Section 8.4: Determining Wave Speed

Tutorial 1 Practice, page 389

1. Given: f = 230 Hz; λ = 2.3 m

Required: v

Analysis: $v = f\lambda$

Solution:
$$v = f\lambda$$
$$= (230\text{ Hz})(2.3\text{ m})$$
$$v = 530\text{ m/s}$$

Statement: The speed of the wave is 530 m/s.

2. Given: v = 1500 m/s; f = 11 Hz

Required: λ

Analysis:
$$v = f\lambda$$
$$\lambda = \frac{v}{f}$$

Solution:
$$\lambda = \frac{v}{f}$$
$$= \frac{1500\text{ m/s}}{11\text{ Hz}}$$
$$\lambda = 140\text{ m}$$

Statement: The wavelength is 140 m.

3. Given: v = 405 m/s; λ = 2.0 m

Required: f

Analysis:
$$v = f\lambda$$
$$f = \frac{v}{\lambda}$$

Solution:
$$f = \frac{v}{\lambda}$$
$$= \frac{405\ \frac{\cancel{\text{m}}}{\text{s}}}{2.0\ \cancel{\text{m}}}$$
$$f = 2.0 \times 10^2\text{ Hz}$$

Statement: The frequency of the wave is 2.0×10^2 Hz, or 200 Hz.

Tutorial 2 Practice, page 391

1. Given: L = 2.5 m; F_T = 240 N; v = 300 m/s

Required: m

Analysis:
$$v = \sqrt{\frac{F_T}{\mu}}$$
$$v^2 = \frac{F_T}{\mu}$$
$$\mu = \frac{F_T}{v^2}$$
$$\mu = \frac{m}{L}$$
$$m = \mu L$$
$$m = \frac{F_T}{v^2}L$$

Solution:
$$m = \frac{F_T}{v^2}L$$
$$= \frac{240\text{ N}}{(300\text{ m/s})^2}(2.5\text{ m})$$
$$= \frac{240\ \frac{\text{kg}\bullet\cancel{\text{m}}}{\cancel{\text{s}^2}}}{90\ 000\ \frac{\cancel{\text{m}^2}}{\cancel{\text{s}^2}}}(2.5\ \cancel{\text{m}})$$
$$m = 6.7 \times 10^{-3}\text{ kg}$$

Statement: The mass of the string is 6.7×10^{-3} kg, or 6.7 g.

2. Given: μ = 0.2 kg/m; v = 200 m/s

Required: F_T

Analysis:
$$v = \sqrt{\frac{F_T}{\mu}}$$
$$v^2 = \frac{F_T}{\mu}$$
$$F_T = \mu v^2$$

Solution:
$$F_T = \mu v^2$$
$$= (0.2\text{ kg/m})(200\text{ m/s})^2$$
$$= \left(0.2\ \frac{\text{kg}}{\cancel{\text{m}}}\right)\left(40\ 000\ \frac{\text{m}^{\cancel{2}}}{\text{s}^2}\right)$$
$$= 8 \times 10^3\ \frac{\text{kg}\bullet\text{m}}{\text{s}^2}$$
$$F_T = 8 \times 10^3\text{ N}$$

Statement: The tension required is 8×10^3 N, or 8000 N.

3. Given: μ = 0.011 kg/m; F_T = 250 N

Required: v

Analysis: $v = \sqrt{\frac{F_T}{\mu}}$

Solution:
$$v = \sqrt{\frac{F_T}{\mu}}$$
$$= \sqrt{\frac{250\text{ N}}{0.011\text{ kg/m}}}$$
$$= \sqrt{\frac{250\ \frac{\cancel{\text{kg}}\bullet\text{m}}{\text{s}^2}}{0.011\ \frac{\cancel{\text{kg}}}{\text{m}}}}$$
$$v = 1.5 \times 10^2\text{ m/s}$$

Statement: The wave speed is 1.5×10^2 m/s, or 150 m/s.

Section 8.4 Questions, page 391

1. Given: v = 123 m/s; f = 230 Hz

Required: λ

Analysis: $v = f\lambda$

$$\lambda = \frac{v}{f}$$

Solution: $\lambda = \frac{v}{f}$

$$= \frac{123 \text{ m/s}}{230 \text{ Hz}}$$

$$\lambda = 0.53 \text{ m}$$

Statement: The wavelength is 0.53 m.

2. Given: F_T = 37 N;
μ = 0.03 g/m = 3×10^{-5} kg/m

Required: v

Analysis: $v = \sqrt{\frac{F_T}{\mu}}$

Solution: $v = \sqrt{\frac{F_T}{\mu}}$

$$= \sqrt{\frac{37 \text{ N}}{3 \times 10^{-5} \text{ kg/m}}}$$

$$= \sqrt{\frac{37 \ \frac{\cancel{\text{kg}} \bullet \text{m}}{s^2}}{3 \times 10^{-5} \ \frac{\cancel{\text{kg}}}{\text{m}}}}$$

$$= 1.11 \times 10^3 \text{ m/s}$$

$$v = 1000 \text{ m/s}$$

Statement: The speed of sound along this string is 1000 m/s.

3. Given: $T = 1.20 \times 10^{-3}$ s; $v = 3.40 \times 10^2$ m/s

Required: λ

Analysis: $v = f\lambda$

$$\lambda = \frac{v}{f}$$

$$\lambda = vT$$

Solution: $\lambda = vT$

$$= \left(3.40 \times 10^2 \ \frac{\text{m}}{\cancel{\text{s}}}\right)\left(1.20 \times 10^{-3} \ \cancel{\text{s}}\right)$$

$$\lambda = 0.408 \text{ m}$$

Statement: The wavelength is 0.408 m.

4. (a) P-waves:

Given: v = 8.0 km/s; $\Delta d = 2.4 \times 10^3$ km

Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = v\Delta d$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{2.4 \times 10^3 \ \cancel{\text{km}}}{8.0 \ \frac{\cancel{\text{km}}}{\text{s}}}$$

$$= \left(300 \ \cancel{\text{s}}\right)\left(\frac{1 \text{ min}}{60 \ \cancel{\text{s}}}\right)$$

$$\Delta t = 5 \text{ min}$$

Statement: The P-wave should arrive in 5 min.

S-waves:

Given: v = 4.5 km/s; $\Delta d = 2.4 \times 10^3$ km

Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = v\Delta d$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{2.4 \times 10^3 \ \cancel{\text{km}}}{4.5 \ \frac{\cancel{\text{km}}}{\text{s}}}$$

$$= \left(533.3 \ \cancel{\text{s}}\right)\left(\frac{1 \text{ min}}{60 \ \cancel{\text{s}}}\right)$$

$$\Delta t = 8.9 \text{ min}$$

Statement: The S-wave should arrive in 8.9 min.

(b) Transverse waves are called secondary waves because they arrive after the longitudinal wave.

(c) Answers may vary. Sample answer:
Observing these waves helps geophysicists analyze the structure of the Earth's interior. By collecting data from around the world, they can determine the location of a liquid core and the composition of the layers of Earth. The information is based on which waves arrive at various stations and how long it takes for them to get there.

5. The wavelength is halved. The speed stays the same because the tension and linear density remain the same. That means that when the value of f doubles in the equation $v = f\lambda$, the value of λ must be divided by two.

6. The speed is doubled. Given the equation $v = f\lambda$, when frequency is doubled, for the left side of the equation to equal the right side, the velocity should also be doubled.

7. You would have to multiply the tension by a factor of 4 to double the speed. Double the speed and see how it changes the tension (linear density remains constant):

$$\begin{aligned} v &= 2\sqrt{\frac{F_\mathrm{T}}{\mu}} \\ &= \sqrt{4\frac{F_\mathrm{T}}{\mu}} \\ v &= \sqrt{\frac{(4F_\mathrm{T})}{\mu}} \end{aligned}$$

So, when velocity is doubled, the tension should be multiplied by so that the left side of the equation equals the right side.

8. Start with the equation for force: $F_\mathrm{T} = ma$.

Substitute for $a = \frac{v}{\Delta t}$: $F_\mathrm{T} = m\frac{v}{\Delta t}$. Substitute for Δt knowing that the velocity is the length of the divided by the time:

$$F_\mathrm{T} = m\frac{v}{\left(\frac{L}{v}\right)}$$

$$F_\mathrm{T} = \frac{mv^2}{L}$$

Substitute in $\mu = \frac{m}{L}$, then rearrange to get the equation for wave speed on a string:

$$\begin{aligned} F_\mathrm{T} &= m\frac{v}{\left(\frac{L}{v}\right)} \\ F_\mathrm{T} &= v^2\mu \\ v^2 &= \frac{F_\mathrm{T}}{\mu} \\ v &= \sqrt{\frac{F_\mathrm{T}}{\mu}} \end{aligned}$$