Section 7.3: Half-Life

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A. This is beta-negative decay. Nitrogen (the daughter atom) has one more proton than carbon (the parent atom). A neutron has changed into a proton and an electron.

B. The nuclear reaction equation is

$${}^{15}_{6}\mathrm{C} \rightarrow {}^{15}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e}$$

C. The mass of carbon-15 is decreasing exponentially, while the mass of nitrogen-15 is increasing exponentially. Since carbon-15 is decaying and becoming nitrogen-15, it makes sense that these graphs are the inverse of each other.

D. The point of intersection is the point at which half the carbon-15 has decayed into nitrogen-15. The *x*-coordinate of this point represents the half-life of carbon-15. The *y*-coordinate of this point represents half the mass of the original carbon-15.

Tutorial 1 Practice, page 332

1. (a) Given: h = 3.6 s; t = 10 s

Required: percent of initial sample remaining

Analysis:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$\frac{A}{A_0}\times 100 = \text{percent remaining}$$

Solution:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$= \left(\frac{1}{2}\right)^{\frac{10\,\cancel{s}}{3.6\,\cancel{s}}}$$

$$\frac{A}{A_0} = 0.1458$$

$$\frac{A}{A_0}\times 100 = 15\ \%$$

Statement: There would be 15 % of the sample remaining after 10 s.

(b) Given: h = 3.6 s; t = 10 min = 600 s

Required: percent of initial sample remaining

Analysis:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$\frac{A}{A_0}\times 100 = \text{percent remaining}$$

Solution:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$= \left(\frac{1}{2}\right)^{\frac{600\,\cancel{s}}{3.6\,\cancel{s}}}$$

$$\frac{A}{A_0} = 6.734\times 10^{-51}$$

$$\frac{A}{A_0}\times 100 = 6.7\times 10^{-49}\ \%$$

Statement: There would be 6.7×10^{-49} % of the sample remaining after 10 min.

2. Given: t = 10 years; A_0 = 100 mg; A = 81 mg

Required: h

Analysis:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Solution:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$\frac{81\ \cancel{\text{mg}}}{100\ \cancel{\text{mg}}} = \left(\frac{1}{2}\right)^{\frac{10\text{ years}}{h}}$$

$$0.81 = \left(\frac{1}{2}\right)^{\frac{10\text{ years}}{h}}$$

Use a table to estimate the value of the exponent.

Exponent	Final mass
1	$\left(\frac{1}{2}\right)^{1} = 0.5$
0.5	$\left(\frac{1}{2}\right)^{0.5} = 0.707...$
0.3	$\left(\frac{1}{2}\right)^{0.3} \doteq 0.812...$
0.303	$\left(\frac{1}{2}\right)^{0.303} \doteq 0.810...$

Solve for h.

$$\left(\frac{1}{2}\right)^{\frac{10}{h}} = \left(\frac{1}{2}\right)^{0.303}$$

$$\frac{10}{h} = 0.303$$

$$h = \frac{10}{0.303}$$

$$h = 33$$

Statement: The half-life of argon-42 is approximately 33 years.

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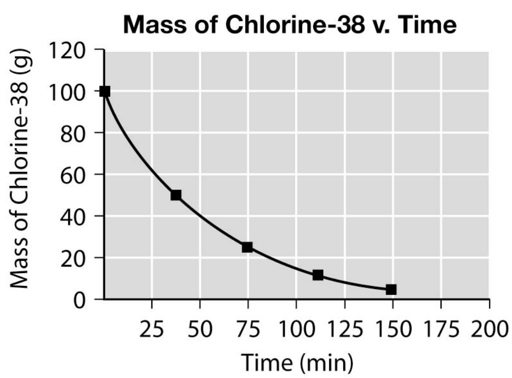
1. (a)

Time (min)	Mass remaining (g)
0	100
37.24	50
74.48	25
111.72	12.5
148.96	6.25

(b)



(c) The atomic number of chlorine is 17.

$${}^{38}_{17}\text{Cl} \rightarrow {}^{38}_{18}\text{Ar} + {}^{0}_{-1}\text{e}$$

Chlorine-38 will decay into argon-38.

2. (a)

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{2.6\text{ day}}}$$

(b) (i) Given: h = 2.6 days, t = 1 day

Required: percent of sample remaining

Analysis:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{2.6\text{ day}}}$$

$$\frac{A}{A_0} \times 100 = \text{percent remaining}$$

Solution:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{2.6\text{ day}}}$$

$$= \left(\frac{1}{2}\right)^{\frac{1\ \cancel{\text{day}}}{2.6\ \cancel{\text{day}}}}$$

$$\frac{A}{A_0} = 0.7398$$

$$\frac{A}{A_0} \times 100 = 74\ \%$$

Statement: There would be 74 % of the sample remaining after 1 day.

(ii) Given: h = 2.6 days, t = 1 week = 7 days

Required: percent of sample remaining

Analysis:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{2.6\text{ day}}}$$

$$\frac{A}{A_0} \times 100 = \text{percent remaining}$$

Solution:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{2.6\text{ day}}}$$

$$= \left(\frac{1}{2}\right)^{\frac{7\ \cancel{\text{day}}}{2.6\ \cancel{\text{day}}}}$$

$$\frac{A}{A_0} = 0.1547$$

$$\frac{A}{A_0} \times 100 = 15\ \%$$

Statement: There would be 15 % of the sample remaining after 1 week.

3. (a) Given: A_0 = 50 g; h = 5.3 years; t = 6 months = 0.5 years

Required: A_1; A_2

Analysis:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Solution:

$$A_1 = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}},$$

$$= 50\text{ g}\left(\frac{1}{2}\right)^{\frac{0.5\ \cancel{\text{years}}}{5.3\ \cancel{\text{years}}}}$$

$$= 46.83\text{ g}$$

$$A_1 = 47\text{ g}$$

Statement: There would be 47 g of the sample remaining after 6 months.

(b) Given: $A_0 = 50$ g; $h = 5.3$ years; $t = 5$ years
Required: A; A_2

Analysis: $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

Solution:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$= 50\ \text{g}\left(\frac{1}{2}\right)^{\frac{5\ \cancel{\text{years}}}{5.3\ \cancel{\text{years}}}}$$

$$A = 26.00\ \text{g}$$

Statement: There would be 26 g of the sample remaining after 5 years.

4. Beta-negative decay is involved in carbon dating. One neutron decays into one proton and one electron. The equation is as follows:

$${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}$$

The ratio of carbon-14 to the more stable carbon-12 is relatively constant in living things, and the half-life of carbon-14 is 5730 years. This means that scientists can measure the ratio of carbon-14 to carbon-12 in a fossil, then compare the value to the expected ratios in fossils of various ages, and determine how long ago the living creature died.

5. Given: $\frac{A}{A_0} \times 100 = 70\ \%$; $h = 5730$ years

Required: t

Analysis:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Solution:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$70\ \% = \left(\frac{1}{2}\right)^{\frac{t}{5730\ \text{years}}}$$

$$0.70 = \left(\frac{1}{2}\right)^{\frac{t}{5730\ \text{years}}}$$

Use a table to estimate the value of the exponent.

Exponent	Final mass
1	$\left(\frac{1}{2}\right)^{1} = 0.5$
0.5	$\left(\frac{1}{2}\right)^{0.5} = 0.707...$
0.514	$\left(\frac{1}{2}\right)^{0.514} \doteq 0.700...$

Solve for t.

$$\left(\frac{1}{2}\right)^{0.514} = \left(\frac{1}{2}\right)^{\frac{t}{5730\ \text{years}}}$$

$$0.514 = \frac{t}{5730\ \text{years}}$$

$$t = (0.514)(5730\ \text{years})$$

$$= 2945.22\ \text{years (three extra digits carried)}$$

$$t = 2950\ \text{years}$$

Statement: The creature died approximately 2950 years ago.

6. (a) Magnesium has one less proton than aluminum. In beta-positive decay, one proton decays into one neutron and one positron. So, aluminum-26 undergoes beta-positive decay.

(b) No, aluminum-26 does not decay in the same way as carbon-14. Carbon-14 undergoes beta-negative decay, which is different from the beta-positive decay undergone by aluminum-26. In beta-negative decay, one neutron decays into one proton and one electron. In beta-positive decay, one proton decays into one neutron and one positron.

7. (a) Given: $\frac{A}{A_0} = 3$;; $h = 720\ 000$ years

Required: t

Analysis: $\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$

Solution:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$3 = \left(\frac{1}{2}\right)^{\frac{t}{h}} \times 100$$

$$\frac{3}{100} = \left(\frac{1}{2}\right)^{\frac{t}{720\ 000\ \text{years}}}$$

$$0.03 = \left(\frac{1}{2}\right)^{\frac{t}{720\ 000\ \text{years}}}$$

Use a table to estimate the value of the exponent.

Exponent	Final mass
1	$\left(\frac{1}{2}\right)^{1} = 0.5$
5	$\left(\frac{1}{2}\right)^{0.51} \doteq 0.702...$
5.03	$\left(\frac{1}{2}\right)^{5.05} = 0.03...$

Solve for t.

$$\left(\frac{1}{2}\right)^{\frac{t}{720\ 000\ \text{years}}} = \left(\frac{1}{2}\right)^{5.05}$$

$$\frac{t}{720\ 000\ \text{years}} = 5.05$$

$$t = 3\ 636\ 000\ \text{years}$$

$$t = 3\ 600\ 000\ \text{years}$$

Statement: The moon rock is 3 600 000 years old.

(b) Answers may vary: Assumptions may include that the rate of aluminum-26 decay has been constant since the moon rock was formed, and that no new aluminum-26 has formed in the rock since it was originally made.

8. Each fold in this model can be used to represent one half-life. The area of the paper after each fold can be used to represent the amount of mass remaining. The area will decrease by half with each fold, just as the amount of mass remaining will decrease by half with each half-life.