Section 10.2: Musical Instruments

Tutorial 1 Practice, page 457

1. Given: L_1 = 63 cm = 0.63 m; f_1 = 110 Hz; f_2 = 150 Hz
Required: L_2
Analysis: $f_1L_1 = f_2L_2$

$$L_2 = \frac{f_1L_1}{f_2}$$

Solution: $L_2 = \frac{f_1L_1}{f_2}$

$$= \frac{(110\ \cancel{\text{Hz}})(0.63\ \text{m})}{(150\ \cancel{\text{Hz}})}$$

$$L_2 = 0.46\ \text{m}$$

Statement: The string should be 0.46 m or 46 cm long.

2. (a) Given: L = 22 cm = 0.22 m; v = 140 m/s
Required: f_0
Analysis: $f_0 = \frac{v}{\lambda}$

$$f_0 = \frac{v}{2L}$$

Solution: $f_0 = \frac{v}{2L}$

$$= \frac{\left(140\ \frac{\cancel{\text{m}}}{\text{s}}\right)}{2(0.22\ \cancel{\text{m}})}$$

$$f_0 = 320\ \text{Hz}$$

Statement: The fundamental frequency of the string is 320 Hz.

(b) Given: L = (0.85)(22 cm) = 0.187 m; v = 140 m/s
Required: f_0
Analysis: $f_0 = \frac{v}{\lambda}$

$$f_0 = \frac{v}{2L}$$

Solution: $f_0 = \frac{v}{2L}$

$$= \frac{\left(140\ \frac{\cancel{\text{m}}}{\text{s}}\right)}{2(0.187\ \cancel{\text{m}})}$$

$$f_0 = 370\ \text{Hz}$$

Statement: The fundamental frequency of the string increases to 370 Hz.

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1. (a) The highest pitch is represented by the shortest wavelength: (ii).
(b) The greatest loudness is represented by the greatest amplitude: (iii).
(c) The highest quality is represented by the waves with multiple harmonics: (iii).
2. Answers may vary. Sample answer:
The string is not connected to any resonator so the quality and loudness of the sound produced is very low.
3. Answers may vary. Sample answer:
The sonometer contains a sounding board which impacts the loudness and quality of the sound produced. Additionally, as the string's length and tension is varied, different frequencies of sound are produced.
4. Given: L_1 = 0.66 m; f_1 = 140 Hz; L_2 = 0.66 m – 0.11 m = 0.55 m
Required: f_2
Analysis: $f_1L_1 = f_2L_2$

$$f_2 = \frac{f_1L_1}{L_2}$$

Solution: $f_2 = \frac{f_1L_1}{L_2}$

$$= \frac{(140\ \text{Hz})(0.66\ \cancel{\text{m}})}{(0.55\ \cancel{\text{m}})}$$

$$L_2 = 170\ \text{Hz}$$

Statement: The new fundamental frequency of the string is 170 Hz.
5. (a) The string is vibrating at its fundamental frequency, so the wavelength is double the length of the string.
2(0.60 m) = 1.2 m
The wavelength of the string is 1.2 m.
(b) The string is fixed at both ends, so the other resonant frequencies are multiples of the fundamental frequency.
2(120 Hz) = 240 Hz
3(120 Hz) = 360 Hz
Two other resonant frequencies are 240 Hz and 360 Hz.
(c) Another way to improve the quality of the sound is to increase the loudness by increasing the amplitude.
6. Answers may vary. Sample answer:
(a) As the plastic tube is spun, air vibrates over the opening, as in a flute, producing a musical note.
(b) The pitch increases as the frequency of rotation increases.

7. (a) Diagrams should resemble parts (a) and (b) from Figure 8 on page 458 of the Student Book or Figure 8 from page 424 of the Student Book.
If the column is open at both ends, then there is an antinode at both ends when the air column resonates. The wavelength of the fundamental frequency is double the length of the air column. The wavelength of any resonant frequency is determined using the calculation for standing waves:

$$L = \frac{n\lambda_n}{2}$$
$$\lambda_n = \frac{2L}{n}$$

Now, use the universal wave equation with the constant speed of sound:

$$v = f_0\lambda_0$$
$$v = f_n\lambda_n$$
$$f_0\lambda_0 = f_n\lambda_n$$
$$f_0(2L) = f_n\left(\frac{2L}{n}\right)$$
$$nf_0 = f_n$$

The resonant frequency is a whole-number multiple of the fundamental frequency.
(b) Diagrams should resemble Figure 9 on page 424 of the Student Book.
If the column is open at only one end, then there is an antinode at one end and a node at the other end when the air column resonates. The wavelength of the fundamental frequency is four times the length of the air column. The wavelength of any resonant frequency is determined using the calculation for standing waves:

$$L = \frac{(2n-1)\lambda_n}{4}$$
$$\lambda_n = \frac{4L}{2n-1}$$

Now, use the universal wave equation with the constant speed of sound:

$$v = f_0\lambda_0$$
$$v = f_n\lambda_n$$
$$f_0\lambda_0 = f_n\lambda_n$$
$$f_0(4L) = f_n\left(\frac{4L}{2n-1}\right)$$
$$(2n-1)f_0 = f_n$$

The resonant frequency is an odd-number multiple of the fundamental frequency.

8. (a) Fundamental frequency increases when velocity increases and velocity increases when temperature increases. So when you go from inside to outside, the fundamental frequency decreases.
(b) (i) Given: T = 22.0 °C; L = 0.800 m; both ends open
Required: f_0
Analysis: $f_0 = \frac{v}{\lambda}$

$$f_0 = \frac{v}{2L}$$

Solution: Determine the speed of sound at 22.0 °C:

$$v_{\text{sound}} = 331.4\ \text{m/s} + (0.606\ \text{m/s/°C})T$$
$$= 331.4\ \text{m/s} + \left(0.606\ \frac{\text{m/s}}{\cancel{°\text{C}}}\right)(22.0\ \cancel{°\text{C}})$$
$$v_{\text{sound}} = 344.7\ \text{m/s}$$

Determine the fundamental frequency:

$$f_0 = \frac{v}{2L}$$
$$= \frac{344.7\ \frac{\cancel{\text{m}}}{\text{s}}}{2(0.800\ \cancel{\text{m}})}$$
$$f_0 = 215\ \text{Hz}$$

Statement: The fundamental frequency of the instrument inside is 215 Hz.
(ii) Given: T = 11.0 °C; L = 0.800 m; both ends open
Required: f_0
Analysis: $f_0 = \frac{v}{\lambda}$

$$f_0 = \frac{v}{2L}$$

Solution: Determine the speed of sound at 11.0 °C:

$$v_{\text{sound}} = 331.4\ \text{m/s} + (0.606\ \text{m/s/°C})T$$
$$= 331.4\ \text{m/s} + \left(0.606\ \frac{\text{m/s}}{\cancel{°\text{C}}}\right)(11.0\ \cancel{°\text{C}})$$
$$v_{\text{sound}} = 338.1\ \text{m/s}$$

Determine the fundamental frequency:

$$f_0 = \frac{v}{2L}$$
$$= \frac{338.1\ \frac{\cancel{\text{m}}}{\text{s}}}{2(0.800\ \cancel{\text{m}})}$$
$$f_0 = 211\ \text{Hz}$$

Statement: The fundamental frequency of the instrument outside is 211 Hz.

9. (a) Given: T = 24 °C; f_0 = 420 Hz; both ends open

Required: L

Analysis: $f_0 = \frac{v}{\lambda}$

$$f_0 = \frac{v}{2L}$$

$$L = \frac{v}{2f_0}$$

Solution: Determine the speed of sound at 24 °C:

$$\begin{aligned} v_{\text{sound}} &= 331.4 \text{ m/s} + (0.606 \text{ m/s/°C})T \\ &= 331.4 \text{ m/s} + \left(0.606 \frac{\text{m/s}}{\cancel{°\text{C}}}\right)(24 \cancel{°\text{C}}) \\ v_{\text{sound}} &= 345.9 \text{ m/s} \end{aligned}$$

Determine the length:

$$\begin{aligned} L &= \frac{v}{2f_0} \\ &= \frac{345.9 \text{ m/s}}{2(420 \text{ Hz})} \\ L &= 0.41 \text{ m} \end{aligned}$$

Statement: The minimum length of the box is 0.41 m.

(b) Given: T = 24 °C; f_0 = 420 Hz; open and closed ends

Required: L

Analysis: $f_0 = \frac{v}{\lambda}$

$$f_0 = \frac{v}{4L}$$

$$L = \frac{v}{4f_0}$$

Solution: Determine the speed of sound at 24 °C:

$$\begin{aligned} v_{\text{sound}} &= 331.4 \text{ m/s} + (0.606 \text{ m/s/°C})T \\ &= 331.4 \text{ m/s} + \left(0.606 \frac{\text{m/s}}{\cancel{°\text{C}}}\right)(24 \cancel{°\text{C}}) \\ v_{\text{sound}} &= 345.9 \text{ m/s} \end{aligned}$$

Determine the length:

$$\begin{aligned} L &= \frac{v}{4f_0} \\ &= \frac{345.9 \text{ m/s}}{4(420 \text{ Hz})} \\ L &= 0.21 \text{ m} \end{aligned}$$

Statement: The minimum length of the box is 0.21 m.

10. (a) The wavelength of the sound at the fundamental frequency for each pipe is four times the length of the pipe (since the pipe is open at one end and closed at the other).

Pipe 1: 4(23.0 cm) = 92.0 cm = 0.920 m

Pipe 2: 4(30.0 cm) = 120 cm = 1.20 m

Pipe 3: 4(38.0 cm) = 152 cm = 1.52 m

So, the wavelengths of the sound produced by the pipes are 0.920 m for pipe 1, 1.20 m for pipe 2, and 1.52 m for pipe 3.

(b) To calculate the fundamental frequency for each pipe, divide the speed of sound by the wavelength.

Pipe 1:

$$f = \frac{342 \frac{\cancel{\text{m}}}{\text{s}}}{0.920 \cancel{\text{m}}}$$

$$f = 372 \text{ Hz}$$

Pipe 2:

$$f = \frac{342 \frac{\cancel{\text{m}}}{\text{s}}}{1.20 \cancel{\text{m}}}$$

$$f = 285 \text{ Hz}$$

Pipe 3:

$$f = \frac{342 \frac{\cancel{\text{m}}}{\text{s}}}{1.52 \cancel{\text{m}}}$$

$$f = 225 \text{ Hz}$$

So, the fundamental frequencies produced by the pipe are 372 Hz for pipe 1, 285 Hz for pipe 2, and 225 Hz for pipe 3.