Applying Newton's Laws of Motion

2.3

As you read in Section 2.2, Newton's laws of motion describe how objects move as a result of different forces. In this section, you will apply Newton's laws to objects subjected to various forces in two dimensions, as well as objects that are accelerating.

For example, **Figure 1** shows a skier moving downhill. You can draw an FBD of all the forces acting on the skier. Earth's gravity acts directly downward and has components parallel and perpendicular to the slope. The normal force acts perpendicular to the hill and cancels the component of gravity perpendicular to the hill. Finally, friction acts parallel to the hill, opposing the skier's motion. You can use the sum of these forces and Newton's laws to learn about the motion of the skier.

Figure 1 The forces on this skier are gravity, the normal force, and friction. Compared to the other forces acting on the skier, air resistance is negligible here. These forces can be broken into components parallel and perpendicular to the hillside to analyze the motion of the skier.

Objects in Equilibrium

When the net force on an object is zero, that object is said to be in **equilibrium**. As discussed in Section 2.2, an object with no net force acting on it will not accelerate. So, an object in equilibrium will remain at rest or remain moving at a constant velocity until a force acts on it. Mathematically, an object is in equilibrium when $\Sigma\vec{F} = 0$, or, when you break the forces down into their components, both $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

When solving problems involving objects in equilibrium, you can set the positive *x*-axis in any direction, but you should draw the FBD first and then pick the most convenient direction. By "convenient" we mean the direction that will give you the fewest components.

The Tutorial on the next page shows you how to solve problems when an object is in equilibrium.

equilibrium a state in which an object has no net force acting on it

Investigation 2.3.1

Static Equilibrium of Forces (page 95)

In this investigation, you will analyze the conditions for equilibrium using vector components.

Tutorial 1 Solving Problems for Objects That Are in Equilibrium

This Tutorial shows how to solve problems for objects in equilibrium when acted on by two-dimensional forces.

Sample Problem 1: Calculating Tension and the Normal Force

A sled has a mass of 14 kg and is on a hill that is inclined 25° to the horizontal, as shown in **Figure 2(a)**. The hill is very icy (negligible friction), and the sled is held at rest by a rope attached to a post. The rope is parallel to the hill as shown. **Figure 2(b)** shows the FBD.

(a) Calculate the magnitude of the tension in the rope.

(b) Calculate the magnitude of the normal force acting on the sled.

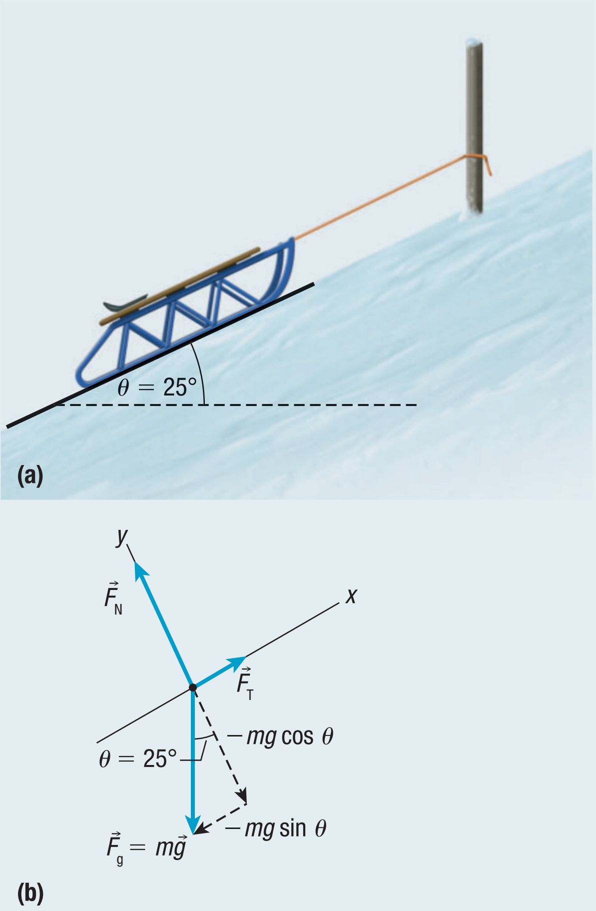


Figure 2

Solution

(a) **Given:** $m = 14$ kg; $\theta = 25°$

Required: F_T

Analysis: Tension is parallel to the hillside, and the normal force is perpendicular to the hillside. So let the positive *x*-axis point up the hillside as shown in Figure 2(b). Determine the components of gravity, and resolve the forces in the *x*-direction to solve for tension. The sled is in equilibrium, so the net force is zero.

Solution:

$$\Sigma F_x = F_T + (-F_{gx})$$
$$\Sigma F_x = F_T + (-mg\sin\theta)$$
$$0 = F_T - mg\sin\theta$$
$$F_T = mg\sin\theta$$
$$= (14\text{ kg})(9.8\text{ m/s}^2)\sin 25°$$
$$F_T = 58\text{ N}$$

Statement: The magnitude of the tension in the rope is 58 N.

(b) **Given:** $m = 14$ kg; $\theta = 25°$

Required: F_N

Analysis: Resolve the forces in the *y*-direction to solve for the normal force. The sled is in equilibrium, so the net force is zero.

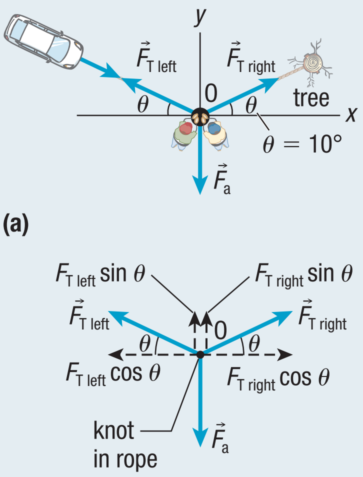
Solution:

$$\Sigma F_y = F_N + (-F_{gy})$$
$$\Sigma F_y = F_N + (-mg\cos\theta)$$
$$0 = F_N - mg\cos\theta$$
$$F_N = mg\cos\theta$$
$$= (14\text{ kg})(9.8\text{ m/s}^2)\cos 25°$$
$$F_N = 1.2 \times 10^2\text{ N}$$

Statement: The magnitude of the normal force on the sled is 1.2×10^2 N.

Sample Problem 2: Force Applied at an Angle

Your car is stuck in the mud, and you ask a friend to help you pull it free using a rope. You tie one end of the rope to your car and then pull on the other end with a force of 10^3 N. Unfortunately, the car does not move. Your friend then suggests that you make a knot in the middle of the rope, tie the other end of the rope to a tree, and then pull on the knot. Although you are skeptical that your friend's idea will help, you try it anyway. You make a knot in the middle of the rope. You leave one end of the rope attached to the car and tie the other end to a tree at an angle $\theta = 10°$. Then you and your friend pull on the knot in the direction indicated by $\vec{F}_a$ in **Figure 3(a)**. **Figure 3(b)** shows the FBD with the forces acting on the knot at point O. You discover that when a 10^3 N force is applied to the knot in the middle of the rope in the direction shown in Figure 3(a), you are just able to free the car at a slow constant velocity. Why does this work?



(a)

(b)

Figure 3

Given: $F_{\text{a}} = 10^3$ N; angle, θ, of the rope to the x-axis is 10°

Required: F_{T}

Analysis: Calculate the magnitude of the tension in the rope given the 10^3 N force exerted by you and your friend at the point where the car has just started to move at a slow constant velocity.

Since the car is just on the verge of moving, you can apply the conditions for equilibrium to this situation. Consider the forces acting on the rope at point 0 (the point at which you and your friend exert your force) to determine the tension in terms of the applied force.

Apply the conditions for static equilibrium to the rope at point 0. Use the x–y coordinate system to calculate the components of the three forces along x and y, and then apply the condition for equilibrium along y. The rope is continuous and the angles on the two sides are equal, so the tensions in the left and right portions of the rope are the same, $F_{\text{T right}} = F_{\text{T left}} = F_{\text{T}}$.

Solution: $\Sigma F_y = +F_{\text{T right}} \sin \theta + F_{\text{T left}} \sin \theta - F$

$$0 = +F_{\text{T}} \sin \theta + F_{\text{T}} \sin \theta - F$$

$$F = 2F_{\text{T}} \sin \theta$$

$$F_{\text{T}} = \frac{F}{2 \sin \theta}$$

$$= \frac{10^3 \text{ N}}{2 \sin 10°}$$

$$F_{\text{T}} = 3 \times 10^3 \text{ N}$$

Statement: This arrangement multiplies the applied force. The tension in the rope is able to pull the car out because it is 3 times the applied force ($3F_{\text{a}}$).

Practice

1. The static friction on one block is holding another block up, as shown in **Figure 4**. Block A has a weight of 6.5 N, sits on a table, and is connected to a wall by a string. Block B has a weight of 2.8 N, is attached to a string, and is connected to block A's string at point P. The string from block A to point P is horizontal. The magnitude of the force of friction on block A is 1.4 N. K/U T/I C A
   (a) Draw an FBD for block B. Determine the magnitude of the tension in the vertical rope. [ans: 2.8 N]
   (b) Draw an FBD for block A. Determine the magnitude of the tension in the horizontal rope and the magnitude of the normal force acting on block A. [ans: 1.4 N; 6.5 N]
   (c) Draw an FBD of point P. Calculate the tension (the magnitude and the angle θ) in the third rope. [ans: 3.1 N [right 63° up]]
2. A 62 kg rock climber is attached to a rope that is allowing him to hang horizontally with his feet against the wall. The tension in the rope is 7.1×10^2 N, and the rope makes an angle of 32° with the horizontal. Determine the force exerted by the wall on the climber's feet. K/U T/I A [ans: 6.5×10^2 N [left 21° up]]
3. The three forces shown in **Figure 5** act on an object. The object is in equilibrium. Calculate the magnitude of the force F_3 and the angle θ_3. T/I [ans: 78 N; W 9.8° S]

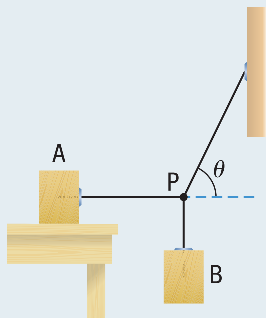


Figure 4

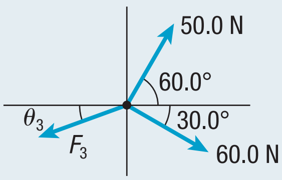


Figure 5

UNIT TASK **BOOKMARK**

You can apply what you have learned about forces and acceleration to the Unit Task on page 146.

Accelerating Objects

If an object is not in equilibrium, then it is accelerating in some direction. You can use Newton's second law, $\Sigma\vec{F} = m\vec{a}$, to determine the acceleration from the net force on the object, $\Sigma\vec{F}$.

When solving problems that involve accelerating objects, set the positive x-axis in the direction of the net force (acceleration). This will ensure that the net force has no additional y-component, which will simplify the solution. If you do not know the direction of the net force, then just set the positive x-axis in the direction that is the most convenient to solve the problem.

In the following Tutorial, you will use Newton's second law of motion to calculate velocity, acceleration, and tension for objects acted on by two-dimensional forces.

Tutorial 2 Solving Problems for Objects That Are Accelerating

The Sample Problems model how to calculate velocity, acceleration, and tension for objects that are accelerating when acted on by two-dimensional forces.

Sample Problem 1: Velocity Due to Acceleration

A sled is at the top of a hill, which makes an angle of 18° with the horizontal, as shown in **Figure 6(a)**. **Figure 6(b)** shows the FBD for the sled. The height of the hill is 25 m. Calculate the speed of the sled as it reaches the bottom of the hill. Assume that no friction acts on the sled.

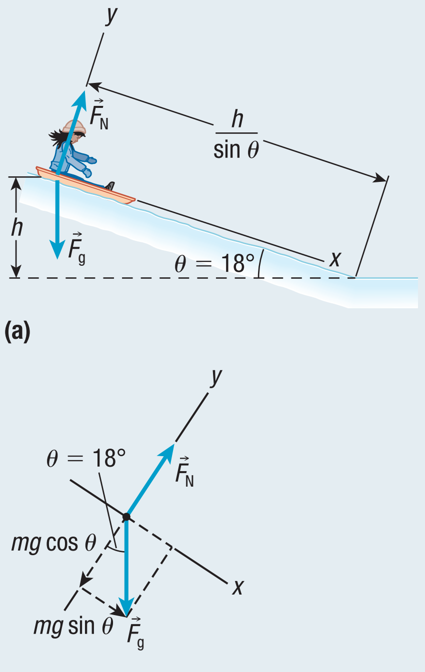


Figure 6

Given: $\Delta d_y = 25$ m; $\theta = 18°$, $v_i = 0$

Required: v_f

Analysis: The sled will accelerate down the hill, so the net force is down the hill according to Newton's second law. Therefore, make the positive x-axis down the hill. This means that the normal force is in the direction of the positive y-axis. Determine the components of the force of gravity in the x- and y-directions as defined by the coordinate axes in Figure 6(b). Use Newton's second law of motion to determine the acceleration along x; then apply $v_{fx}^2 = v_{ix}^2 + 2a_x\Delta d_x$ to calculate the final speed.

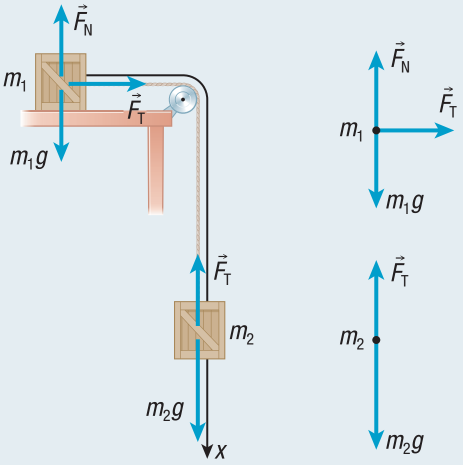
Solution:

$$\Sigma F_x = mg\sin\theta$$
$$\cancel{m}a_x = \cancel{m}g\sin\theta$$
$$a_x = g\sin\theta$$
$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta d_x$$
$$\Delta d_x = \frac{h}{\sin\theta}$$
$$v_{fx}^2 = 0^2 + 2(g\cancel{\sin\theta})\left(\frac{h}{\cancel{\sin\theta}}\right)$$
$$v_{fx} = \sqrt{2gh}$$
$$= \sqrt{2(9.8\ \text{m/s}^2)(25\ \text{m})}$$
$$v_{fx} = 22\ \text{m/s}$$

Statement: The speed of the sled at the bottom of the hill is 22 m/s.

Sample Problem 2: Acceleration and Tension

A crate with a mass of 32.5 kg sits on a frictionless surface and is connected to a second crate by a string that passes over a pulley, as shown in **Figure 7(a)**. The second crate has a mass of 40.0 kg. The pulley is frictionless and has no mass. The string also has no mass. FBDs are shown in **Figure 7(b)**. Determine the acceleration of the system of crates and the magnitude of the tension in the string.



(a) **(b)**

Figure 7

Given: $m_1 = 32.5$ kg; $m_2 = 40.0$ kg

Required: the acceleration of the crates, a; the magnitude of the tension in the string, F_T

Analysis: Apply Newton's laws to determine the acceleration of the crates, considering all the forces acting on them. The FBDs in Figure 7(b) show all these forces. The positive x-direction for each FBD is determined by the direction of the acceleration of each mass: right for mass 1 and down for mass 2. Write Newton's second law for each crate, and solve for the unknown values. To determine the magnitude of the tension, use the FBD for the crate on the surface. The accelerations of both masses are equal because they are tied together and the string does not stretch.

Solution:

$$\Sigma F_x = +F_T \quad \text{(For crate 1)}$$
$$m_1 a = F_T \quad \text{(Equation 1)}$$
$$\Sigma F_x = m_2 g - F_T \quad \text{(For crate 2)}$$
$$m_2 a = m_2 g - F_T \quad \text{(Equation 2)}$$
$$m_1 a + m_2 a = +F_T + m_2 g - F_T \quad \text{(Equation 1 + Equation 2)}$$
$$m_1 a + m_2 a = m_2 g$$
$$a = \frac{m_2 g}{m_1 + m_2} = \frac{40.0\ \text{kg}\,(9.8\ \text{m/s}^2)}{32.5\ \text{kg} + 40.0\ \text{kg}}$$
$$a = 5.41\ \text{m/s}^2 \text{ (one extra digit carried)}$$
$$\Sigma F_x = +F_T$$
$$m_1 a = F_T$$
$$F_T = (32.5\ \text{kg})(5.41\ \text{m/s}^2)$$
$$F_T = 1.8 \times 10^2\ \text{N}$$

Statement: The acceleration of the crates is 5.4 m/s^2, and the magnitude of the tension in the string is 1.8×10^2 N.

Practice

1. Two blocks are fastened onto strings inside an elevator, as shown in **Figure 8**. The mass of the top block is 1.2 kg, and the mass of the bottom block is 1.8 kg. The elevator is accelerating up at 1.2 m/s^2. K/U T/I A

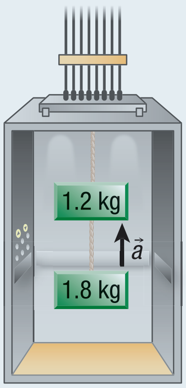


Figure 8

(a) Calculate the tension in each string. [ans: top string: 33 N; bottom string: 20 N]

(b) The maximum tension the strings can withstand is 38 N. Determine the maximum acceleration of the elevator that will not break the strings. [ans: 2.9 m/s^2 [up]]

2. A skier with a mass of 63 kg glides with negligible friction down a hill covered with hard-packed snow. The hill is inclined at an angle of 14° above the horizontal. K/U T/I A
   (a) Determine the magnitude of the normal force on the skier. [ans: 6.0×10^2 N]
   (b) Determine the magnitude of the skier's acceleration. (Hint: Remember to choose the $+x$-direction as the direction of the acceleration, parallel to the hillside.) [ans: 2.4 m/s^2]
3. A child on a toboggan slides down a hill with an acceleration of magnitude 1.9 m/s^2. Friction is negligible. Determine the angle between the hill and the horizontal. K/U T/I A [ans: 11°]
4. You pull a desk across a horizontal floor by exerting a force of 82 N, at an angle of 17° above the horizontal. The normal force exerted by the floor on the desk is 213 N. The acceleration of the desk across the floor is 0.15 m/s^2. K/U T/I A
   (a) Determine the mass of the desk. [ans: 24 kg]
   (b) Determine the magnitude of the friction force on the desk. [ans: 75 N]
5. A store clerk pulls three loaded shopping carts connected with two horizontal cords to help customers load their cars (**Figure 9**). Cart 1 has a mass of 9.1 kg, cart 2 has a mass of 12 kg, and cart 3 has a mass of 8.7 kg. Friction is negligible. A third cord, which pulls on cart 1 and is at an angle of 23° above the horizontal, has a tension of magnitude 29 N. K/U T/I A

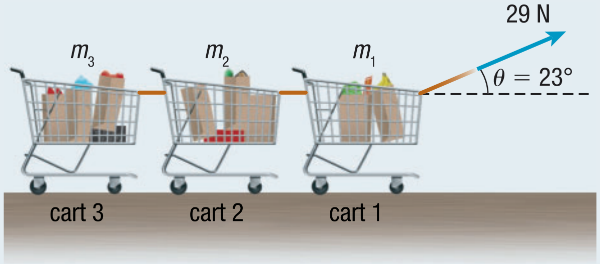


Figure 9

   (a) Determine the magnitude of the acceleration of the carts. [ans: 0.90 m/s^2]
   (b) Determine the magnitude of the tension in the cord between m_3 and m_2. [ans: 7.8 N]
   (c) Determine the magnitude of the tension in the cord between m_2 and m_1. [ans: 19 N]
6. Block A, with a mass of 4.2 kg, is suspended from a vertical string as shown in **Figure 10**. The string passes over a pulley and is attached to block B. The mass of block B is 1.8 kg. The pulley and the surface of the ramp are essentially frictionless. Calculate (a) the acceleration of the blocks and (b) the tension in the string. T/I [ans: (a) 5.3 m/s^2; (b) 19 N]

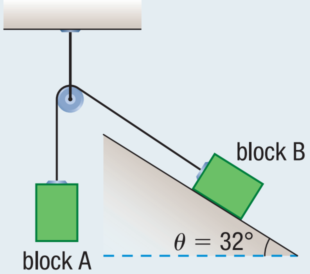


Figure 10

2.3 Review

Summary

- An object is in equilibrium when the net force on it is zero.
- For objects experiencing forces in two dimensions, break the motion into perpendicular components, which can be analyzed independently.
- Once you have determined the net force using components, use Newton's second law to determine the acceleration.

Questions

1. In **Figure 11**, two ropes are pulling on a skater, and they exert forces on her as shown in the figure. Calculate the magnitude and direction of the total force exerted by the ropes on the skater. T/I

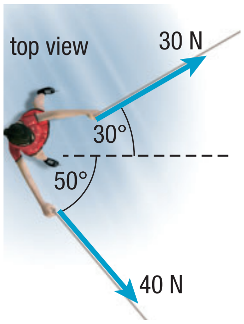


Figure 11

2. Determine the tensions in all three cables in **Figure 12**. T/I

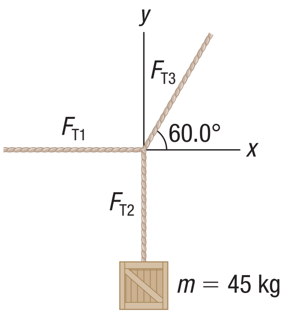


Figure 12

3. A flag of mass 2.5 kg is supported by a single rope as shown in **Figure 13**. A strong horizontal wind exerts a force of 12 N on the flag. Calculate the tension in the rope and the angle, θ, the rope makes with the horizontal. T/I

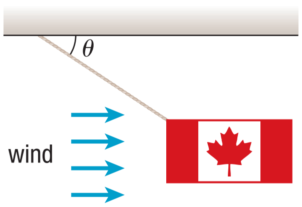


Figure 13

4. A car is parked on a slippery hill (**Figure 14**). The hill is at an angle of 15° to the horizontal. To keep it from sliding down the hill, the owner attaches a cable at the back of the car and to a post. The mass of the car is 1.41×10^3 kg. K/U T/I C

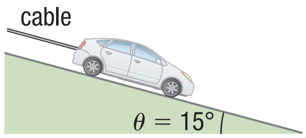


Figure 14

   (a) Draw an FBD showing the forces on the car.
   (b) Write the equations for the conditions for static equilibrium along the horizontal and vertical directions.
   (c) Calculate the tension in the cable. Assume there is no friction between the road and the tires.

5. A student pushes on a lawn mower from rest parallel to the handle of the mower. The student pushes with a force of magnitude 42 N. The handle makes an angle of 35° to the horizontal. The mower accelerates across a level driveway with negligible friction on the mower toward the lawn, 5.0 m away. The mass of the lawn mower is 18 kg. K/U T/I C
   (a) Draw the FBD of the mower.
   (b) Calculate the acceleration of the mower.
   (c) Calculate the normal force acting on the mower.
   (d) Calculate the velocity of the mower when it reaches the lawn.

6. In a physics experiment, a 1.3 kg dynamics cart is placed on a ramp inclined at 25° to the horizontal. The cart is initially at rest but is then pulled up the ramp with a force sensor. The force sensor exerts a force on the cart parallel to the ramp. Negligible friction acts on the cart. T/I
   (a) What force is required to pull the cart up the ramp at a constant velocity?
   (b) What force is required to pull the cart up the ramp at an acceleration of 2.2 m/s^2?