleration when pushing the crate.

(b) Calculate the acceleration when pulling the crate.

(c) Evaluate your answers to (a) and (b). Does it matter whether the worker pushes or pulls the crate? Explain your answer.



(a) **(b)**

Figure 4

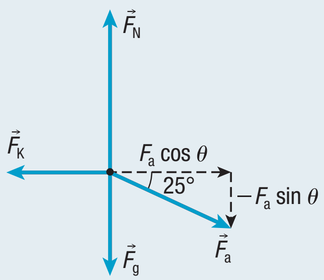
Solution

(a) **Given:** $F = 3.6 \times 10^2$ N; $m = 45$ kg; $\theta = 25°$; $\mu_K = 0.36$

Required: the acceleration when pushing on the crate, a_1

Analysis: Draw an FBD for the push. The force in the *y*-direction must be zero because the crate is not accelerating upward or downward. Calculate the normal force on the crate as well as the *y*-components of all the forces on the crate. The worker is pushing, so make the *y*-component downward. Use the horizontal components to calculate the acceleration.

Solution:



$$
\begin{aligned}
\Sigma F_y &= +F_N + (-F_g) + (-F_a \sin\theta) \\
0 &= F_N - F_g - F_a \sin\theta \\
F_N &= F_g + F_a \sin\theta \\
&= mg + F_a \sin\theta \\
&= (45\ \text{kg})(9.8\ \text{m/s}^2) + (3.6 \times 10^2\ \text{N}) \sin 25° \\
F_N &= 593.1\ \text{N}\ \ \text{(two extra digits carried)} \\
\Sigma F_x &= F\cos\theta + (-\mu_K F_N) \\
&= (3.6 \times 10^2\ \text{N})\cos 25° - (0.36)(593.1\ \text{N}) \\
\Sigma F_x &= 112.8\ \text{N}\ \ \text{(two extra digits carried)} \\
a_1 &= \frac{\Sigma F_x}{m} \\
&= \frac{112.8\ \text{N}}{45\ \text{kg}} \\
a_1 &= 2.5\ \text{m/s}^2
\end{aligned}
$$

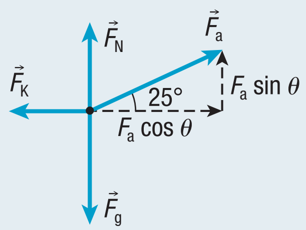
Statement: The acceleration when pushing the crate is 2.5 m/s^2.

(b) **Given:** $F = 3.6 \times 10^2$ N; $m = 45$ kg; $\theta = 25°$; $\mu_K = 0.36$

Required: the acceleration when pulling the crate, a_2

Analysis: In this case, the *x*-component of the force is the same, but the worker is now pulling so the *y*-component of the worker's force changes: it is now upward instead of downward. Draw an FBD for the pull.

Solution:



$$
\begin{aligned}
\Sigma F_y &= +F_N + (-F_g) + (+F_a \sin\theta) \\
0 &= F_N - F_g + F_a \sin\theta \\
F_N &= F_g - F_a \sin\theta \\
&= mg - F_a \sin\theta \\
&= (45\ \text{kg})(9.8\ \text{m/s}^2) - (3.6 \times 10^2\ \text{N}) \sin 25° \\
F_N &= 288.9\ \text{N}\ \ \text{(two extra digits carried)} \\
\Sigma F_x &= F_a\cos\theta + (-\mu_K F_N) \\
&= (3.6 \times 10^2\ \text{N})\cos 25° - (0.36)(288.9\ \text{N}) \\
\Sigma F_x &= 222.3\ \text{N}\ \ \text{(two extra digits carried)}
\end{aligned}
$$

$$a_2 = \frac{\Sigma F_x}{m} = \frac{222.3\text{ N}}{45\text{ kg}}$$
$$a_2 = 4.9\text{ m/s}^2$$

Statement: The acceleration when pulling the crate is 4.9 m/s^2.

(c) **Statement:** The worker should pull the crate because pulling decreases the normal force on the crate. Consequently, the force of friction is less, which produces a greater acceleration.

Sample Problem 2: Overcoming Static Friction

A crate is placed on an inclined board as shown in **Figure 5**. One end of the board is hinged so that the angle θ is adjustable. The coefficient of static friction between the crate and the board is 0.30. Determine the angle at which the crate just begins to slip.

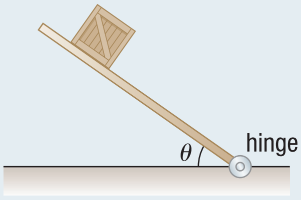


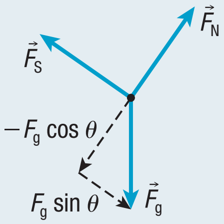
Figure 5

Given: $\mu_S = 0.30$

Required: θ

Analysis: The force of static friction on the crate can be as large as $F_S = \mu_S F_N$, where F_N is the normal force. First draw the FBD of the crate. Then calculate the normal force using the y-components. Then calculate the angle θ using the x-components and the fact that the object is in equilibrium.

Solution:



$$\Sigma F_y = +F_N + (-F_g \cos\theta)$$
$$0 = F_N - F_g \cos\theta$$
$$F_N = F_g \cos\theta$$

$$F_S = \mu_S F_N = \mu_S (F_g \cos\theta)$$
$$F_S = \mu_S F_g \cos\theta$$

$$\Sigma F_x = +F_g \sin\theta - F_S$$
$$0 = F_g \sin\theta - F_S$$
$$F_g \sin\theta = F_S$$
$$\cancel{F_g} \sin\theta = \mu_S \cancel{F_g} \cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = \mu_S$$
$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\tan\theta = \mu_S$$
$$\theta = \tan^{-1}\mu_S = \tan^{-1}(0.30)$$
$$\theta = 17°$$

Statement: The angle at which the crate just begins to slip is 17°.

Sample Problem 3: Calculating Mass in Friction Problems

Figure 6 shows two blocks joined with a rope that runs over a pulley. The mass of m_2 is 5.0 kg, and the incline is 35°. The coefficient of static friction between m_1 and the inclined plane is 0.25. Determine the largest mass for m_1 such that both blocks remain at rest.

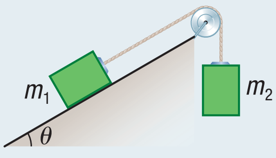


Figure 6

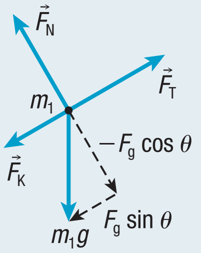
Given: $m_2 = 5.0$ kg; $\theta = 35°$; $\mu_S = 0.25$

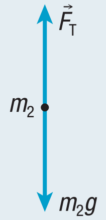
Required: m_1

Analysis: Draw an FBD of the situation. As long as the blocks are at rest, the tension in the rope is equal to the force of gravity on m_2, $F_T = F_{g2} = m_2 g$.

Now consider m_1. To remain at rest, the net force must also be zero. First, use the y-components to determine the normal force; then use the x-components to calculate m_2.

Solution:





$$\begin{aligned}
\Sigma F_y &= +F_N + (-F_{g1} \cos\theta) \\
0 &= F_N - F_{g1} \cos\theta \\
F_N &= F_{g1} \cos\theta
\end{aligned}$$

$$\begin{aligned}
F_S &= \mu_S F_N \\
&= \mu_S (F_{g1} \cos\theta) \\
&= \mu_S F_{g1} \cos\theta \\
F_S &= \mu_S m_1 g \cos\theta \\
\Sigma F_x &= +F_g \sin\theta + (-F_S) + (-F_T) \\
0 &= m_1 \cancel{g} \sin\theta - \mu_S m_1 \cancel{g} \cos\theta - m_2 \cancel{g} \\
-m_1 \sin\theta + \mu_S m_1 \cos\theta &= -m_2 \\
m_1(\mu_S \cos\theta - \sin\theta) &= -m_2 \\
m_1 &= \frac{-m_2}{\mu_S \cos\theta - \sin\theta} \\
&= \frac{-(5.0 \text{ kg})}{(0.25) \cos 35° - \sin 35°} \\
m_1 &= 14 \text{ kg}
\end{aligned}$$

Statement: The largest mass for m_1 for the blocks to remain at rest is 14 kg.

Practice

1. A small textbook is resting on a larger textbook on a horizontal desktop. You apply a horizontal force to the bottom book and both books accelerate together. The top book does not slip on the lower book. K/U T/I C A
   (a) Draw an FBD of the top book during its acceleration.
   (b) What force causes the top book to accelerate horizontally?
2. A stack of dinner plates on a kitchen counter is accelerating horizontally at 2.7 m/s^2. Determine the smallest coefficient of static friction between the dinner plates that will prevent slippage. K/U T/I [ans: 0.28]
3. A rope exerts a force of magnitude 28 N, at an angle 29° above the horizontal, on a box at rest on a horizontal floor. The coefficients of friction between the box and the floor are $\mu_S = 0.45$ and $\mu_K = 0.41$. The box remains at rest. Determine the smallest possible mass of the box. K/U T/I [ans: 6.9 kg]
4. A sled takes off from the top of a hill inclined at 6.0° to the horizontal. The sled's initial speed is 12 m/s. The coefficient of kinetic friction between the sled and the snow is 0.14. Determine how far the sled will slide before coming to rest. K/U T/I A [ans: 2.1×10^2 m]
5. You are pulling a 39 kg box on a level floor by a rope attached to the box. The rope makes an angle of 21° with the horizontal. The coefficient of kinetic friction between the box and the floor is 0.23. Calculate the magnitude of the tension in the rope needed to keep the box moving at a constant velocity. (Hint: The normal force is not equal in magnitude to the force of gravity.) K/U T/I A [ans: 87 N]
6. A 24 kg box is tied to a 14 kg box with a horizontal rope. The coefficient of friction between the boxes and the floor is 0.32. You pull the larger box forward with a force of 1.8×10^2 N at an angle of 25° above the horizontal. Calculate (a) the acceleration of the boxes and (b) the tension in the rope. K/U T/I [ans: (a) 1.8 m/s^2 [forward]; (b) 59 N]
7. The coefficient of kinetic friction between a refrigerator and the floor is 0.20. The mass of the refrigerator is 100.0 kg, and the coefficient of static friction is 0.25. Determine the acceleration when you apply the minimum force needed to get the refrigerator to move. K/U T/I A [ans: 0.49 m/s^2]
8. You are given the job of moving a stage prop with a mass of 110 kg across a horizontal floor. The coefficient of static friction between the stage prop and the floor is 0.25. Calculate the minimum force required to just set the stage prop into motion. T/I [ans: 2.7×10^2 N [horizontal]]

2.4 Review

Summary

- The coefficients of static friction and kinetic friction relate the force of friction between two objects to the normal force acting at the surfaces of the objects. These coefficients have no units and depend on the nature of the surfaces.
- Frictional force increases as the normal force increases.
- The force of static friction, $F_S \leq \mu_S F_N$, opposes the force applied to an object, increasing as the applied force increases, until the maximum static friction is reached. At that instant, the object begins to move and kinetic friction, $F_K = \mu_K F_N$, opposes the motion.

Questions

1. A car is moving with a speed of 20 m/s when the brakes are applied. The wheels lock (stop spinning). After travelling 40 m, the car stops. Determine the coefficient of kinetic friction between the tires and the road. K/U T/I A
2. A hockey puck slides with an initial speed of 50.0 m/s on a large frozen lake. The coefficient of kinetic friction between the puck and the ice is 0.030. Determine the speed of the puck after 10.0 s. K/U T/I A
3. You are trying to slide a sofa across a horizontal floor. The mass of the sofa is 2.0×10^2 kg, and you need to exert a force of 3.5×10^2 N to make it just begin to move. K/U T/I A
   (a) Calculate the coefficient of static friction between the floor and the sofa.
   (b) After it starts moving, the sofa reaches a speed of 2.0 m/s after 5.0 s. Calculate the coefficient of kinetic friction between the sofa and the floor.
4. A crate is placed on an adjustable, inclined board. The coefficient of static friction between the crate and the board is 0.29. T/I
   (a) Calculate the value of θ at which the crate just begins to slip.
   (b) Determine the acceleration of the crate down the incline at this angle when the coefficient of kinetic friction is 0.26.
5. Friction can be helpful in some situations but cause problems in other situations. K/U T/I A
   (a) Describe two situations in which friction is helpful for an object moving on a horizontal surface.
   (b) Describe two situations in which it would be ideal if there were no friction when an object moves across a horizontal surface.
6. Two blocks are connected by a massless string that passes over a frictionless pulley (**Figure 7**). The coefficient of static friction between m_1 and the table is 0.45. The coefficient of kinetic friction is 0.35. Mass m_1 is 45 kg, and m_2 is 12 kg. K/U T/I

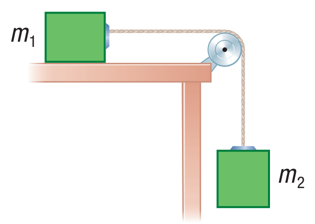


Figure 7

   (a) Is this system in static equilibrium? Explain.
   (b) Determine the tension in the string.
   (c) A mass of 20.0 kg is added to m_2. Calculate the acceleration.
7. A block of rubber is placed on an adjustable inclined plane and released from rest. The angle of the incline is gradually increased. T/I
   (a) The block does not move until the incline makes an angle of 42° to the horizontal. Calculate the coefficient of static friction.
   (b) The block stops accelerating when the incline is at an angle of 35° to the horizontal. Determine the coefficient of kinetic friction.
8. Two masses, connected by a massless string, hang over a pulley that connects two inclines (**Figure 8**). Mass m_1 is 8.0 kg, and mass m_2 is 12 kg. The coefficient of kinetic friction for both inclines is 0.21. Determine the acceleration of the two masses. T/I

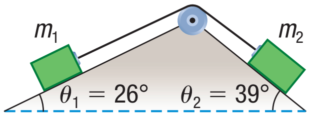


Figure 8