## Displacement in Two Dimensions

So far, you have learned about motion in one dimension. This is adequate for learning basic principles of kinematics, but it is not enough to describe the motions of objects in real life. Cars and buses do not always move in a straight line because streets do not always follow straight lines (Figure 1). Even train tracks change directions, and airplanes have both vertical and horizontal displacements.


Figure 1 Streets typically change direction, as this image shows. The position of a vehicle on such streets requires coordinates in more than one dimension.

In this section, you will learn how to combine vectors to describe the position of an object in two dimensions. From this, you will be able to determine the object's two-dimensional displacement using different methods. This basic but important skill will prepare you for describing two-dimensional velocity and acceleration using vector addition. CAREER LINK

## Displacement Vectors and Their Properties

In Sections 1.1 and 1.2, you reviewed how some quantities, such as speed, are described solely in terms of magnitude (scalars), and other quantities, such as displacement and velocity, are described in terms of both magnitude and direction (vectors).

As with equations, in diagrams and figures an arrow represents a vector quantity. The arrow's length indicates the magnitude of the vector (for example, how fast a car is moving), and the direction of the arrow indicates the direction of the vector relative to a chosen coordinate system (for example, which way a car is moving). In most cases, we use reference coordinates that are perpendicular to each other (such as $x$ and $y$, or north and east) and then describe the vector in two dimensions with respect to the coordinate system.

In Sections 1.1 and 1.2, our vector notation described situations in one dimension. Now that we are describing situations in two dimensions, we need to slightly modify the notation. For example, suppose you walk 15 m toward the west. Your displacement will be $15 \mathrm{~m}[W]$. Now suppose you turn and walk 15 m in a direction that is west $35^{\circ}$ north. You express this displacement as $15 \mathrm{~m}\left[\mathrm{~W} 35^{\circ} \mathrm{N}\right]$ (Figure 2). The direction [W $35^{\circ} \mathrm{N}$ ] can be read as "point west, and then turn $35^{\circ}$ toward north."


Figure 2 This scale diagram represents a displacement of $15 \mathrm{~m}\left[\mathrm{~W} 35^{\circ} \mathrm{N}\right]$.

## Determining Total Displacement

Often, we will want to determine the total displacement of an object that has changed direction during its motion. In such cases, we will treat the object's linear displacement in each direction as a separate vector. Below are three different methods to determine the total displacement, $\Delta \vec{d}_{\mathrm{T}}$ : the scale diagram method, the cosine and sine laws method, and the perpendicular components of a vector method.

## The Scale Diagram Method

Representing vectors as arrows is a convenient way to add them together and determine the total displacement. Drawing a scale diagram is the most direct way of doing this. Simply draw the vectors, making sure that you draw the magnitudes to scale with respect to each other, and orient their directions correctly with respect to the coordinate system using a protractor. This approach to solving the problem lacks accuracy, but it makes it easier to visualize the vector addition.

Figure 3 shows two displacements, $\Delta \vec{d}_{1}$ and $\Delta \vec{d}_{2}$, drawn to scale and added together. Each vector has a magnitude and a direction, and both the lengths and the directions of the arrows are important. When you draw each vector, you can choose where you want to locate the arrow, just as long as it has the same length and the same direction. If the arrow keeps its properties of magnitude and direction, it is still the same vector. You can write this addition of vectors as

$$
\Delta \vec{d}_{\mathrm{T}}=\Delta \vec{d}_{1}+\Delta \vec{d}_{2}
$$



Figure 3 Drawing the tail of $\Delta \vec{d}_{2}$ at the tip of $\Delta \vec{d}_{1}$ is a convenient way to add vectors. The total displacement $\Delta \vec{d}_{\mathrm{T}}$ equals $\Delta \vec{d}_{1}+\Delta \vec{d}_{2}$.

Note that the addition of $\Delta \vec{d}_{1}$ and $\Delta \vec{d}_{2}$ and the total displacement $\Delta \vec{d}_{\mathrm{T}}$ form a triangle, with $\Delta \vec{d}_{1}$ and $\Delta \vec{d}_{2}$ directed one way around the triangle (counterclockwise in this case) and $\Delta \vec{d}_{\mathrm{T}}$ directed the other way. The triangle concept can be difficult to understand: Think of walking around a triangle on the ground. The two individual displacements, $\Delta \vec{d}_{1}$ and $\Delta \vec{d}_{2}$, indicate one way you could walk around the triangle. The total displacement vector, $\Delta \vec{d}_{\mathrm{T}}$, indicates the other way you could go. Also note that the order in which you add the vectors does not matter; that is, the addition is commutative. As long as you scale the arrows properly, orient them in the right directions, and add the tail of one vector to the tip of the other, the total displacement arrow will extend from the tail of the first displacement to the tip of the second displacement. This is true of all vectors, not just displacements. You must also remember to convert the answer you obtain from the scale diagram back into the actual answer using the scale.

## The Cosine and Sine Laws Method

Another method to determine the sum of two vectors is to use the cosine and sine laws. These trigonometric relations allow you to calculate the length of the total displacement vector and its angle of orientation with respect to the coordinate system. This trigonometric method only works when adding two vectors at a time, but the result is more accurate than that of a scale diagram. In Tutorial 1, you will practise adding displacement vectors to determine total displacement by drawing scale diagrams and by using trigonometry.

## Tutorial 1 Adding Displacement Vectors

This Tutorial models two methods of adding two displacement vectors to determine the total displacement. In Sample Problem 1, you will use the scale diagram method. In Sample Problem 2, you will use the cosine and sine laws method.

## Sample Problem 1: Vector Addition by Scale Diagram

Suppose you walk to a friend's house, taking a shortcut across an open field. Your first displacement is 140 m [E 35 $\left.{ }^{\circ} \mathrm{N}\right]$ across the field. Then you walk 200.0 m [E] along the sidewalk. Determine your total displacement using a scale diagram.
Given: $\Delta \vec{d}_{1}=140 \mathrm{~m}\left[\mathrm{E} 35^{\circ} \mathrm{N}\right] ; \Delta \vec{d}_{2}=200.0 \mathrm{~m}[\mathrm{E}]$
Required: $\Delta \vec{d}_{\mathrm{T}}$; the angle for $\Delta \vec{d}_{\mathrm{T}}, \theta$
Analysis: $\Delta \vec{d}_{T}=\Delta \vec{d}_{1}+\Delta \vec{d}_{2}$. Decide on a scale so that you can draw each vector to scale with the correct direction with respect to the coordinate axes.

## Solution:

Step 1. Choose a suitable scale. In this case, set the scale so that $1 \mathrm{~cm}: 40 \mathrm{~m}$. Then, determine the lengths of the arrows for $\Delta \vec{d}_{1}$ and $\Delta \vec{d}_{2}$.

$$
\Delta \vec{d}_{1}: \frac{140 \mathrm{nt}}{40 \mathrm{my} / \mathrm{cm}}=3.5 \mathrm{~cm} ; \Delta \vec{d}_{2}: \frac{200.0 \mathrm{mr}}{40 \mathrm{mp} / \mathrm{cm}}=5.0 \mathrm{~cm}
$$

Step 2. Using a ruler and a protractor, draw the two vectors, placing the tail of $\Delta \vec{d}_{2}$ at the tip of $\Delta \vec{d}_{1}$, as shown in Figure 4.


Step 3. Draw the total displacement vector $\Delta \vec{d}_{\mathrm{T}}$ from the tail of $\Delta \vec{d}_{1}$ to the tip of $\Delta \vec{d}_{2}$, measure the length of the vector, and measure the angle the displacement vector makes to the horizontal, as shown in Figure 5. The measured length of the total displacement vector is 8.1 cm . Convert to metres:
$8.1 \mathrm{~cm} \times 40 \frac{\mathrm{~m}}{\mathrm{~cm}}=324 \mathrm{~m}$
To two significant digits, the total displacement is 320 m . The measured angle between east and the total displacement vector is about $14^{\circ}$.


Figure 5 Draw the total displacement vector, $\Delta \vec{d}_{\mathrm{T}}$.
Statement: The total displacement $\Delta \vec{d}_{\mathrm{T}}$ is approximately $320 \mathrm{~m}\left[\mathrm{E} 14^{\circ} \mathrm{N}\right]$.

Figure 4 Set the scale, and draw vectors $\Delta \vec{d}_{1}$ and $\Delta \vec{d}_{2}$.

## Sample Problem 2: Vector Addition Using the Cosine and Sine Laws

Using your solution diagram for Sample Problem 1 (Figure 5), determine the total displacement using the cosine and sine laws.
Given: $\Delta \vec{d}_{1}=140 \mathrm{~m}\left[\mathrm{E} 35^{\circ} \mathrm{N}\right] ; \Delta \vec{d}_{2}=200.0 \mathrm{~m}[\mathrm{E}]$
Required: $\Delta \vec{d}_{\mathrm{T}}$; the angle for $\Delta \vec{d}_{\mathrm{T}}, \theta$

Analysis: $\Delta \vec{d}_{\mathrm{T}}=\Delta \vec{d}_{1}+\Delta \vec{d}_{2}$. To determine the magnitude of the displacement, use the cosine law. To calculate the angle of $\Delta \vec{d}_{\mathrm{T}}$ with respect to the horizontal axis E , use the sine law.

Solution: The second displacement is parallel to the E axis, and $\theta_{2}+35^{\circ}=180^{\circ}$, so the angle $\theta_{2}$ is $145^{\circ}$ (Figure 6).

$$
\begin{aligned}
\frac{\sin \theta_{3}}{\left|\Delta \vec{d}_{2}\right|} & =\frac{\sin \theta_{2}}{\left|\Delta \vec{d}_{\mathrm{T}}\right|} \\
\sin \theta_{3} & =\frac{\left|\Delta \vec{d}_{2}\right| \sin \theta_{2}}{\left|\Delta \vec{d}_{\mathrm{T}}\right|} \\
& =\frac{(200.0 \mathrm{~m})\left(\sin 145^{\circ}\right)}{324.8 \mathrm{~m}} \\
\theta_{3} & =20.7^{\circ} \text { (one extra digit carried) }
\end{aligned}
$$

The angle $\theta$ between east and the total displacement is therefore $35^{\circ}-20.7^{\circ}=14.3^{\circ}$.
Statement: Using the cosine and sine laws, the displacement is $\Delta \vec{d}_{\mathrm{T}}=320 \mathrm{~m}\left[\mathrm{E} 14^{\circ} \mathrm{N}\right]$.

## Figure 6

$$
\begin{aligned}
a^{2} & =b^{2}+c^{2}-2 b c \cos A \\
\left|\Delta \vec{d}_{T}\right|^{2} & =\left|\Delta \vec{d}_{1}\right|^{2}+\left|\Delta \vec{d}_{2}\right|^{2}-2\left|\Delta \vec{d}_{1}\right|\left|\Delta \vec{d}_{2}\right| \cos \theta_{2} \\
& =(140 \mathrm{~m})^{2}+(200.0 \mathrm{~m})^{2}-2(140 \mathrm{~m})(200.0 \mathrm{~m})\left(\cos 145^{\circ}\right) \\
\left|\Delta \vec{d}_{T}\right|^{2} & =105473 \mathrm{~m}^{2} \\
\left|\Delta \vec{d}_{T}\right| & =324.8 \mathrm{~m} \text { (two extra digits carried) }
\end{aligned}
$$

## Practice

1. A car starts from a parking lot and travels 1.2 km south and then 3.1 km in a direction $53^{\circ}$ north of east. Relative to the parking lot, what is the car's total displacement? Solve the problem using the scale diagram method, and express your answer in terms of distance and an angle. kJU TITM [ans: $\left.2.3 \mathrm{~km}\left[\mathrm{E} 34^{\circ} \mathrm{N}\right]\right]$
2. A boater travels across a river from one point on the western shore to a point 95.0 m south on the eastern shore. The river is 77.0 m wide as measured directly from west to east. Calculate the boater's total displacement. $k \mathbb{T N T}$ [ans: $\left.122 \mathrm{~m}\left[\mathrm{E} 51.0^{\circ} \mathrm{S}\right]\right]$
3. A helicopter flies $65 \mathrm{~km}\left[\mathrm{~N} 32^{\circ} \mathrm{E}\right]$ and then $42 \mathrm{~km}\left[\mathrm{E} 21^{\circ} \mathrm{N}\right]$. Determine the total displacement of the helicopter. KUT TTM [ans: $\left.1.0 \times 10^{2} \mathrm{~km}\left[\mathrm{E} 44^{\circ} \mathrm{N}\right]\right]$

## The Perpendicular Components of a Vector Method

A more straightforward way of adding two or more displacement vectors is to resolve, or separate, each vector into perpendicular components. For two dimensions, the perpendicular components of a vector are the parts of the vector that lie along either the $x$-axis or the $y$-axis. This makes it easy to add the components of several vectors that are all parallel. We obtain the components of the total displacement vector by adding the parallel displacement components.

To do this, we need to be familiar with obtaining the components of a vector, which requires trigonometry. For example, suppose you walked a distance of 5.0 km in a direction that is east $37^{\circ}$ toward north, or $\Delta \vec{d}=5.0 \mathrm{~km}\left[\mathrm{E} 37^{\circ} \mathrm{N}\right]$, as shown in Figure 7. In this example, we will use the convention that east and north are positive. To draw the $x$-component, you position the tail of the $\Delta d_{x}$ vector at the origin and make a vector going east. You draw this one first because it is indicated as the first direction in $\left[\mathrm{E} 37^{\circ} \mathrm{N}\right.$ ]. From the tip of the $x$-component you draw the $y$-component, $\Delta d_{y}$, directly north and stop at the tip of $\Delta \vec{d}$, as shown in Figure 7. You draw the $y$-component second because it is the second direction listed in $\left[\mathrm{E} 37^{\circ} \mathrm{N}\right]$. Then the components of this vector can be determined using the sine and cosine trigonometric ratios:

$$
\begin{array}{rlrl}
\cos \theta & =\frac{\text { adjacent }}{\text { hypotenuse }} & \sin \theta=\frac{\text { opposite }}{\text { hypotenuse }} \\
\cos \theta & =\frac{\Delta d_{x}}{|\Delta \vec{d}|} & \sin \theta & =\frac{\Delta d_{y}}{|\Delta \vec{d}|} \\
\Delta d_{x} & =|\Delta \vec{d}| \cos \theta & \Delta d_{y} & =|\Delta \vec{d}| \sin \theta
\end{array}
$$

component of a vector in two dimensions, either of the $x$-vector and $y$-vector that are combined into an overall vector


Figure 7 The displacement vector can be broken down into its perpendicular $x$ - and $y$-components.

For the situation described in Figure 7, the components are

$$
\begin{aligned}
\Delta d_{x} & =|\Delta \vec{d}| \cos \theta & \Delta d_{y} & =|\Delta \vec{d}| \sin \theta \\
& =(5.0 \mathrm{~km})\left(\cos 37^{\circ}\right) & & =(5.0 \mathrm{~km})\left(\sin 37^{\circ}\right) \\
& =4.0 \mathrm{~km} & & =3.0 \mathrm{~km} \\
\Delta d_{x} & =4.0 \mathrm{~km}[\mathrm{E}] & \Delta d_{y} & =3.0 \mathrm{~km}[\mathrm{~N}]
\end{aligned}
$$

When drawing these displacement components, be sure to place the tail of the vertical displacement $\left(\Delta d_{y}\right)$ at the tip of the horizontal component $\left(\Delta d_{x}\right)$. Keep in mind that $\theta$ will not always be the angle between the $x$-axis and the displacement. Sometimes, $\theta$ will be situated between the $y$-axis and the displacement. For this reason, always consider which component is opposite $\theta$ and which one is adjacent to $\theta$ to determine the components. To add and subtract vectors using components, you must first become adept at determining the components of vectors. Tutorial 2 models how to do this.

## Tutorial 2 Determining the Components of a Displacement Vector

This Tutorial shows how to use trigonometry to determine the components of a displacement vector.

## Sample Problem 1: Determining Vector Components Using Trigonometry

A polar bear walks toward Churchill, Manitoba. The polar bear's displacement is 15.0 km [S $\left.60.0^{\circ} \mathrm{E}\right]$. Determine the components of the displacement.
Given: $\Delta \vec{d}=15.0 \mathrm{~km}\left[\mathrm{~S} 60.0^{\circ} \mathrm{E}\right]$
Required: $\Delta d_{x} ; \Delta d_{y}$
Analysis: Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.

Solution:

$\Delta d_{x}=+|\Delta \vec{d}| \sin \theta$ (positive because the component points east)
$=(15.0 \mathrm{~km})\left(\sin 60.0^{\circ}\right)$
$\Delta d_{x}=12.99 \mathrm{~km}$ (one extra digit carried)
$\Delta d_{y}=-|\Delta \vec{d}| \cos \theta$ (negative because thecomponent points south)
$=-(15.0 \mathrm{~km})\left(\cos 60.0^{\circ}\right)$
$=-7.50 \mathrm{~km}$
$\Delta d_{y}=7.50 \mathrm{~km}[\mathrm{~S}]$

To check these results, use the magnitudes of the components to determine the magnitude of the total displacement, $|\Delta \vec{d}|=\sqrt{\left(\Delta d_{x}\right)^{2}+\left(\Delta d_{y}\right)^{2}}$, and then calculate the angle using the equation $\theta=\tan ^{-1}\left(\frac{\left|\Delta d_{x}\right|}{\left|\Delta d_{y}\right|}\right)$.

$$
\begin{aligned}
|\Delta \vec{d}| & =\sqrt{\left(\Delta d_{x}\right)^{2}+\left(\Delta d_{y}\right)^{2}} \\
& =\sqrt{(12.99 \mathrm{~km})^{2}+(7.50 \mathrm{~km})^{2}} \\
|\Delta \vec{d}| & =15.0 \mathrm{~km} \\
\theta & =\tan ^{-1}\left(\frac{\left|\Delta d_{x}\right|}{\left|\Delta d_{y}\right|}\right) \\
& =\tan ^{-1}\left(\frac{12.99 \mathrm{~km}}{7.50 \mathrm{~km}}\right) \\
\theta & =60.0^{\circ}
\end{aligned}
$$

The magnitude of the displacement and the angle are the same as those given in the Sample Problem.
Statement: The components of the polar bear's displacement are $\Delta d_{x}=13.0 \mathrm{~km}[\mathrm{E}]$ and $\Delta d_{y}=7.50 \mathrm{~km}[\mathrm{~S}]$.

## Practice

1. Determine the perpendicular vector components for each of the following displacements. ITI $A$
(a) $25.0 \mathrm{~km}\left[\mathrm{E} 45.0^{\circ} \mathrm{N}\right]$ [ans: $\left.\Delta d_{x}=17.7 \mathrm{~km}[\mathrm{E}] ; \Delta d_{y}=17.7 \mathrm{~km}[\mathrm{~N}]\right]$
(b) $355 \mathrm{~km}\left[\mathrm{~N} 42.0^{\circ} \mathrm{W}\right]$ [ans: $\left.\Delta d_{x}=238 \mathrm{~km}[\mathrm{~W}] ; \Delta d_{y}=264 \mathrm{~km}[\mathrm{~N}]\right]$
(c) $32.3 \mathrm{~m}\left[\mathrm{E} 27.5^{\circ} \mathrm{S}\right]$ [ans: $\left.\Delta d_{x}=28.7 \mathrm{~m}[\mathrm{E}] ; \Delta d_{y}=14.9 \mathrm{~m}[\mathrm{~S}]\right]$
(d) $125 \mathrm{~km}\left[\mathrm{~S} 31.2^{\circ} \mathrm{W}\right]$ [ans: $\left.\Delta d_{x}=64.8 \mathrm{~km}[\mathrm{~W}] ; \Delta d_{y}=107 \mathrm{~km}[\mathrm{~S}]\right]$

## Adding Vectors Algebraically

Now that you know how to separate a vector into its perpendicular components, you can add two or more displacement vectors by separating each vector into its horizontal and vertical components. Once you have these components, add all the horizontal components as one-dimensional vectors $\left(\Delta d_{1 x}+\Delta d_{2 x}+\Delta d_{3 x}+\cdots\right)$. Then do the same with the vertical displacement components $\left(\Delta d_{1 y}+\Delta d_{2 y}+\Delta d_{3 y}+\cdots\right)$. In both cases, you obtain a total component, $\Delta d_{x}$ and $\Delta d_{y}$, in each direction (Figure 8). These components represent the horizontal and vertical components of the total vector $\Delta \vec{d}_{\mathrm{T}}$.


Figure 8 To calculate the sum of two vectors, add their components. Here, the $x$-components of $\Delta \vec{d}_{1}$ and $\Delta \vec{d}_{2}$ are added to get the total $x$-component $\Delta d_{x}$. The same procedure with the $y$-components yields $\Delta d_{y}$.

The magnitude of $\Delta \vec{d}_{\mathrm{T}}$ is given by the Pythagorean theorem,

$$
\left|\Delta \vec{d}_{\mathrm{T}}\right|=\sqrt{\left(\Delta d_{x}\right)^{2}+\left(\Delta d_{y}\right)^{2}}
$$

and the angle between the total vector and the positive horizontal axis is given by

$$
\theta=\tan ^{-1}\left(\frac{\left|\Delta d_{y}\right|}{\left|\Delta d_{x}\right|}\right)
$$

In Tutorial 3, you will practise adding vectors algebraically.

## Tutorial 3 Adding Vectors Algebraically

This Tutorial models how to add vectors algebraically to determine the total displacement.

## Sample Problem 1: Determining Displacement by Adding Vectors Algebraically

An airplane flies 250 km [ $\mathrm{E} 25^{\circ} \mathrm{N}$ ], and then flies 280 km [S $13^{\circ} \mathrm{W}$ ]. Using components, calculate the airplane's total displacement.
Given: $\Delta \vec{d}_{1}=250 \mathrm{~km}\left[\mathrm{E} 25^{\circ} \mathrm{N}\right] ; \Delta \vec{d}_{2}=280 \mathrm{~km}\left[\mathrm{~S} 13^{\circ} \mathrm{W}\right]$ Required: $\Delta \vec{d}_{T}$

Analysis: Draw a simple scale vector diagram of each vector, and determine the corresponding components. Add the $x$-components and then the $y$-components to determine the total displacement in these directions. Use north and east as positive, and south and west as negative. Then use the Pythagorean theorem and the tangent ratio to determine the total displacement.

Solution: For the first vector:

$$
\begin{aligned}
& \xrightarrow[\Delta \Delta d_{1 x}=+\left|\Delta \overrightarrow{d_{1}}\right| \cos \theta]{\substack{\Delta \vec{d}_{1}}} \underset{\Delta}{\Delta \vec{d}_{1 y}}=+\left|\Delta d_{1}\right| \sin \theta \\
& \Delta d_{1 x}=\left|\Delta d_{1}\right| \cos \theta \\
& =(250 \mathrm{~km})\left(\cos 25^{\circ}\right) \\
& \Delta d_{1 x}=226.6 \mathrm{~km} \text { (two extra digits carried) } \\
& \Delta d_{1 y}=\left|\Delta \vec{d}_{1}\right| \sin \theta \\
& =(250 \mathrm{~km})\left(\sin 25^{\circ}\right) \\
& \Delta d_{1 y}=+105.7 \mathrm{~km} \text { (two extra digits carried) }
\end{aligned}
$$

For the second vector:


$$
\begin{aligned}
\Delta d_{2 x} & =-\left|\Delta \vec{d}_{2}\right| \sin \theta \\
& =-(280 \mathrm{~km})\left(\sin 13^{\circ}\right) \\
\Delta d_{2 x} & =-62.99 \mathrm{~km} \quad(\text { two extra digits carried }) \\
\Delta d_{2 y} & =-\left|\Delta \vec{d}_{2}\right| \cos \theta \\
& =-(280 \mathrm{~km})\left(\cos 13^{\circ}\right) \\
\Delta d_{2 y} & =-272.8 \mathrm{~km}(\text { two extra digits carried })
\end{aligned}
$$

Add the horizontal components:

$$
\begin{aligned}
\Delta d_{x} & =\Delta d_{1 x}+\Delta d_{2 x} \\
& =226.6 \mathrm{~km}+(-62.99 \mathrm{~km}) \\
\Delta d_{x} & =163.6 \mathrm{~km}[\mathrm{E}]
\end{aligned}
$$

Add the vertical components:

$$
\begin{aligned}
\Delta d_{y} & =\Delta d_{1 y}+\Delta d_{2 y} \\
& =105.7 \mathrm{~km}+(-272.8 \mathrm{~km})[\mathrm{N}] \\
& =-167.1 \mathrm{~km}[\mathrm{~N}] \\
\Delta d_{y} & =167 \mathrm{~km}[\mathrm{~S}]
\end{aligned}
$$

Combine the total displacement components to determine the total displacement.


Statement: The airplane's total displacement is $230 \mathrm{~km}\left[\mathrm{E} 46^{\circ} \mathrm{S}\right]$.

## Practice

1. An airplane flies 276.9 km [W $76.70^{\circ} \mathrm{S}$ ] from Edmonton to Calgary and then continues 675.1 km [W $11.45^{\circ} \mathrm{S}$ ] from Calgary to Vancouver. Using components, calculate the plane's total displacement. [TI AA [ans: $\left.830.0 \mathrm{~km}\left[\mathrm{~W} 29.09^{\circ} \mathrm{S}\right]\right]$
2. A person drives $120 \mathrm{~km}\left[\mathrm{~N} 32^{\circ} \mathrm{W}\right.$ ] in a car to a friend's place to visit and then drives 150 km [ $\mathrm{W} 24^{\circ} \mathrm{N}$ ] to visit family. Determine the total displacement of the trip. TTII [ans: $\left.260 \mathrm{~km}\left[\mathrm{~W} 39^{\circ} \mathrm{N}\right]\right]$
3. In a helicopter ride, the pilot first flies $12 \mathrm{~km}[\mathrm{~N}]$, then $14 \mathrm{~km}\left[\mathrm{~N} 22^{\circ} \mathrm{E}\right]$, and then $11 \mathrm{~km}[\mathrm{E}]$.

Determine the total displacement. [ans: $\left.3.0 \times 10^{1} \mathrm{~km}\left[\mathrm{E} 57^{\circ} \mathrm{N}\right]\right]$

When adding displacement vectors, you can choose the method that you find convenient. The scale diagram method ("tip-to-tail") gives you a visual sense of how the displacements relate, but it is not the most accurate way of determining the total displacement. The trigonometric method using the sine and cosine laws is accurate, but you can only combine two vectors at a time. The algebraic component method is less familiar but is accurate, and you can use it for any number of displacement vectors. In time, you will find it easier to use as well. For these reasons, it is the preferred approach in physics for determining the total displacement or adding a number of vectors of any type.

### 1.3 Review

## Summary

- To add displacement vectors using the scale diagram method, draw the vectors tip to tail and to the proper scale on a graph. Then determine the total displacement by measuring the length of the displacement vector and its angle. Finally, rescale the answer to the appropriate units.
- To calculate the sum of two vectors using the cosine and sine laws, first draw the two vectors tip to tail. Then apply the cosine and sine laws.
- Two-dimensional displacement vectors can be resolved into two perpendicular component vectors, one parallel to the $x$-axis and one parallel to the $y$-axis.
- You can use the component method to determine two perpendicular components using the sine and cosine ratios. The $x$-component of the total displacement equals the sum of the individual $x$-displacements. The $y$-component of the total displacement equals the sum of the individual $y$-displacements.
- Using the perpendicular vector component method, you can add two or more displacement vectors by separating each vector into its components and adding the components to obtain a total component in each direction. The total component represents a component of the total displacement vector $\Delta \vec{d}_{\mathrm{T}}$.


## Questions

1. Consider the displacements $\Delta \vec{d}_{1}=7.81 \mathrm{~km}\left[\mathrm{E} 50^{\circ} \mathrm{N}\right]$ and $\Delta \vec{d}_{2}=5.10 \mathrm{~km}\left[\mathrm{~W} 11^{\circ} \mathrm{N}\right]$. TWIC
(a) Draw a scale diagram of these two vectors, and determine the total displacement using the scale diagram method.
(b) Determine the total displacement mathematically using a different method.
(c) Compare the two answers by determining the percent difference.
2. Consider the following three vectors:
$\Delta \vec{d}_{1}=5.0 \mathrm{~cm}\left[\mathrm{E} 30.0^{\circ} \mathrm{N}\right], \Delta \vec{d}_{2}=7.5 \mathrm{~cm}[\mathrm{E}]$, and $\Delta \vec{d}_{3}=15.0 \mathrm{~cm}\left[\mathrm{E} 10.0^{\circ} \mathrm{S}\right]$. Add these vectors using the scale diagram method to determine the total displacement. 페
3. Determine the components of the vector $2.50 \mathrm{~m}\left[\mathrm{~N} 38.0^{\circ} \mathrm{W}\right]$. TTI
4. A student walks 25.0 m [E $30.0^{\circ} \mathrm{N}$ ]. Determine the components of the student's displacement. TIII
5. A vector has the following components: $\Delta d_{x}=54 \mathrm{~m}[\mathrm{E}]$ and $\Delta d_{y}=24 \mathrm{~m}[\mathrm{~N}]$.
(a) Calculate the length of the total displacement vector.
(b) Determine the vector's direction.
6. A driver travels 15.0 km to the west and then turns and drives 45.0 km to the south. Finally, she travels $32 \mathrm{~km}\left[\mathrm{~N} 25^{\circ} \mathrm{W}\right]$. Determine the driver's total displacement. TIT $A$
7. Using components, determine the total displacement from the following individual displacements: $\Delta \vec{d}_{1}=2.5 \mathrm{~m}\left[\mathrm{~W} 30.0^{\circ} \mathrm{S}\right]$, $\Delta \vec{d}_{2}=3.6 \mathrm{~m}[\mathrm{~S}]$, and $\Delta \vec{d}_{3}=4.9 \mathrm{~m}\left[\mathrm{E} 38.0^{\circ} \mathrm{S}\right]$. 지 |TTI
8. A boat travelling on the St. Lawrence River moves $2.70 \mathrm{~km}\left[\mathrm{E} 25.0^{\circ} \mathrm{N}\right]$ and then changes direction and moves 4.80 km [E $45.0^{\circ} \mathrm{S}$ ]. Determine (a) the components of the total displacement and (b) the total displacement of the boat. TII
9. An airplane flies 1512.0 km [W $19.30^{\circ} \mathrm{N}$ ] from Toronto to Winnipeg, continues $571.0 \mathrm{~km}\left[\mathrm{~W} 4.35^{\circ} \mathrm{N}\right.$ ] from Winnipeg to Regina, and then changes direction again and flies 253.1 km [W $39.39^{\circ} \mathrm{N}$ ] from Regina to Saskatoon. Determine the total displacement of the plane. ITI
10. A car takes a trip consisting of two displacements. The first displacement is $25 \mathrm{~km}[\mathrm{~N}]$, and the total displacement is 62 km [ $\mathrm{N} 38^{\circ} \mathrm{W}$ ]. Determine the second displacement. ITIIA
11. A plane flies 450 km [W] and then another 220 km in an unknown direction. Determine the maximum and minimum displacement of the plane. Explain your reasoning. KTU ITIIA
12. Figure 3 on page 23 shows, among other things, that vector addition is commutative. What does this mean? Make a similar diagram (or two separate diagrams) that shows that vector addition is commutative when using vector components. $\mathbb{K}|\mathbb{T I T}|$
