1.1 Motion and Motion Graphs

Cars drive by on the street, people walk and cycle past us, and garbage cans blow in a high wind. From quiet suburbs to busy highways, different kinds of motion happen all the time during a normal day (**Figure 1**). We often take this motion for granted, but we react to it instinctively: We dodge out of the way of objects flying or swerving toward us. We change our own motion to avoid hitting objects in our way or to get to school on time.

Figure 1 A highway is a good example of the physics of motion in action.

Kinematics is the study of motion without considering the forces that produce the motion. **Dynamics**, which is the topic of study in Chapters 2 and 3, is the study of the causes of motion. An understanding of kinematics and dynamics is essential in understanding motion.

kinematics the study of motion without considering the forces that produce the motion

dynamics the study of the causes of motion

Kinematics Terminology

In physics, the terms we use to describe motion—displacement, distance, speed, velocity, and acceleration—all have specific definitions and equations that connect them. We can divide the mathematical quantities we use to describe the motion of objects into two categories: scalars and vectors. **Scalars** are quantities that have only a magnitude, or numerical value. **Vectors** have both a magnitude and a direction.

scalar a quantity that has magnitude (size) but no direction

vector a quantity that has both magnitude (size) and direction

Position and Displacement

To start, consider motion along a straight line—one-dimensional motion—such as a hockey puck sliding on a horizontal icy surface. **Figure 2(a)** shows what a multiple-exposure image of a hockey puck in motion might look like. The dots represent evenly spaced intervals of time. The puck is moving the same distance in each interval, so the puck is moving at a constant speed. Here, we measure **position**—the distance and direction of an object from a reference point—as the distance from the origin on the horizontal axis to the centre of the hockey puck. For one-dimensional motion, distance specifies the position of the object. We can use the information in Figure 2(a) to construct a graph of the puck's position as a function of time, as shown in **Figure 2(b)**. Notice in Figure 2(b) that the distance axis is now vertical as we plot the position, $\vec{d}$, as a function of time, *t*. In such a position–time plot, it is conventional to plot time along the horizontal axis. The change in the puck's position, in one direction, is its **displacement**. Mathematically, for one-dimensional motion, displacement is written as

position ($\vec{d}$) the straight-line distance and direction of an object from a reference point

displacement ($\Delta\vec{d}$) the change in position of an object

$$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$$

where $\vec{d}_1$ is the object's initial position and $\vec{d}_2$ is the object's final position. While displacement tells us how far an object moves, it does not tell us how fast it moved.

Vectors provide two pieces of information, so we need a specific way of presenting this information clearly and unambiguously. For example, a displacement of 15 m [E] clearly identifies a magnitude of 15 and a direction of east.

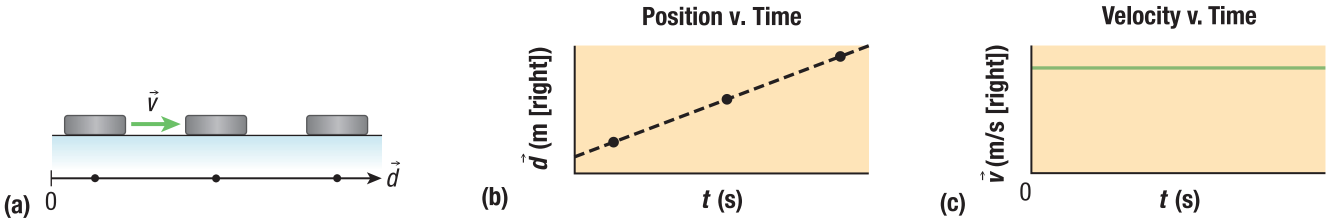


Figure 2 (a) If we took multiple exposures of a hockey puck travelling across an icy surface at a constant speed, the photo might look like this. (b) Each dot in the graph corresponds to a position of the puck in (a). (c) The velocity of the puck versus time is a straight line in this case.

Speed and Velocity

Speed and velocity are related quantities, but they are not the same. Many people use them interchangeably in everyday language, but strictly speaking this is incorrect in scientific terms. Speed tells how fast an object is moving, and speed is always a positive quantity or zero. The distance between adjacent dots in Figure 2(a) shows how far the puck has moved during each time interval. The **average speed**, v_{av}, of an object is the total distance travelled divided by the total time to travel that distance. Speed is a scalar quantity; it does not have a direction. The SI unit for speed is metres per second (m/s). You can calculate the average speed using the equation

average speed (v_{av}) the total distance travelled divided by the total time to travel that distance

$$v_{av} = \frac{\Delta d}{\Delta t}$$

Velocity, the change in position divided by the time interval, is a vector quantity, so it is written with the vector arrow, $\vec{v}$. The direction of $\vec{v}$ indicates the direction of the motion. In the puck example, the direction of the velocity is to the right. But it could have been negative (motion to the left, toward smaller or more negative values of $\vec{d}$) or even zero. So, velocity can be positive, negative, or zero.

velocity ($\vec{v}$) the change in position divided by the time interval

How does an object's velocity relate to its position? For a particular time interval that begins at time t_1 and ends at time t_2, the time interval is $\Delta t = t_2 - t_1$. You can then calculate the average velocity using the equation

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

The hockey puck in Figure 2(a) is sliding at a constant speed, so its velocity also has a constant value, as shown in the velocity–time graph in **Figure 2(c)**. In this case, the velocity is positive, which means that the direction of motion is toward increasing values of $\vec{d}$ (that is, toward the right).

When an object moves with a constant speed, the **average velocity**, $\vec{v}_{av}$—the displacement divided by the time interval for that change—is constant throughout the motion, and the position–time graph has a constant slope, as in Figure 2(b). For more general cases, the average velocity is the slope of the line segment that connects the positions at the beginning and end of the time interval, called the **secant**. This is illustrated in the hypothetical position–time graph in **Figure 3**. In Tutorial 1, you will solve two simple problems related to average velocity and average speed.

average velocity ($\vec{v}_{av}$) the displacement divided by the time interval for that change; the slope of a secant on a position–time graph

secant a straight line connecting two separate points on a curve

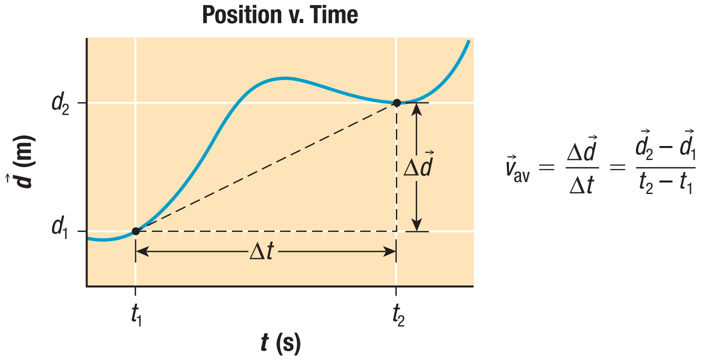


Figure 3 The average velocity during the time interval from t_1 to t_2 is the slope of the dashed line connecting the two corresponding points on the curve.

Tutorial 1 Distinguishing between Average Speed and Average Velocity

The following Sample Problem reviews how to calculate average speed and average velocity.

Sample Problem 1: Calculating Average Velocity and Average Speed

A jogger takes 25.1 s to run a total distance of 165 m by running 140 m [E] and then 25 m [W]. The displacement is 115 m [E].

(a) Calculate the jogger's average velocity.

(b) Calculate the jogger's average speed.

Solution

(a) **Given:** $\Delta\vec{d} = 115$ m [E]; $\Delta t = 25.1$ s

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$

Solution:
$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{115\text{ m [E]}}{25.1\text{ s}}$$
$$\vec{v}_{av} = 4.58\text{ m/s [E]}$$

Statement: The jogger's average velocity is 4.58 m/s [E].

(b) **Given:** $\Delta d = 165$ m; $\Delta t = 25.1$ s

Required: v_{av}

Analysis: $v_{av} = \dfrac{\Delta d}{\Delta t}$

Solution:
$$v_{av} = \frac{\Delta d}{\Delta t} = \frac{165\text{ m}}{25.1\text{ s}}$$
$$v_{av} = 6.57\text{ m/s}$$

Statement: The jogger's average speed is 6.57 m/s.

Practice

1. A woman leaves her house to walk her dog. They stop a few times along a straight path. They walk a distance of 1.2 km [E] from their house in 24 min. In another 24 min, they turn around and take the same path home. Give your answers to the following questions in kilometres per hour. T/I A
   (a) Determine the average speed of the woman and her dog for the entire route. [ans: 3.0 km/h]
   (b) Calculate the average velocity from their house to the farthest position from the house. [ans: 3.0 km/h [E]]
   (c) Calculate the average velocity for the entire route. [ans: 0.0 km/h]
   (d) Are your answers for (b) and (c) different? Explain why or why not.
2. A bus driver begins a descent down a steep hill and suddenly sees a deer about to cross the road. He applies the brakes. During the bus driver's 0.32 s reaction time, the bus maintains a constant velocity of 27 m/s [forward]. Determine the displacement of the bus during the time the driver takes to react. T/I [ans: 8.6 m [forward]]
3. Drivers at the Daytona 500 Speedway in Florida must complete 200 laps of a track that is 4.02 km long. Calculate the average speed, in kilometres per hour, of a driver who completes 200 laps in 6.69 h. T/I [ans: 1.20×10^2 km/h]
4. A student in a mall walks 140 m [E] in 55 s to go to his favourite store. The store is not open yet, so he walks 45 m [W] in 21 s to go to another store. Calculate his
   (a) average speed [ans: 2.4 m/s]
   (b) average velocity T/I [ans: 1.2 m/s [E]]
5. A delivery truck heads directly south for 62 km, stopping for an insignificant amount of time, and then travels 78 km directly north. The average speed for the entire trip is 55 km/h. K/U T/I C
   (a) Determine the average velocity for the entire trip in kilometres per hour. [ans: 6.3 km/h [N]]
   (b) Why is the average velocity of the truck so much smaller than the average speed?

Graphical Interpretation of Velocity

Consider another example of one-dimensional motion: a rocket-powered car travelling on a straight, flat road (**Figure 4(a)**). Assume the car is initially at rest; "initially" means that the car is not moving when the clock reads zero. At $t = 0$, the driver turns on the rocket engine and the car begins to move forward in a straight-line path. **Figure 4(b)** is a motion diagram showing the position of the car at evenly spaced instants in time. **Figure 4(c)** shows the corresponding position–time graph for the car, where again we use dots to mark the car's position at evenly spaced time intervals. Notice that the dots are not equally spaced along the position axis. Instead, their spacing increases as the car travels. This means that the car moves farther during each successive and equal time interval, so the velocity of the car increases with time.

In this example, the car moves toward increasing values of position, so the velocity is again positive and increases smoothly with time, as shown in **Figure 4(d)**.

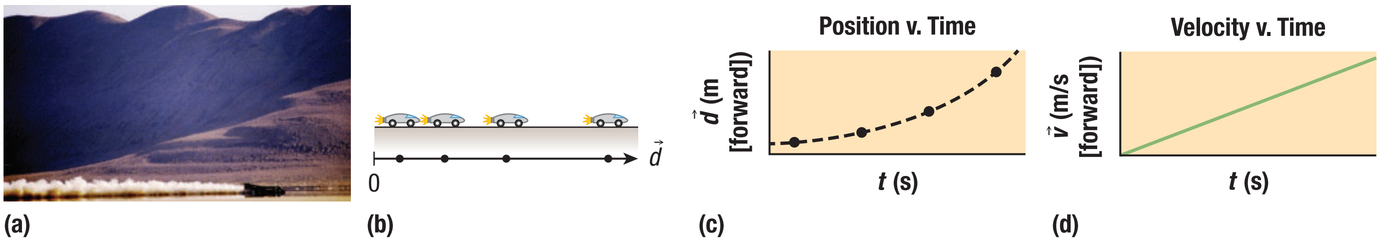


Figure 4 (a) In 1997, the rocket-powered Thrust SSC became the first car to break the sound barrier by travelling at a speed of over 1200 km/h. (b) A time-lapse image of a rocket-propelled car travelling along a horizontal road might look like this. (c) Position versus time for the rocket-powered car. (d) The velocity–time graph indicates that the car's velocity is not constant.

Instantaneous Velocity

Figure 5 is a position–time graph for an object moving a certain displacement over a short time period. Again, we use dots to mark the position at the beginning and end of a particular time interval, which starts at $t = 1.0$ s and ends at $t = 2.0$ s. The average velocity during this time interval is just the displacement during the interval divided by the length of the time interval. Figure 5(a) shows that this average velocity is the slope of the secant connecting the two points on the curve.

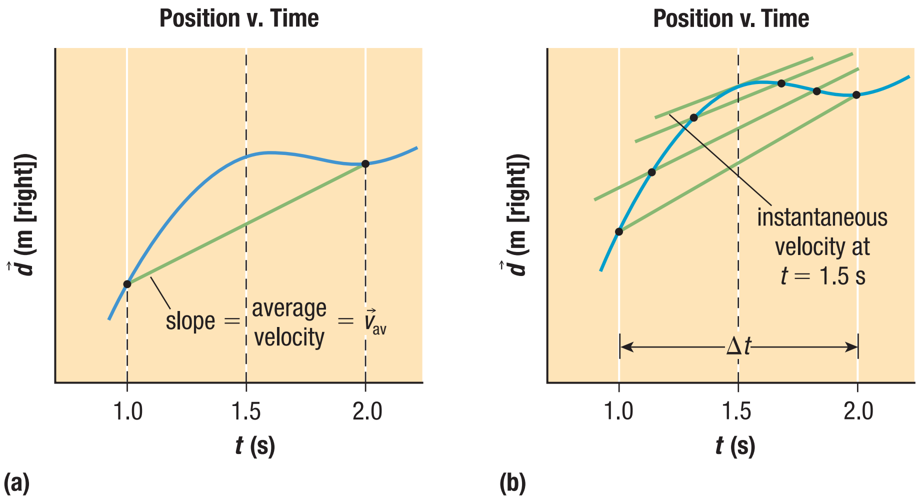


Figure 5 (a) The average velocity during a particular time interval is the slope of the line connecting the start of the interval to the end of the interval. (b) The instantaneous velocity at a particular time is the slope of the position–time curve at that time. The instantaneous velocity in the middle of a time interval is not necessarily equal to the average velocity during the interval.

With this approach, though, we lose the details about what happens in the middle of the interval. In Figure 5(a), the slope of the position–time curve varies considerably as we move through the interval from $t = 1.0$ s to $t = 2.0$ s. If we want to get a more accurate description of the object's motion at a particular instant within this time interval, say at $t = 1.5$ s, it is better to use a smaller interval. How small an interval should we use? In Figure 5(b), we consider slopes over a succession of smaller time intervals. Intuitively, we expect that using a smaller interval will give a better measure of the motion at a particular instant in time.

tangent a straight line that intersects a curve at a point and has the same slope as the curve at the point of intersection

instantaneous velocity ($\vec{v}$) the velocity of an object at a particular instant; the slope of the tangent to a position–time graph

instantaneous speed (v) the speed of an object at a particular instant; the magnitude of the slope of the tangent to a position–time graph

From Figure 5(b), we see that as we take smaller and smaller time intervals, we are actually approximating the slope of the position–time curve at the point of interest ($t = 1.5$ s) using a tangent. A **tangent** is a straight line that intersects a curve at a point and has the same slope as the curve at the point of intersection. The slope of a tangent to a position–time curve is called the **instantaneous velocity**, $\vec{v}$, which is the velocity of an object at a certain instant of time.

In some cases, a moving object changes its speed during its motion, so we need to clarify the difference between average speed and instantaneous speed: As mentioned earlier in this section, average speed is the total distance divided by the total time. **Instantaneous speed**, v, refers to the speed of an object at any given instant in time and is defined as the magnitude of the slope of the tangent to a position–time graph.

Moving objects do not always travel with changing speeds. Objects often move at a steady rate with a constant speed. The difference between the average and instantaneous values can be understood using the analogy of a car's speedometer. The speedometer reading gives your instantaneous speed, which is the magnitude of your instantaneous velocity at a particular moment in time. If you are taking a long drive, your average speed will generally be different because the average value will include periods when you are stopped in traffic, accelerating to pass other cars, and so on. In many cases, such as in discussions with a police officer, the instantaneous value will be of greater interest. CAREER LINK

The instantaneous velocity gives a mathematically precise measure of how the position is changing at a particular moment, making it much more useful than the average velocity. For this reason, from now on in this textbook we refer to the instantaneous velocity as simply the "velocity," and we denote it by $\vec{v}$.

In Tutorial 2, you will analyze a position–time graph, use the graph to determine average velocity, and then create a velocity–time graph.

Tutorial 2 Working with Motion Graphs

In the following Sample Problem, we will calculate average velocity from a position–time graph. Then, we will analyze a position–time graph and use the data to make a velocity–time graph.

Sample Problem 1: Calculating Average Velocity and Sketching a Velocity–Time Graph

The position–time graph in **Figure 6** shows the details of how an object moved.

(a) Calculate the average velocity during the time interval $t_1 = 1.0$ s to $t_2 = 2.5$ s.

(b) Analyze the position–time graph in Figure 6. Use your analysis to sketch a qualitative velocity–time graph of the object's motion.

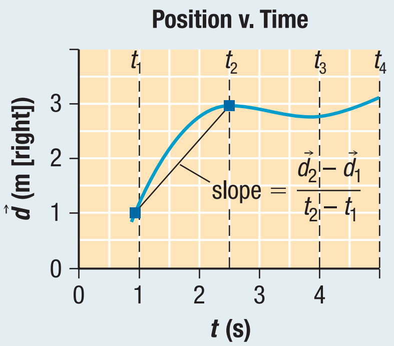


Figure 6

Solution

(a) **Given:** $t_1 = 1.0$ s; $t_2 = 2.5$ s

Required: $\vec{v}_{av}$ over the interval $t_1 = 1.0$ s to $t_2 = 2.5$ s

Analysis: To determine $\vec{v}_{av}$ between t_1 and t_2, calculate the slope of the line between t_1 and t_2.

Solution: Reading the values from the graph,

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$
$$= \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$
$$= \frac{3.0 \text{ m [right]} - 1.0 \text{ m [right]}}{2.5 \text{ s} - 1.0 \text{ s}}$$
$$\vec{v}_{av} = 1.3 \text{ m/s [right]}$$

Statement: The average velocity during the time interval $t_1 = 1.0$ s to $t_2 = 2.5$ s is 1.3 m/s [right].

(b) **Step 1.** To generate the data for the velocity–time graph, analyze the motion of the object in Figure 6.

This object is initially moving to the right. The object reverses direction near 2.5 s (t_2) and 4.0 s (t_3).

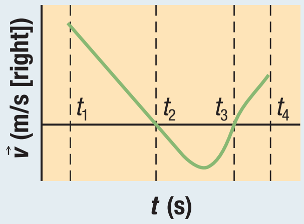
At t_1, the slope is large and positive, so v is large and positive at t_1.

At t_2, the slope is approximately zero, so v is near zero.

Between t_2 and t_3, the object is moving toward smaller values of position, so the slope of the position–time curve and, hence, the velocity, are negative. At t_3, the slope is zero, so the velocity is zero.

Finally, at t_4, the object is again moving to the right because d is increasing with time. So, v is again positive.

Step 2. After estimating the position–time slope at these places, we can construct a qualitative velocity–time graph.



Practice

1. Examine the position–time graphs in **Figure 7**. K/U A
   (a) In which graph(s) does the velocity increase with time? [ans: (c)]
   (b) In which graph(s) does the velocity decrease with time? [ans: (b)]

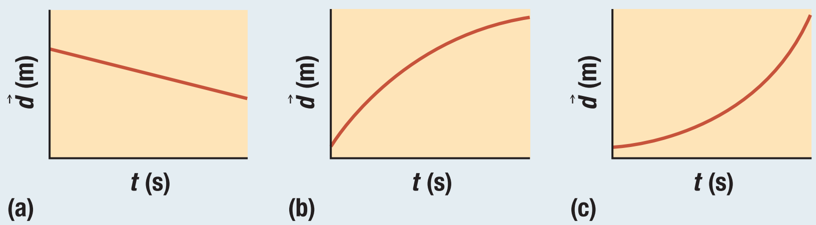


Figure 7

2. Analyze the graphs in **Figure 8**. Create a corresponding velocity–time graph for each graph. T/I C

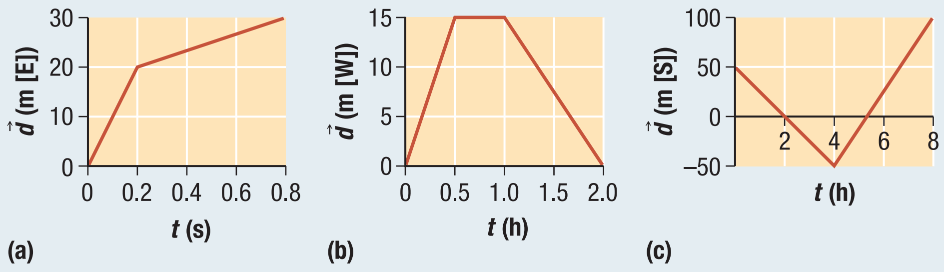


Figure 8

The average velocity is the slope of a secant on a position–time graph. To analyze the velocity, we take approximate values by drawing lines tangent to the position–time curve at several places and calculating their slopes. The instantaneous velocity at a particular time is always equal to the slope of the position–time curve at that time.

Acceleration

Acceleration is a measure of how velocity changes with time. The SI unit for acceleration is metres per second squared (m/s^2). Sometimes objects move at constant velocity, but usually the velocities we observe are changing. When an object's velocity is changing, that object is accelerating.

We can study acceleration using velocity–time graphs. These graphs display time values on the horizontal axis and velocity on the vertical axis. Velocity–time graphs can be useful when studying objects moving with uniform (constant) velocity (zero acceleration) or uniform acceleration (velocity changing, but at a constant rate). The velocity–time graphs for both uniform velocity and uniform acceleration are always straight lines. By contrast, the position–time graph of an accelerated motion is curved.

average acceleration ($\vec{a}_{av}$) the change in velocity divided by the time interval for that change

When an object's velocity changes by $\Delta\vec{v}$ over time Δt, the **average acceleration**, $\vec{a}_{av}$, or the change in velocity divided by the time interval for that change, during this interval is

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

instantaneous acceleration ($\vec{a}$) the acceleration at a particular instant in time

As with velocity, we often need to know the acceleration at a particular instant in time, or the **instantaneous acceleration**, $\vec{a}$. The instantaneous acceleration equals the slope of the velocity–time graph at a particular instant in time. If the velocity–time graph is straight during a time interval, then the acceleration is constant. This means that the instantaneous acceleration is equal to the average acceleration, and we can omit the subscript "av" in the equation above. You can now apply these concepts by completing Tutorial 3.

Tutorial 3 Working with Motion Graphs

In the following Sample Problem, we will create an acceleration–time graph from a velocity–time graph and analyze the graph to determine the maximum acceleration.

Sample Problem 1: Creating an Acceleration–Time Graph and Calculating the Maximum Acceleration

Suppose the sprinter in **Figure 9(a)** is running a 100 m dash. The sprinter's time and distance data have been recorded and used to make the velocity–time graph in **Figure 9(b)**.

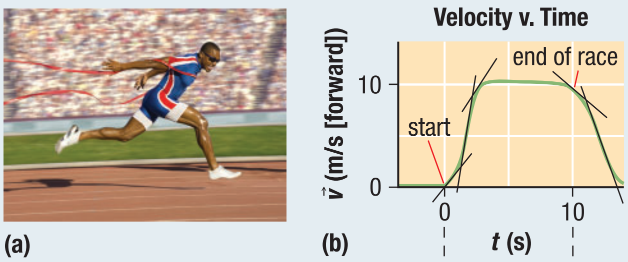


Figure 9

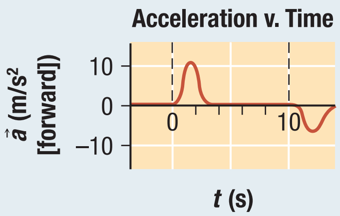
(a) Analyze the velocity–time graph in Figure 9(b). Use your analysis to make a sketch of an acceleration–time graph of the sprinter's motion.

(b) From your acceleration–time graph, determine the maximum acceleration and the time at which it occurs.

Solution

(a) **Step 1.** Acceleration is the slope of the velocity–time graph, so first we must estimate this slope at several different values of *t* to be able to graph the sprinter's acceleration as a function of time. Figure 9(b) shows several lines tangent to the velocity–time curve at various times. The slopes of these tangent lines give the acceleration. Estimate the slopes.

Step 2. Use the slope estimations to sketch the acceleration–time graph. The resulting acceleration–time graph is only qualitative (approximate). More accurate results would be possible if we had started with a more detailed graph of the velocity.



(b) The largest value of the acceleration is approximately 11 m/s^2. This occurs near the start of the race, around $t = 1.5$ s, when the sprinter is gaining speed. Note that this is where the slope of the velocity–time graph is largest. At the end of the race, as the runner crosses the finish line, he slows down and eventually comes to a stop with $v = 0$.

Practice

1. **Figure 10** shows a graph of the motion of a car along a straight road. K/U T/I C

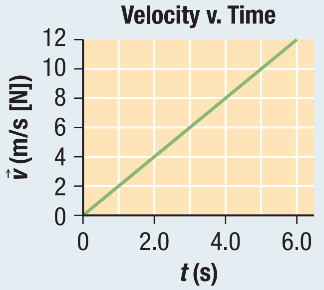


Figure 10

   (a) How can you tell from the graph that the car has a constant acceleration?
   (b) Describe the motion of the car.
   (c) Determine the acceleration of the car. [ans: 2.0 m/s^2 [N]]

2. Examine the velocity–time graph in **Figure 11**. K/U C A

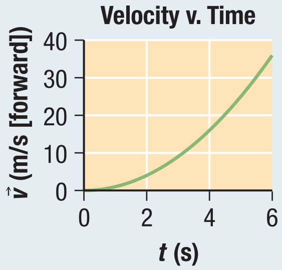


Figure 11

   (a) Determine the average acceleration for the entire trip. [ans: 6 m/s^2 [forward]]
   (b) Determine the instantaneous acceleration at 3 s and at 5 s. [ans: 6 m/s^2 [forward]; 10 m/s^2 [forward]]
   (c) Draw a reasonable acceleration–time graph of the motion.

3. Examine the velocity–time graph in **Figure 12**. T/I C

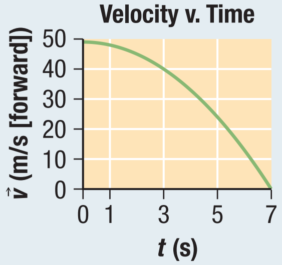


Figure 12

   (a) Determine the average acceleration for the entire trip. [ans: 7 m/s^2 [backward]]
   (b) Determine the instantaneous acceleration at 2 s, 4 s, and 6 s. [ans: 4 m/s^2 [backward]; 8 m/s^2 [backward]; 12 m/s^2 [backward]]
   (c) Draw a reasonable acceleration–time graph of the motion.

Acceleration is the slope of a velocity–time graph. Therefore, given a velocity–time graph, we can describe the behaviour of an object's acceleration. An interesting feature of the graphs in Sample Problem 1, on page 14, is that the maximum velocity and the maximum acceleration do not occur at the same time. It is tempting to think that if the "motion" is large, both v and a will be large, but this notion is incorrect. Acceleration is the slope—the rate of change—of the velocity versus time. The time at which the rate of change in velocity is greatest may not be the time at which the velocity itself is greatest.

1.1 Review

Summary

- The equation for average speed is $v_{av} = \frac{\Delta d}{\Delta t}$, and the equation for average velocity is $\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$. The slope of an object's position–time graph gives the velocity of the object.
- Acceleration describes how quickly an object's velocity changes over time. The equation for average acceleration is $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$. The slope of an object's velocity–time graph gives the object's acceleration.

Questions

1. A cardinal flies east for 2.9 s in a horizontal plane for a distance of 22 m from a fence post to a bush. It then flies north another 11 m to a bird feeder for 1.5 s. T/I
   (a) Calculate the total distance travelled.
   (b) Calculate the cardinal's average speed.
   (c) Calculate the cardinal's average velocity.
2. At a sled race practice field in North Bay, Ontario, a dogsled team covers a single-lap distance of 2.90 km at an average speed of 15.0 km/h. T/I
   (a) Calculate the average speed in metres per second.
   (b) Calculate the time, in seconds, needed to complete the lap.
3. A skater travels straight across a circular pond with a diameter of 16 m. It takes her 2.1 s. T/I
   (a) Determine the skater's average speed.
   (b) How long would it take the skater to skate around the edge of the pond at the same average speed?
4. An airplane flies 450 km at a compass heading of 85° for 45 min. T/I
   (a) Calculate the airplane's average speed.
   (b) Calculate the airplane's average velocity.
5. A model rocket accelerates from rest to 96 km/h [W] in 4.1 s. Determine the average acceleration of the rocket. T/I
6. A batter hits a baseball in a batting-practice cage. The ball undergoes an average acceleration of 1.37×10^3 m/s^2 [W] in 3.12×10^{-2} s before it hits the cage wall. Calculate the velocity of the baseball when it hits the wall. T/I A
7. A track runner begins running at the starting whistle and reaches a velocity of 9.3 m/s [forward] in 3.9 s. Calculate the runner's average acceleration. T/I
8. The position–time graph in **Figure 13** represents the motion of a race car moving along a straight road. K/U T/I C

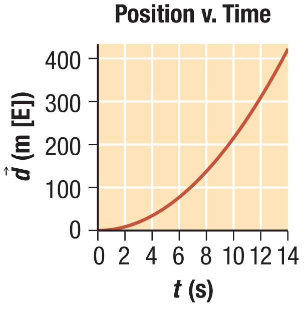


Figure 13

   (a) Determine the average velocity for the entire trip.
   (b) Determine the average velocity for the last 10 s of the motion. Why are the two average velocities different?
   (c) Determine the instantaneous velocity at 4.0 s, 8.0 s, and 12.0 s.
   (d) Sketch a qualitative velocity–time graph for the motion of the car.
9. Study the graph in **Figure 14**. K/U T/I C

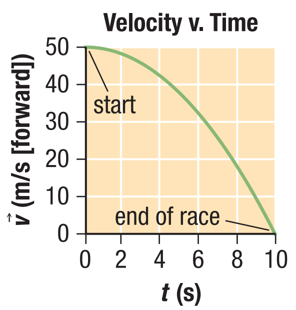


Figure 14

   (a) Determine the average acceleration for the entire trip.
   (b) Determine the instantaneous acceleration at 3.0 s, 6.0 s, and 9.0 s.
   (c) Sketch a qualitative acceleration–time graph of the motion.