1.6 Relative Motion

Suppose that you are flying in an airplane at a constant velocity south. Your view from the window might be similar to the view in **Figure 1(a)**. How would you describe the view? At first, the answer might seem simple and you would describe the clouds and the ground. After a little thought, you would realize that you can see much more. From the point of view of the plane, the ground and the clouds appear to be moving north and the plane appears to be stationary. However, from the point of view of an observer on the ground (**Figure 1(b)**), the plane is moving south, the ground is stationary, and the clouds are moving with the wind. CAREER LINK

Figure 1 An airplane in flight is an excellent example of relative motion. (a) The view from an airplane window provides one perspective of an airplane's motion. (b) The view from the ground provides a completely different perspective of an airplane's motion.

Relative Velocity

frame of reference a coordinate system relative to which motion is described or observed

relative velocity the velocity of an object relative to a specific frame of reference

The airplane scenario above is an example of relative motion. The pilot and passengers in the plane are in one frame of reference (or have one point of view), and the observer on the ground is in another frame of reference. A **frame of reference** is a coordinate system relative to which motion is described or observed. The velocity of an object relative to a specific frame of reference is called the **relative velocity**.

When analyzing relative velocity problems, we will use the vector symbol, $\vec{v}$, with two subscripts in capital letters. The first subscript represents the moving object, and the second subscript represents the frame of reference. For example, suppose an airplane (P) is travelling at 450 km/h [N] relative to the frame of reference from Earth (E). Then $\vec{v}_{PE}$ is the velocity of the airplane relative to Earth, and we write the relative velocity like this: $\vec{v}_{PE} = 450$ km/h [N].

Now suppose we analyze the scenario further to be more realistic: At the altitudes at which airplanes fly, the air (A) often moves very fast relative to the ground. So, we must also consider the velocity of the plane relative to the air, $\vec{v}_{PA}$, and the velocity of the air relative to the ground, $\vec{v}_{AE}$, in addition to the velocity of the plane relative to Earth, $\vec{v}_{PE}$. The relationship that connects these three relative velocities is

$$\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$$

This equation applies whether the motion is in one, two, or three dimensions. In one dimension, solving the equation is straightforward. For example, if the plane is moving relative to the air at $\vec{v}_{PA} = 450$ km/h [N], but the air velocity relative to Earth is $\vec{v}_{AE} = 40$ km/h [N] (a tailwind), then the velocity of the plane relative to Earth is

$$\begin{aligned}
\vec{v}_{PE} &= \vec{v}_{PA} + \vec{v}_{AE} \\
&= 450\text{ km/h [N]} + 40\text{ km/h [N]} \\
\vec{v}_{PE} &= 490\text{ km/h [N]}
\end{aligned}$$

So, the ground speed increases with a tailwind, which makes sense. What would happen if the wind were a headwind? The airplane's ground speed would decrease to 410 km/h [N] if the magnitude of the wind's velocity were the same. This also makes sense because airplanes slow down relative to Earth because of headwinds.

In two dimensions, for example, when there is a crosswind, then the solution is also straightforward but requires more steps. You will work through relative velocity problems in one and two dimensions in Tutorial 1. But before you start the Tutorial, make sure you understand the patterns of the subscripts in the equation for relative velocity. In general, the equation takes the form

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

In the above equation, note that the outside subscripts on the right side of the equation (A and C) are in the same order as the subscripts on the left side of the equation, and the inside subscripts on the right side of the equation are the same (B).

If we add another frame of reference, the equation becomes

$$\vec{v}_{AD} = \vec{v}_{AB} + \vec{v}_{BC} + \vec{v}_{CD}$$

Tutorial 1 Solving Relative Motion Problems

A variety of situations involve relative motion. This Tutorial models a few examples in both one and two dimensions.

Sample Problem 1: Relative Motion in One Dimension

Passengers on a cruise ship are playing shuffleboard (**Figure 2**). The shuffleboard disc's velocity relative to the ship is 4.2 m/s [forward], and the ship is travelling in the same direction as the disc at 4.6 km/h relative to Earth when the water is stationary.

Figure 2

(a) Determine the disc's velocity relative to Earth.

(b) Determine the disc's velocity relative to Earth when the disc is moving in a direction opposite to that of the ship.

(c) Determine the disc's velocity relative to Earth when the water is moving at 1.1 m/s [forward].

Solution

(a) **Given:** Use the subscripts D for the disc, S for the ship, and E for Earth. $\vec{v}_{DS} = 4.2$ m/s [forward]; $\vec{v}_{SE} = 4.6$ km/h [forward]

Required: $\vec{v}_{DE}$

Analysis: Use the equation for relative velocity, $\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$, but first convert the ship's velocity to metres per second.

Solution:

$$\vec{v}_{SE} = \left(4.6\,\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\,[\text{forward}]\right)\left(\frac{1\,\cancel{\text{h}}}{60\,\cancel{\text{min}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\text{ s}}\right)\left(\frac{1000\text{ m}}{1\,\cancel{\text{km}}}\right)$$

$$= \frac{4.6 \times 1000\text{ m }[\text{forward}]}{60 \times 60\text{ s}}$$

$$\vec{v}_{SE} = 1.278\text{ m/s }[\text{forward}]\ (\text{two extra digits carried})$$

$$\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$$

$$= 4.2\text{ m/s }[\text{forward}] + 1.278\text{ m/s }[\text{forward}]$$

$$\vec{v}_{DE} = 5.5\text{ m/s }[\text{forward}]$$

Statement: The velocity of the disc relative to Earth is 5.5 m/s [forward].

(b) **Given:** Use the subscripts D for the disc, S for the ship, and E for Earth. $\vec{v}_{DS} = 4.2$ m/s [backward]; $\vec{v}_{SE} = 4.6$ km/h [forward] = 1.278 m/s [forward]

Required: $\vec{v}_{DE}$

Analysis: Use the equation for relative velocity, $\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$. For the ship's velocity, use the value from part (a) after converting to metres per second.

Solution:

$$\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$$

$$= 4.2\text{ m/s }[\text{backward}] + 1.278\text{ m/s }[\text{forward}]$$

$$= 4.2\text{ m/s }[\text{backward}] - 1.278\text{ m/s }[\text{backward}]$$

$$\vec{v}_{DE} = 2.9\text{ m/s }[\text{backward}]$$

Statement: The velocity of the disc relative to Earth is 2.9 m/s [backward].

(c) **Given:** Use the subscripts D for the disc, S for the ship, W for water, and E for Earth. $\vec{v}_{DS} = 4.2$ m/s [forward]; $\vec{v}_{SE} = 4.6$ km/h [forward] $= 1.278$ m/s [forward]; $\vec{v}_{WE} = 1.1$ m/s [forward]

Required: $\vec{v}_{DE}$

Analysis: Use the equation for relative velocity, $\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SW} + \vec{v}_{WE}$. For the ship's velocity, use the value from part (a) after converting to metres per second.

Solution:

$$\begin{aligned}\vec{v}_{DE} &= \vec{v}_{DS} + \vec{v}_{SW} + \vec{v}_{WE}\\ &= 4.2\text{ m/s [forward]} + 1.278\text{ m/s [forward]}\\ &\quad + 1.1\text{ m/s [forward]}\\ \vec{v}_{DE} &= 6.6\text{ m/s [forward]}\end{aligned}$$

Statement: The velocity of the disc relative to Earth is 6.6 m/s [forward].

Sample Problem 2: Relative Motion in Two Dimensions at Right Angles

The boat in **Figure 3** is heading due north as it crosses a wide river. The velocity of the boat is 10.0 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the boat's velocity relative to an observer on the riverbank.

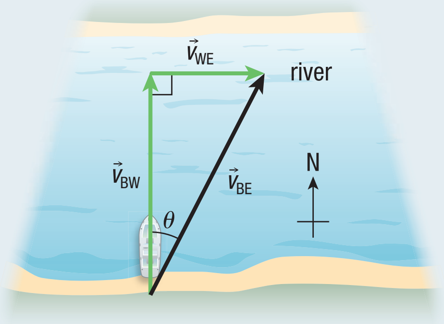


Figure 3

Given: Use the subscripts B for boat, W for water, and E for Earth. $\vec{v}_{BW} = 10.0$ km/h [N]; $\vec{v}_{WE} = 5.00$ km/h [E]

Required: $\vec{v}_{BE}$

Analysis: $\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$. This problem involves vectors in two dimensions, so we will use components to solve it. The vectors form a right-angled triangle, so the solution is straightforward. To determine the magnitude of the velocity of the boat relative to the ground, we can use the Pythagorean theorem. Then we can use the inverse tangent ratio to determine the direction.

Solution:

$$\begin{aligned}|\vec{v}_{BE}| &= \sqrt{|\vec{v}_{BW}|^2 + |\vec{v}_{WE}|^2}\\ &= \sqrt{(10.0\text{ km/h})^2 + (5.00\text{ km/h})^2}\\ |\vec{v}_{BE}| &= 11.2\text{ km/h}\\ \tan\theta &= \frac{|\vec{v}_{WE}|}{|\vec{v}_{BW}|}\\ &= \frac{5.00\text{ km/h}}{10.0\text{ km/h}}\\ \theta &= 26.6^\circ\end{aligned}$$

Statement: The velocity of the boat relative to an observer on the riverbank is 11.2 km/h [N 26.6° E].

Sample Problem 3: Relative Motion in Two Dimensions

The driver of the boat in Sample Problem 2 moves with the same speed of 10.0 km/h relative to the water but now wants to arrive across the water at a location that is due north of his present location. The river is flowing east at 5.00 km/h.
In which direction should he head? What is the speed of the boat, according to an observer on the shore?

Given: Use the subscripts B for boat, W for water, and E for Earth. $\vec{v}_{BW} = 10.0$ km/h [?]; $\vec{v}_{WE} = 5.00$ km/h [E]; $\vec{v}_{BE} = ?$ [N]

Required: $|\vec{v}_{BE}|$; the heading of the boat, θ

Analysis: $\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$. This problem involves vectors in two dimensions, but we do not know two complete vectors. So, first we will draw the vector triangle and then resolve the triangle: draw $\vec{v}_{BW}$ in a northwest direction as shown in **Figure 4**, and then add $\vec{v}_{WE}$ head to tail. The sum of these two vectors, $\vec{v}_{BE}$, must be directed north as shown. Determine the heading of the boat using the definition of sine. To calculate the magnitude of the velocity of the boat relative to an observer on shore, use the Pythagorean theorem.

Solution:

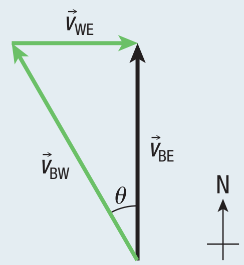


Figure 4

Determine the heading of the boat:

$$\begin{aligned}\sin\theta &= \frac{|\vec{v}_{WE}|}{|\vec{v}_{BW}|}\\ &= \frac{5.00\text{ km/h}}{10.0\text{ km/h}}\\ \theta &= 30.0^\circ\end{aligned}$$

$$|\vec{v}_{BE}| = \sqrt{|\vec{v}_{BW}|^2 - |\vec{v}_{WE}|^2}$$
$$= \sqrt{(10.0 \text{ km/h})^2 - (5.00 \text{ km/h})^2}$$
$$|\vec{v}_{BE}| = 8.66 \text{ km/h}$$

Statement: The heading of the boat is N 30.0° W, and the speed of the boat relative to the shore is 8.66 km/h.

Sample Problem 4: Using Trigonometry with Relative Motion

The air velocity of a small plane is 230 km/h [N 35° E] when the wind is blowing at 75 km/h [W 25° S]. Determine the velocity of the plane relative to the ground.

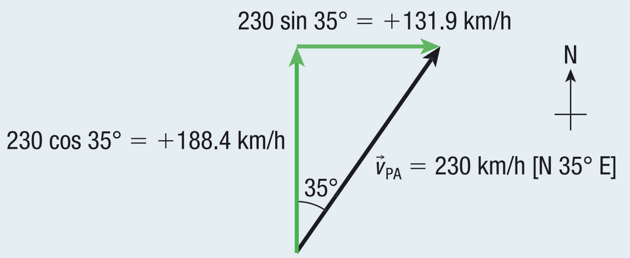
Given: Use the subscripts P for plane, A for air, and E for Earth.
$\vec{v}_{PA} = 230$ km/h [N 35° E]; $\vec{v}_{AE} = 75$ km/h [W 25° S]

Required: $\vec{v}_{PE}$

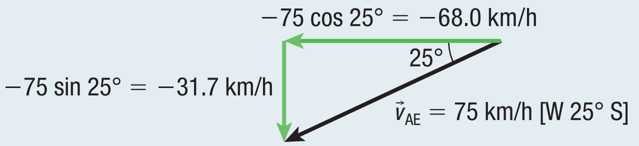
Analysis: $\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$. This problem involves vectors in two dimensions, so we will use components to solve it.

First, determine the components of each vector—the vector for the airplane and the vector for the wind—using the $+y$-direction as north and the $+x$-direction as east. Then use the Pythagorean theorem to calculate the speed of the plane and the inverse tangent ratio to calculate the direction of the plane.

Solution:
Airplane components:



Wind components:



Determine the vertical components, where the $+y$-direction is north:

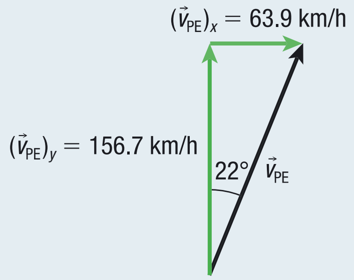
$$(\vec{v}_{PE})_y = (\vec{v}_{PA})_y + (\vec{v}_{AE})_y$$
$$= 188.4 \text{ km/h} + (-31.7 \text{ km/h})$$
$$(\vec{v}_{PE})_y = 156.7 \text{ km/h}$$

Determine the horizontal components, where the $+x$-direction is east:

$$(\vec{v}_{PE})_x = (\vec{v}_{PA})_x + (\vec{v}_{AE})_x$$
$$= 131.9 \text{ km/h} + (-68.0 \text{ km/h})$$
$$(\vec{v}_{PE})_x = 63.9 \text{ km/h}$$

Calculate the magnitude of the velocity of the plane relative to Earth and then the direction of the plane:

$$|\vec{v}_{PE}| = \sqrt{|(\vec{v}_{PE})_y|^2 + |(\vec{v}_{PE})_x|^2}$$
$$= \sqrt{(156.7 \text{ km/h})^2 + (63.9 \text{ km/h})^2}$$
$$|\vec{v}_{PE}| = 170 \text{ km/h}$$



$$\tan\theta = \frac{|(\vec{v}_{PE})_x|}{|(\vec{v}_{PE})_y|}$$
$$= \frac{63.9 \text{ km/h}}{156.7 \text{ km/h}}$$
$$\theta = 22°$$

Statement: The velocity of the plane relative to the ground is 170 km/h [N 22° E].

Practice

1. A group of teenagers on a ferry boat walk on the deck with a velocity of 1.1 m/s relative to the deck. The ship is moving forward with a velocity of 2.8 m/s relative to the water.
K/U T/I A
   (a) Determine the velocity of the teenagers relative to the water when they are walking to the bow (front). [ans: 3.9 m/s [forward]]
   (b) Determine the velocity of the teenagers relative to the water when they are walking to the stern (rear). [ans: 1.7 m/s [forward]]

2. An airplane flies due north over Sudbury with a velocity relative to the air of 235 km/h and with a wind velocity of 65 km/h [NE]. Calculate the speed and direction of the airplane. K/U T/I A [ans: 280 km/h [E 81° N]]

3. A helicopter flies with an air speed of 175 km/h, heading south. The wind is blowing at 85 km/h to the east relative to the ground. Calculate the speed and direction of the helicopter. K/U T/I A [ans: 190 km/h [E 64° S]]

4. Suppose you are the pilot of a small plane flying due south between northern Ontario and Barrie. You want to reach the airport in Barrie in 3.0 h. The airport is 450 km away, and the wind is blowing from the west at 50.0 km/h. Determine the heading and air speed you should use to reach your destination on time. K/U T/I A [ans: 160 km/h [S 18° W]]

5. A large ferry boat is moving north at 4.0 m/s [N] with respect to the shore, while a child is running on the deck at a speed of 3.0 m/s. Determine the velocity of the child relative to Earth when the child is running in the following directions with respect to the deck of the boat:
   (a) north [ans: 7.0 m/s [N]]
   (b) south [ans: 1.0 m/s [N]]
   (c) east K/U T/I A [ans: 5.0 m/s [N 37° E]]

6. A plane is travelling with a velocity relative to the air of 3.5×10^2 km/h [N 35° W] as it passes over Hamilton. The wind velocity is 62 km/h [S]. K/U T/I A
   (a) Determine the velocity of the plane relative to the ground. [ans: 3.0×10^2 km/h [N 42° W]]
   (b) Determine the displacement of the plane after 1.2 h. [ans: 3.6×10^2 km [N 42° W]]

7. A person decides to swim across a river 84 m wide that has a current moving with a velocity of 0.40 m/s [E]. The person swims at 0.70 m/s [N] relative to the water. K/U T/I A
   (a) What is the velocity of the person with respect to Earth? [ans: 0.81 m/s [N 30° E]]
   (b) How long will it take to cross? [ans: 1.2×10^2 s]
   (c) How far downstream will the person land? [ans: 48 m]
   (d) In what direction should she swim if she lands at a point directly north of her starting position? [ans: [N 35° W]]

8. Two canoeists paddle with the same speed relative to the water, but one moves upstream at −1.2 m/s and the other moves downstream at +2.9 m/s, both relative to Earth. K/U T/I A
   (a) Determine the speed of the water relative to Earth. [ans: 0.85 m/s]
   (b) Determine the speed of each canoe relative to the water. [ans: 2.0 m/s]

9. An airplane maintains a velocity of 630 km/h [N] relative to the air as it makes a trip to a city 750 km away to the north. K/U T/I A
   (a) How long will the trip take when the wind velocity is 35 km/h [S]? [ans: 1.3 h]
   (b) How long will the same trip take when there is a tailwind of 35 km/h [N] instead? Why does the answer change? [ans: 1.1 h]
   (c) What will the pilot do if the wind velocity is 35 km/h [E] instead? How long will the trip take in this case? [ans: 1.2 h]

1.6 Review

Summary

- Relative motion is motion observed from a specific perspective or frame of reference. Each frame of reference has its own coordinate system. Relative velocity is the velocity of an object observed from a specific frame of reference.
- The relative velocity equation is $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$, where A is the object moving relative to the frame of reference C, which is moving relative to the frame of reference B.

Questions

1. A river has a steady current of 0.50 m/s [E]. A person can swim at 1.2 m/s in still water. The person swims upstream 1.0 km and then back to the starting point. K/U T/I A
   (a) How long does the trip take?
   (b) Will the time change if he swims downstream 1.0 km and then back instead? Explain your reasoning.
   (c) How much time is required to complete the same trip in still water? Why does the trip take longer when there is a current?
2. An airplane has an air velocity of 200 m/s [W]. The wind velocity relative to the ground is 60 m/s [N]. K/U T/I A
   (a) Determine the velocity of the airplane relative to the ground.
   (b) The airplane now faces a headwind of 60 m/s [E]. Calculate how long it takes the airplane to fly between two cities 300 km apart.
3. A helicopter travels at a velocity of 62 m/s [N] with respect to the air. Calculate the velocity of the helicopter with respect to Earth when the wind velocity is as follows: K/U T/I A
   (a) 18 m/s [N]
   (b) 18 m/s [S]
   (c) 18 m/s [W]
   (d) 18 m/s [N 42° W]
4. A person can swim 0.65 m/s in still water. She heads directly south across a river 130 m wide and lands at a point 88 m [W] downstream. K/U T/I A
   (a) Determine the velocity of the water relative to the ground.
   (b) Determine the swimmer's velocity relative to Earth.
   (c) Determine the direction she should swim to land at a point directly south of the starting point.
5. A pilot is required to fly directly from London, United Kingdom, to Rome, Italy, in 3.4 h. The displacement is 1.4×10^3 km [S 43° E]. The wind velocity reported from the ground is 55 km/h [S]. Determine the required velocity of the plane relative to the air. K/U T/I A
6. A pilot is flying to a destination 220 km [N] of her present position. An air traffic controller on the ground tells her the wind velocity is 42 km/h [N 36° E]. She knows her plane cruises at a speed of 230 km/h relative to the air. K/U T/I A
   (a) Determine the heading of the plane.
   (b) How long will the trip take?
7. An airplane flies 5.0×10^3 km from Boston to San Francisco at an air speed of 250 m/s. On the way to San Francisco, the airplane faces a headwind of 50.0 m/s blowing from west to east, and a tailwind of the same speed on the way back. K/U T/I A
   (a) Calculate the average speed of the airplane relative to the ground on the way west.
   (b) Calculate the average speed of the airplane relative to the ground on the way east.
8. A group of people on vacation on a cruise ship decide to go up to the top floor. Some decide to take an elevator, which moves at 2.0 m/s, while others climb the stairs at 2.0 m/s. The stairs are at an angle of elevation of 38° up from the east direction. The boat is cruising at a velocity of 3.2 m/s [E] relative to the water. K/U T/I A
   (a) Calculate the velocity of the people in the elevator relative to the water.
   (b) Calculate the velocity of the people taking the stairs relative to the water.
9. A car travels due east with a speed of 60.0 km/h relative to the ground. Raindrops are falling at a constant speed vertically relative to Earth. The traces of the rain on the side windows of the car make an angle of 70.0° with the vertical. Calculate the velocity of the rain relative to (a) the car and (b) Earth. K/U T/I A
10. A plane must reach a destination N 30.0° W of its present position. The wind velocity is 48 km/h [W], and the plane moves at 260 km/h relative to the air. Determine (a) the heading of the plane and (b) the speed of the plane relative to the ground. K/U T/I A