Section 7.4: Nuclear Fission and Nuclear Power Generation

Tutorial 1 Practice, page 336

1. (a) Given: m = 4.002 613 u;
m_p = 1.007 276 u; m_n = 1.008 665 u;
m_e = 0.000 549 u
Required: mass defect
Analysis: Δm = atomic mass – actual atomic mass; Determine the combined mass of protons, neutrons, and electrons, and then determine the mass defect.
Solution:

$$2m_p + 2m_n + 2m_e = 2(1.007\ 276\ \text{u}) + 2(1.008\ 665\ \text{u}) + 2(0.000\ 549\ \text{u})$$
$$= 4.032\ 980\ \text{u}$$
$$\Delta m = 4.032\ 980\ \text{u} - 4.002\ 613\ \text{u}$$
$$\Delta m = 0.030\ 367\ \text{u}$$

Statement: The mass defect of helium-4 is 0.030 367 u.

(b) Note: After the first printing, a note was added to this question asking students to give their answer in MeV. The correct answer is still 28 MeV.
Given: m = 4.002 613 u; $c = 3.0 \times 10^8$ m/s
Required: E
Analysis: $E = \Delta mc^2$
Solution:

$$\Delta m = (0.030\ 367\ \cancel{\text{u}})\left(1.66 \times 10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$= 5.040\ 922 \times 10^{-29}\ \text{kg (two extra digits carried)}$$

$$E = \Delta mc^2$$
$$= \left(5.0409 \times 10^{-29}\ \text{kg}\right)\left(3.0 \times 10^8\ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 4.536\ 81 \times 10^{-12}\ \text{J}$$
$$= \left(\frac{4.536\ 81 \times 10^{-12}\ \cancel{\text{J}}}{1.602\ \times 10^{-13}\ \frac{\cancel{\text{J}}}{\text{MeV}}}\right)$$
$$= 28.3197\ \text{MeV}$$
$$E = 28\ \text{MeV}$$

Statement: The binding energy of helium-4 is 28 MeV.

Tutorial 2 Practice, page 338

1. Given: $m_{\text{U-235}}$ = 235.044 u; m_n = 1.009 u; $m_{\text{Zr-94}}$ = 93.906 u; $m_{\text{Te-139}}$ = 138.935 u; $c = 3.0 \times 10^8$ m/s
Required: energy released
Analysis: $E = \Delta mc^2$
Solution:

$$\Delta m = m_{\text{U-235}} + \cancel{m_n} - \left(m_{\text{Zr-94}} + m_{\text{Te-139}} + \overset{2}{\cancel{3}}\, m_n\right)$$
$$= 235.044\ \text{u} - [93.906\ \text{u} + 138.935\ \text{u} + 2(1.009\ \text{u})]$$
$$\Delta m = 0.185\ \text{u}$$

$$\Delta m = (0.185\ \cancel{\text{u}})\left(1.66 \times 10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$\Delta m = 3.071 \times 10^{-28}\ \text{kg (two extra digits carried)}$$

$$E = \Delta mc^2$$
$$= \left(3.071 \times 10^{-28}\ \text{kg}\right)\left(3.0 \times 10^8\ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 2.764 \times 10^{-11}\ \text{J}$$
$$E = 2.8 \times 10^{-11}\ \text{J}$$

Statement: The nuclear fission reaction releases 2.8×10^{-11} J of energy per reaction.

Research This: Breeder Reactors, page 340

A. A breeding chain is a set of successive, nuclear reactions (fission) that transforms one radioactive isotope into another, then another, until the desired product is obtained.

B. Answer may vary. Sample answer:
uranium-238 → uranium-239 → neptunium-239 → plutonium-239

C. ${}^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{239}_{92}\text{U}$, ${}^{239}_{92}\text{U} \rightarrow {}^{239}_{93}\text{Np} + {}^{0}_{-1}\text{e}$,
${}^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + {}^{0}_{-1}\text{e}$

The decay of uranium to neptunium and the decay of neptunium to plutonium represent transmutations because the starting element decays by emitting an alpha particle to become a different element.

D. Plutonium-239 is commonly produced in a breeder reactor through decay of uranium-238. Uranium-233 is produced through the decay of thorium-232.

Section 7.4 Questions, page 341

1. (a) Given: $m_e = 9.109\ 56 \times 10^{-31}$ kg; $c = 3.0 \times 10^8$ m/s
Required: energy equivalent in joules
Analysis: $E = mc^2$
Solution:

$$E = mc^2$$
$$= \left(9.109\ 56 \times 10^{-31}\text{ kg}\right)\left(3.0\times10^8\text{ m/s}\right)^2$$
$$= 8.1886\times10^{-14}\text{ J (three extra digits carried)}$$
$$E = 8.2\times10^{-14}\text{ J}$$

Statement: The energy equivalent, in joules, of an electron is 8.2×10^{-14} J.

(b) Given: $m_p = 1.672\ 614 \times 10^{-27}$ kg; $c = 3.0 \times 10^8$ m/s
Required: energy equivalent in joules
Analysis: $E = mc^2$
Solution:

$$E = mc^2$$
$$= \left(1.672\ 614\times10^{-27}\text{ kg}\right)\left(3.0\times10^8\text{ m/s}\right)^2$$
$$= 1.5053\times10^{-10}\text{ J}$$
$$E = 1.5\times10^{-10}\text{ J}$$

Statement: The energy equivalent, in joules, of a proton is 1.5×10^{-10} J.

2. Given: $E = 4.5 \times 10^{14}$ J; $c = 3.0 \times 10^8$ m/s
Required: m
Analysis:

$$E = mc^2$$
$$m = \frac{E}{c^2}$$

Solution:

$$m = \frac{E}{c^2}$$
$$= \frac{4.5\times10^{14}\text{ J}}{\left(3.0\times10^8\text{ m/s}\right)^2}$$
$$m = 0.50\times10^{-2}\text{ kg}$$

Statement: The mass of the original coal was 0.50×10^{-2} kg, or 5.0 g.

3. Given: $m_{\text{U-236}} = 236.045\ 562$ u; $m_{\text{Th-232}} = 232.038\ 051$ u; $m_{\text{He-4}} = 4.003\ 603$ u; $c = 3.0 \times 10^8$ m/s
Required: energy released
Analysis: $E = \Delta mc^2$
Solution:

$$\Delta m = m_{\text{U+236}} - \left(m_{\text{Th-232}} + m_{\text{He-4}}\right)$$
$$= 236.045\ 562\text{ u} - (232.038\ 051\text{ u} + 4.003\ 603\text{ u})$$
$$\Delta m = 0.003\ 900\text{ u}$$

$$\Delta m = (0.003\ 900\ \cancel{\text{u}})\left(1.66\times10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$\Delta m = 6.474\times10^{-30}\text{ kg (two extra digits carried)}$$

$$E = \Delta mc^2$$
$$= \left(6.474\times10^{-30}\text{ kg}\right)\left(3.0\times10^8\text{ m/s}\right)^2$$
$$= 5.826\times10^{-13}\text{ J}$$
$$E = 5.8\times10^{-13}\text{ J}$$

Statement: The energy released is 5.8×10^{-13} J.

4. Given: $m_{\text{U-235}} = 235.044$ u; $m_{\text{Sr-90}} = 89.908$ u; $m_{\text{Xe-135}} = 134.879$ u; $m_n = 1.008\ 665$ u; $c = 3.0 \times 10^8$ m/s
Required: energy released
Analysis: $E = \Delta mc^2$
Solution:

$$\Delta m = m_{\text{U-235}} + \cancel{m_n} - \left(m_{\text{Sr-90}} + m_{\text{Xe-135}} + \overset{10}{\cancel{11}}\, m_n\right)$$
$$= 235.044\text{ u} - [89.908\text{ u} + 134.879\text{ u} + 10(1.008\ 665\text{ u})]$$
$$\Delta m = 0.170\ 35\text{ u (three extra digits carried)}$$

$$\Delta m = (0.170\ 35\ \cancel{\text{u}})\left(1.66\times10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$\Delta m = 2.8278\times10^{-28}\text{ kg (three extra digits carried)}$$

$$E = \Delta mc^2$$
$$= \left(2.8278\times10^{-28}\text{ kg}\right)\left(3.0\times10^8\text{ m/s}\right)^2$$
$$= 2.545\times10^{-11}\text{ J}$$
$$E = 2.5\times10^{-11}\text{ J}$$

Statement: The energy released is 2.5×10^{-11} J.

5. (a) Stage 1: Two protons and two neutrons are lost.

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

Stage 2: One proton is gained. A neutron must have decayed into one proton and one electron.

$${}^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Pa} + {}^{0}_{-1}\text{e}$$

Stage 3: One proton is gained.

$${}^{234}_{91}\text{Pa} \rightarrow {}^{234}_{92}\text{U} + {}^{0}_{-1}\text{e}$$

Stage 4: Two protons and two neutrons are lost.
$^{234}_{92}U \rightarrow {}^{230}_{90}Th + {}^{4}_{2}He$

Stage 5: Two protons and two neutrons are lost.
$^{2320}_{90}Th \rightarrow {}^{226}_{88}Ra + {}^{4}_{2}He$

(b) In stages 1, 4, and 5, two protons and two neutrons are lost and a helium nucleus is formed. This is alpha decay. In stages 2 and 3, one proton is gained and one electron is emitted. This is beta-negative decay.

6. (a) After the reactions identified in question 5, these reactions are involved in the uranium-lead series:
$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$, $^{222}_{86}Rn \rightarrow {}^{218}_{84}Po + {}^{4}_{2}He$,
$^{218}_{84}Po \rightarrow {}^{214}_{82}Pb + {}^{4}_{2}He$, $^{214}_{82}Pb \rightarrow {}^{214}_{83}Bi + {}^{0}_{-1}e$,
$^{214}_{83}Bi \rightarrow {}^{214}_{84}Po + {}^{0}_{-1}e$, $^{214}_{84}Po \rightarrow {}^{210}_{82}Pb + {}^{4}_{2}He$,
$^{210}_{82}Pb \rightarrow {}^{210}_{83}Bi + {}^{0}_{-1}e$, $^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + {}^{0}_{-1}e$,
$^{210}_{84}Po \rightarrow {}^{206}_{84}Pb + {}^{0}_{-1}He$

(b) The final stable isotope is lead-206.

7. Answers may vary. Answers should include information such as the following: Fission reactors have the potential to expose workers and others to unacceptable levels of radiation. If they malfunction, and safety controls do not work, they can expose large numbers of people, plants, and animals to significant doses of radiation. Fission reactors create nuclear waste that is a potential source of unacceptable radiation and must be stored securely until it no longer emits radiation at significant levels.

8. In CANDU nuclear reactors, the control rods are suspended over the calandria and held by electromagnets. If power is lost, the electromagnetic field is eliminated and the rods descend into the calandria, stopping the nuclear reaction.

9. For $^{239}_{92}U \rightarrow {}^{239}_{93}Np$, one proton is gained, so this is beta-negative decay. The reaction equation is
$^{239}_{92}U \rightarrow {}^{239}_{93}Np + {}^{0}_{-1}e$.

For $^{239}_{93}Np \rightarrow {}^{239}_{94}Pu$, one proton is gained, so this is also beta-negative decay. The reaction equation is
$^{239}_{93}Np \rightarrow {}^{239}_{94}Pu + {}^{0}_{-1}e$.

10. Stage 1: The chemical symbol for thorium-233 is $^{233}_{90}Th$. When the isotope absorbs a neutron, it will become $^{234}_{90}Th$. The equation is
$^{233}_{90}Th + {}^{1}_{0}n \rightarrow {}^{234}_{90}Th$.

Stage 2: In beta-negative decay, one neutron decays into one proton and one electron. The element with one more proton than thorium is protactinium, Pa. The equation is
$^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e$.

Stage 3: Beta-negative decay produces the element with one more proton than protactinium, which is uranium, U. The equation is $^{234}_{91}Pa \rightarrow {}^{234}_{92}U + {}^{0}_{-1}e$.