Section 11.8: Resistors in Circuits

Tutorial 1 Practice, page 527

1. Given: $R_1 = 25.2\ \Omega$; $R_2 = 28.12\ \Omega$

Required: R_{series}

Analysis: $R_{series} = R_1 + R_2$

Solution:
$$\begin{aligned} R_{series} &= R_1 + R_2 \\ &= 25.2\ \Omega + 28.12\ \Omega \\ R_{series} &= 53.3\ \Omega \end{aligned}$$

Statement: The equivalent resistance is 53.3 Ω.

2. Given: $R_1 = 53.0\ \Omega$; $R_2 = 53.0\ \Omega$; $R_3 = 53.0\ \Omega$

Required: R_{series}

Analysis: $R_{series} = R_1 + R_2 + R_3$

Solution:
$$\begin{aligned} R_{series} &= R_1 + R_2 + R_3 \\ &= 53.0\ \Omega + 53.0\ \Omega + 53.0\ \Omega \\ R_{series} &= 159\ \Omega \end{aligned}$$

Statement: The equivalent resistance is 159 Ω.

Tutorial 2 Practice, page 529

1. Given: $R_1 = 120\ \Omega$; $R_2 = 60\ \Omega$

Required: $R_{parallel}$

Analysis: $\dfrac{1}{R_{parallel}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Solution:
$$\begin{aligned} \frac{1}{R_{parallel}} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{120\ \Omega} + \frac{1}{60\ \Omega} \\ &= \frac{1}{120\ \Omega} + \frac{2}{120\ \Omega} \\ \frac{1}{R_{parallel}} &= \frac{3}{120\ \Omega} \\ R_{parallel} &= \frac{120\ \Omega}{3} \\ R_{parallel} &= 40\ \Omega \end{aligned}$$

Statement: The equivalent resistance is 40 Ω.

2. Given: $R_1 = 20\ \Omega$; $R_2 = 20\ \Omega$; $R_3 = 20\ \Omega$; $R_4 = 20\ \Omega$;

Required: $R_{parallel}$

Analysis: $\dfrac{1}{R_{parallel}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4}$

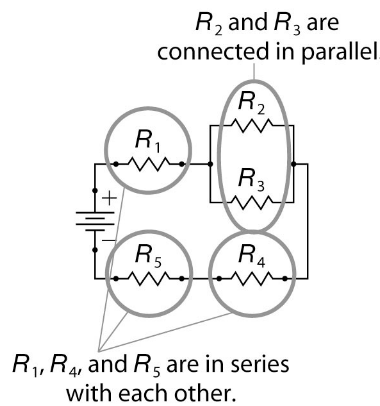
Solution:
$$\begin{aligned} \frac{1}{R_{parallel}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\ &= \frac{1}{20\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{20\ \Omega} \\ \frac{1}{R_{parallel}} &= \frac{4}{20\ \Omega} \\ R_{parallel} &= \frac{20\ \Omega}{4} \\ R_{parallel} &= 5\ \Omega \end{aligned}$$

Statement: The equivalent resistance is 5 Ω.

Tutorial 3 Practice, page 530

1. (a)

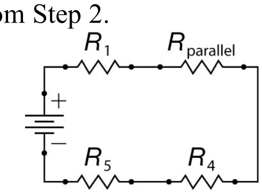
Step 1. Divide the circuit into series and parallel parts.



Step 2. Find the equivalent resistance of the parallel part of the circuit.

$$\begin{aligned} \frac{1}{R_{parallel}} &= \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{5.0\ \Omega} + \frac{1}{5.0\ \Omega} \\ \frac{1}{R_{parallel}} &= \frac{2}{5.0\ \Omega} \\ R_{parallel} &= \frac{5.0\ \Omega}{2} \\ R_{parallel} &= 2.5\ \Omega \end{aligned}$$

Step 3. Redraw the circuit using the equivalent resistance from Step 2.



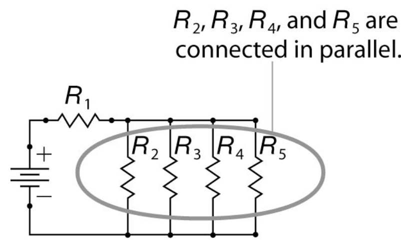
Step 4. Solve to determine the equivalent resistance of the remaining series circuit. Let the equivalent resistance for the complete circuit be R_{total}.

$$\begin{aligned}R_{total} &= R_1 + R_{parallel} + R_4 + R_5 \\ &= 5.0\ \Omega + 2.5\ \Omega + 5.0\ \Omega + 5.0\ \Omega \\ R_{total} &= 17.5\ \Omega\end{aligned}$$

Statement: The total resistance of the mixed circuit is 17.5 Ω.

(b)

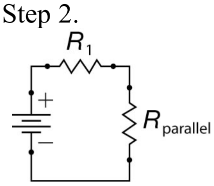
Step 1. Divide the circuit into series and parallel parts.



Step 2. Find the equivalent resistance of the parallel part of the circuit.

$$\begin{aligned}\frac{1}{R_{parallel}} &= \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \\ &= \frac{1}{5.0\ \Omega} + \frac{1}{5.0\ \Omega} + \frac{1}{5.0\ \Omega} + \frac{1}{5.0\ \Omega} \\ \frac{1}{R_{parallel}} &= \frac{4}{5.0\ \Omega} \\ R_{parallel} &= \frac{5.0\ \Omega}{4} \\ R_{parallel} &= 1.3\ \Omega\end{aligned}$$

Step 3. Redraw the circuit using the equivalent resistance from Step 2.



Step 4. Solve to determine the equivalent resistance of the remaining series circuit. Let the equivalent resistance for the complete circuit be R_{total}.

$$\begin{aligned}R_{total} &= R_1 + R_{parallel} \\ &= 5.0\ \Omega + 1.3\ \Omega \\ R_{total} &= 6.3\ \Omega\end{aligned}$$

Statement: The total resistance of the mixed circuit is 6.3 Ω.

Section 11.8 Questions, page 530

1. Start with the equivalent resistance in a series circuit:

$$R_{series} = R_1 + R_2 + R_3 + \cdots$$

Substitute Ohm's Law in the form $R = \frac{V}{I}$:

$$\frac{V_{series}}{I_{series}} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \cdots$$

In a series circuit, the current is constant and the same at all points (KCL). So the currents on the left side will cancel with the currents on the right side:

$$\frac{V_{series}}{I_{series}} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \cdots$$

$$\frac{V_{series}}{\cancel{I_{series}}} = \frac{V_1}{\cancel{I_1}} + \frac{V_2}{\cancel{I_2}} + \frac{V_3}{\cancel{I_3}} + \cdots$$

Therefore, in a series circuit the voltage is given by $V_{series} = V_1 + V_2 + V_3 + \cdots$

This is Kirchhoff's voltage law for a series circuit.

2. Start with the equivalent resistance in a parallel circuit:

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Substitute Ohm's Law in the form $R = \frac{V}{I}$:

$$\frac{1}{\frac{V_{parallel}}{I_{parallel}}} = \frac{1}{\frac{V_1}{I_1}} + \frac{1}{\frac{V_2}{I_3}} + \frac{1}{\frac{V_3}{I_3}} + \cdots$$

$$\frac{I_{parallel}}{V_{parallel}} = \frac{I_1}{V_1} + \frac{I_2}{V_2} + \frac{I_3}{V_3} + \cdots$$

In a parallel circuit, the voltage is constant and the same at all points (KVL). So the voltages on the left side will cancel with the voltages on the right side:

$$\frac{I_{parallel}}{V_{parallel}} = \frac{I_1}{V_1} + \frac{I_2}{V_2} + \frac{I_3}{V_3} + \cdots$$

$$\frac{I_{parallel}}{\cancel{V_{parallel}}} = \frac{I_1}{\cancel{V_1}} + \frac{I_2}{\cancel{V_2}} + \frac{I_3}{\cancel{V_3}} + \cdots$$

$$I_{parallel} = I_1 + I_2 + I_3 + \cdots$$

This is Kirchhoff's current law for a parallel circuit.

3. Suppose the resistance of the two identical resistors is an unknown value r, so that $R_1 = r$ and $R_2 = r$. Since the resistors are in parallel, the equivalent resistance can be found:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{r} + \frac{1}{r}$$
$$\frac{1}{R_{\text{parallel}}} = \frac{2}{r}$$
$$R_{\text{parallel}} = \frac{r}{2}$$

So the equivalent resistance of the two identical resistors in parallel is $\frac{r}{2}$, which is half the resistance of one of the resistors.

4. Answers may vary. Sample answer:
The amount of electric current will increase with each load that is added, since adding a load in parallel causes a decrease in the total resistance of the circuit and an increase in the current. This is a cause for concern because home electrical wiring is designed for low currents, and a high electric current may damage the wires or even begin a fire. In many home electrical systems the dangerous increase in electric current caused by connecting too many loads in parallel is prevented by a device called a circuit breaker.

5. (a) Given: $R_1 = 12.0\ \Omega$; $R_2 = 12.0\ \Omega$; $R_3 = 12.0\ \Omega$; $R_4 = 12.0\ \Omega$
Required: R_{series}
Analysis: $R_{\text{series}} = R_1 + R_2 + R_3 + R_4$
Solution:

$$R_{\text{series}} = R_1 + R_2 + R_3 + R_4$$
$$= 12.0\ \Omega + 12.0\ \Omega + 12.0\ \Omega + 12.0\ \Omega$$
$$R_{\text{series}} = 48.0\ \Omega$$

Statement: The equivalent resistance is 48.0 Ω.

(b) Start by finding the equivalent resistances $R_{1,4}$ and $R_{2,3}$ for the resistors in series in the parallel part of the circuit.
For R_1 and R_4:

$$R_{\text{series}} = R_1 + R_4$$
$$= 12.0\ \Omega + 12.0\ \Omega$$
$$R_{\text{series}} = 24.0\ \Omega$$

$$R_{\text{series}} = R_2 + R_3$$
$$= 12.0\ \Omega + 12.0\ \Omega$$
$$R_{\text{series}} = 24.0\ \Omega$$

Now find the equivalent resistance for the parallel circuit.

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_{1,4}} + \frac{1}{R_{2,3}}$$
$$\frac{1}{R_{\text{parallel}}} = \frac{1}{24.0\ \Omega} + \frac{1}{24.0\ \Omega}$$
$$R_{\text{parallel}} = 12.0\ \Omega$$

Statement: The equivalent resistance is 12.0 Ω.

(c) From part (b), the parallel part of the circuit has an equivalent resistance of 12.0 Ω. Now the total resistance can be found:
R_{parallel} is in series with R_1, so

$$R_{\text{total}} = R_1 + R_{\text{parallel}}$$
$$= 12.0\ \Omega + 12.0\ \Omega$$
$$R_{\text{total}} = 24.0\ \Omega$$

Statement: The equivalent resistance is 24.0 Ω.

(d) From part (b), the parallel part of the circuit has an equivalent resistance of 12.0 Ω. Now the total resistance can be found:
R_{parallel} is in series with R_1 and R_6, so

$$R_{\text{total}} = R_1 + R_{\text{parallel}} + R_6$$
$$= 12.0\ \Omega + 12.0\ \Omega + 12.0\ \Omega$$
$$R_{\text{total}} = 36.0\ \Omega$$

Statement: The equivalent resistance is 36.0 Ω.