Section 6.3: Heat Capacity

Tutorial 1 Practice, page 283

1. Given: m = 2.0 kg; $c_W = 4.18 \times 10^3$ J/(kg • °C); ΔT = 10.0 °C
Required: Q
Analysis: $Q = mc_W\Delta T$
Solution:

$$Q = mc_W\Delta T$$
$$= (2.0\ \text{kg})\left(4.18\times10^3\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(10\ {}^\circ\text{C})$$
$$Q = 8.4\times10^4\ \text{J}$$

Statement: The water requires 8.4×10^4 J of thermal energy to raise its temperature by 10.0 °C.

2. Given: m = 20.0 kg; $c = 8.4 \times 10^2$ J/(kg • °C); T_1 = 32.0 °C; T_2 = 5.0 °C
Required: Q
Analysis: $\Delta T = T_2 - T_1$; $Q = mc\Delta T$
Solution:

$$\Delta T = T_2 - T_1$$
$$= 5.0\ {}^\circ\text{C} - 32.0\ {}^\circ\text{C}$$
$$\Delta T = -27\ {}^\circ\text{C}$$

$$Q = mc\Delta T$$
$$= (20.0\ \text{kg})\left(8.4\times10^2\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(-27\ {}^\circ\text{C})$$
$$Q = -4.5\times10^5\ \text{J}$$

Statement: The glass window releases 4.5×10^5 J of thermal energy into the surroundings at night.

3. Given: $Q = 1.0 \times 10^4$ J; ΔT = 5.0 °C; $c = 9.2 \times 10^2$ J/(kg • °C)
Required: m
Analysis: $Q = mc\Delta T$
Solution:

$$Q = mc\Delta T$$
$$m = \frac{Q}{c\Delta T}$$
$$= \frac{1.0\times10^4\ \text{J}}{\left(9.2\times10^2\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(5.0\ {}^\circ\text{C})}$$
$$m = 2.2\ \text{kg}$$

Statement: The mass of the aluminum block is 2.2 kg.

Tutorial 2 Practice, page 286

1. Given: m_m = 2.0 kg; $c_m = 9.2 \times 10^2$ J/(kg • °C); T_{1m} = 100.0 °C; m_a = 1.5 kg; $c_a = 2.46 \times 10^3 \times$ J/(kg • °C); T_{1a} = 18.0 °C
Required: final temperature of aluminum–ethyl alcohol mixture, T_2
Analysis: $Q_{released} + Q_{absorbed} = 0$; $Q = mc\Delta T$
Solution:

$$0 = Q_{released} + Q_{absorbed}$$
$$0 = m_m c_m \Delta T_m + m_a c_a \Delta T_a$$
$$0 = (2.0\ \text{kg})\left(9.2\times10^2\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(T_2 - 100\ {}^\circ\text{C})$$
$$+ (1.5\ \text{kg})\left(2.46\times10^3\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(T_2 - 18.0\ {}^\circ\text{C})$$
$$0 = \left(1840\ \frac{\text{J}}{{}^\circ\text{C}}\right)(T_2 - 100.0\ {}^\circ\text{C})$$
$$+ \left(3690\ \frac{\text{J}}{{}^\circ\text{C}}\right)(T_2 - 18.0\ {}^\circ\text{C})$$
$$0 = \left(1840\ \frac{\text{J}}{{}^\circ\text{C}}\right)T_2 - 184\ 000\ \text{J}$$
$$+ \left(3690\ \frac{\text{J}}{{}^\circ\text{C}}\right)T_2 - 66\ 420\ \text{J}$$
$$0 = \left(5530\ \frac{\text{J}}{{}^\circ\text{C}}\right)T_2 - 250\ 420\ \text{J}$$
$$T_2 = \frac{250\ 420\ \text{J}}{5530\ \frac{\text{J}}{{}^\circ\text{C}}}$$
$$T_2 = 45\ {}^\circ\text{C}$$

Statement: The final temperature of the mixture is 45 °C.

2. Given: m_m = 4.0 kg; T_{1m} = 100.0 °C; V_w = 500.0 mL; T_{1w} = 20.0 °C; T_2 = 35.0 °C
Required: c_m, specific heat capacity of the metal
Analysis: $Q_{released} + Q_{absorbed} = 0$; $Q = mc\Delta T$
Solution: Since the quantity of heat equation is based on the mass of a substance, first calculate the mass of water, m_w, in kilograms, using the volume of water, V_w, provided and the density of water.

$$m_w = 500\ \text{mL} \times \frac{1\ \text{g}}{1\ \text{mL}} \times \frac{1\ \text{kg}}{1000\ \text{g}}$$
$$m_w = 0.50\ \text{kg}$$

$$0 = Q_{released} + Q_{absorbed}$$
$$0 = m_m c_m \Delta T_m + m_w c_w \Delta T_w$$
$$0 = (4.0\ \text{kg})(c_m)(35.0\ {}^\circ\text{C} - 100\ {}^\circ\text{C})$$
$$+ (0.50\ \text{kg})\left(4.18\times10^3\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(35.0\ {}^\circ\text{C} - 20.0\ {}^\circ\text{C})$$

$$0 = (4.0\text{ kg})(c_m)(-65.0\ °\text{C}) + (0.50\ \cancel{\text{kg}})\left(4.18\times10^3\ \frac{\text{J}}{\cancel{\text{kg}}\bullet\cancel{°\text{C}}}\right)(15.0\ \cancel{°\text{C}})$$

$$0 = (-260\text{ kg}\bullet°\text{C})(c_m) + 31\ 350\text{ J}$$

$$c_m = \frac{31\ 350\text{ J}}{260\text{ kg}\bullet°\text{C}}$$

$$c_m = 1.2\times10^2\ \frac{\text{J}}{\text{kg}\bullet°\text{C}}$$

Statement: The specific heat capacity of the metal is 1.2×10^2 J/(kg • °C).

Section 6.3 Questions, page 287

1. Specific heat capacity is the amount of energy, in joules, required to increase the temperature of 1 kg of a substance by 1 °C. It tells you how much energy a certain substance needs to change its temperature, or how much energy will be released as the substance cools.

2. Given: m = 25.0 g = 0.0250 kg; $c = 2.4 \times 10^2$ J/(kg • °C); T_1 = 50.0 °C; T_2 = 80.0 °C
Required: Q
Analysis: $\Delta T = T_2 - T_1$; $Q = mc\Delta T$
Solution:

$$\Delta T = T_2 - T_1 = 80.0\ °\text{C} - 50.0\ °\text{C}$$
$$\Delta T = 30\ °\text{C}$$

$$Q = mc\Delta T = (0.0250\ \cancel{\text{kg}})\left(2.4\times10^2\ \frac{\text{J}}{\cancel{\text{kg}}\bullet\cancel{°\text{C}}}\right)(30\ \cancel{°\text{C}})$$
$$Q = 1.8\times10^2\text{ J}$$

Statement: The amount of thermal energy required is 1.8×10^2 J.

3. Given: m = 260.0 g = 0.260 kg; $c = 2.1 \times 10^2$ J/(kg • °C); T_1 = –1.0 °C; T_2 = –20.0 °C
Required: Q
Analysis: $\Delta T = T_2 - T_1$; $Q = mc\Delta T$
Solution:

$$\Delta T = T_2 - T_1 = -20.0\ °\text{C} - (-1.0\ °\text{C})$$
$$\Delta T = -19\ °\text{C}$$

$$Q = mc\Delta T = (0.260\ \cancel{\text{kg}})\left(2.1\times10^3\ \frac{\text{J}}{\cancel{\text{kg}}\bullet\cancel{°\text{C}}}\right)(-19\ \cancel{°\text{C}})$$
$$Q = -1.0\times10^4\text{ J}$$

Statement: The amount of thermal energy released is 1.0×10^4 J.

4. Given: Q = –1520 J; m = 50.0 g = 0.0500 kg; T_1 = 100. 0 °C; T_2 = 20.0 °C
Required: c, specific heat capacity of the metal
Analysis: $\Delta T = T_2 - T_1$; $Q = mc\Delta T$
Solution:

$$\Delta T = T_2 - T_1 = 20.0\ °\text{C} - 100.0\ °\text{C}$$
$$\Delta T = -80.0\ °\text{C}$$

$$Q = mc\Delta T$$
$$-1520\text{ J} = (0.0500\text{ kg})(c)(-80.0\ °\text{C})$$
$$-1520\text{ J} = (-4.00\text{ kg}\bullet°\text{C})c$$
$$c = \frac{-1520\text{ J}}{-4.00\text{ kg}\bullet°\text{C}}$$
$$c = 3.8\times10^2\text{ J/(kg}\bullet°\text{C)}$$

Statement: The specific heat capacity for the sample of metal is 3.8×10^2 J/(kg • °C), so the metal is copper.

5. Given: $c = 6.3 \times 10^2$ J/(kg • °C); Q = 302 J; m = 60.0 g = 0.0600 kg; T_1 = 10.0 °C
Required: T_2
Analysis: $\Delta T = T_2 - T_1$; $Q = mc\Delta T$
Solution:

$$Q = mc\Delta T$$
$$-302\text{ J} = (0.06\ \cancel{\text{kg}})\left(6.3\times10^2\ \frac{\text{J}}{\cancel{\text{kg}}\bullet°\text{C}}\right)(T_2 - 10.0\ °\text{C})$$
$$-302\text{ J} = \left(37.8\ \frac{\text{J}}{°\text{C}}\right)(T_2 - 10.0\ °\text{C})$$
$$\frac{-302\ \cancel{\text{J}}}{37.8\ \frac{\cancel{\text{J}}}{°\text{C}}} = T_2 - 10.0\ °\text{C}$$
$$8.0\ °\text{C} = T_2 - 10.0\ °\text{C}$$
$$T_2 = 18\ °\text{C}$$

Statement: The final temperature of the calcium is 18 °C.

6. Given: $c_m = 1.29 \times 10^2$ J/(kg • °C); T_{1m} = 95 °C; V_a = 500 mL; $c_a = 2.46 \times 10^3$ J/(kg • °C); T_{1a} = 25.0 °C; T_2 = 27.0 °C
Required: m_m, the mass of the gold
Analysis: $Q_{released} + Q_{absorbed} = 0$; $Q = mc\Delta T$
Solution: Since the quantity of heat equation is based on the mass of a substance, first calculate the mass of ethyl alcohol, m_a, in kilograms, using the volume of ethyl alcohol, V_a, provided and the density of ethyl alcohol.

$$m_a = 500\ \cancel{\text{mL}} \times \frac{0.789\ \cancel{\text{g}}}{1\ \cancel{\text{mL}}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}$$

$$m_a = 0.395\ \text{kg (two extra digits carried)}$$

$$0 = Q_{\text{released}} + Q_{\text{absorbed}}$$
$$0 = m_m c_m \Delta T_m + m_w c_w \Delta T_w$$
$$0 = (m_m)\left(1.29\times10^2\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(27.0\ {}^\circ\text{C} - 95.0\ {}^\circ\text{C})$$
$$+ (0.395\ \text{kg})\left(2.46\times10^3\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(27.0\ {}^\circ\text{C} - 25.0\ {}^\circ\text{C})$$
$$0 = (m_m)\left(1.29\times10^2\ \frac{\text{J}}{\text{kg}\bullet\cancel{{}^\circ\text{C}}}\right)(-68.0\ \cancel{{}^\circ\text{C}})$$
$$+ (0.395\ \cancel{\text{kg}})\left(2.46\times10^3\ \frac{\text{J}}{\cancel{\text{kg}}\bullet\cancel{{}^\circ\text{C}}}\right)(2.0\ \cancel{{}^\circ\text{C}})$$
$$0 = (m_m)\left(-8.772\times10^3\ \frac{\text{J}}{\text{kg}}\right) + 1.943\times10^3\ \text{J}$$
$$m_m = \frac{1.943\times10^3\ \cancel{\text{J}}}{8.772\times10^3\ \frac{\cancel{\text{J}}}{\text{kg}}}$$
$$= 0.222\ \text{kg}$$
$$m_m = 220\ \text{g}$$

Statement: The mass of the gold is 220 g.

7. Given: $m_m = 2.0$ kg; $T_{1m} = -25.0$ °C; $V_w = 3.0$ L; $T_{1w} = 40.0$ °C; $T_2 = 36.0$ °C

Required: c_m, specific heat capacity of the metal

Analysis: $Q_{\text{released}} + Q_{\text{absorbed}} = 0$; $Q = mc\Delta T$

Solution: Since the quantity of heat equation is based on the mass of a substance, first calculate the mass of water, m_w, in kilograms, using the volume of water, V_w, provided and the density of water.

$$m_w = 3000\ \cancel{\text{mL}} \times \frac{1\ \cancel{\text{g}}}{1\ \cancel{\text{mL}}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}$$
$$m_w = 3.0\ \text{kg}$$

$$0 = Q_{\text{released}} + Q_{\text{absorbed}}$$
$$0 = m_m c_m \Delta T_m + m_w c_w \Delta T_w$$
$$0 = (2.0\ \text{kg})(c_m)(36.0\ {}^\circ\text{C} - (-25.0\ {}^\circ\text{C}))$$
$$+(3.0\ \text{kg})\left(4.18\times10^3\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(36.0\ {}^\circ\text{C} - 40.0\ {}^\circ\text{C})$$
$$0 = (122.0\ \text{kg}\bullet{}^\circ\text{C})(c_m) - 50\ 160\ \text{J}$$
$$c_m = \frac{50\ 160\ \text{J}}{122.0\ \text{kg}\bullet{}^\circ\text{C}}$$
$$c_m = 4.1\times10^2\ \text{J/(kg}\bullet{}^\circ\text{C)}$$

Statement: The specific heat capacity of the metal is 4.1×10^2 J/(kg • °C).

8. Given: $m_m = 1.50 \times 10^2$ g = 0.150 kg; $c_m = 3.80 \times 10^2$ J/(kg • °C); $V_w = 400.0$ mL; $c_w = 4.18 \times 10^3$ J/(kg • °C); $T_{1w} = 27.7$ °C; $T_2 = 28.0$ °C

Required: T_{1m}

Analysis: $Q_{\text{released}} + Q_{\text{absorbed}} = 0$; $Q = mc\Delta T$

Solution: Since the quantity of heat equation is based on the mass of a substance, first calculate the mass of water, m_w, in kilograms, using the volume of water, V_w, provided and the density of water.

$$m_w = 400\ \cancel{\text{mL}} \times \frac{1\ \cancel{\text{g}}}{1\ \cancel{\text{mL}}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}$$
$$m_w = 0.4\ \text{kg}$$

$$0 = Q_{\text{released}} + Q_{\text{absorbed}}$$
$$0 = m_m c_m \Delta T_m + m_w c_w \Delta T_w$$
$$0 = (0.15\ \cancel{\text{kg}})\left(3.80\times10^2\ \frac{\text{J}}{\cancel{\text{kg}}\bullet{}^\circ\text{C}}\right)(28.0\ {}^\circ\text{C} - T_{1m})$$
$$+(0.4\ \text{kg})\left(4.18\times10^3\ \frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(28.0\ {}^\circ\text{C} - 27.7\ {}^\circ\text{C})$$
$$0 = \left(57.00\ \frac{\text{J}}{{}^\circ\text{C}}\right)(28.0\ {}^\circ\text{C} - T_{1m})$$
$$+(0.4\ \cancel{\text{kg}})\left(4.18\times10^3\ \frac{\text{J}}{\cancel{\text{kg}}\bullet\cancel{{}^\circ\text{C}}}\right)(0.3\ \cancel{{}^\circ\text{C}})$$
$$0 = \left(57.00\ \frac{\text{J}}{{}^\circ\text{C}}\right)(28.0\ {}^\circ\text{C} - T_{1m}) + 501.6\ \text{J}$$
$$-501.6\ \text{J} = \left(57.00\ \frac{\text{J}}{{}^\circ\text{C}}\right)(28.0\ {}^\circ\text{C} - T_{1m})$$
$$\frac{-501.6\ \cancel{\text{J}}}{57.00\ \frac{\cancel{\text{J}}}{{}^\circ\text{C}}} = 28.0\ {}^\circ\text{C} - T_{1m}$$
$$T_{1m} = 28.0\ {}^\circ\text{C} + 8.8\ {}^\circ\text{C}$$
$$T_{1m} = 36.8{}^\circ\text{C}$$

Statement: The initial temperature of the brass was 36.8 °C.

9. Answers may vary. Sample answer:
Civil engineers must consider temperature changes when designing building structures so that when the materials expand and contract in hot and cold temperatures, respectively, the building or structure is not damaged. For example, bridges have expansion joints so that when the metal components expand on hot summer days, the pressure will not bend the bridge.

10. Answers may vary. Sample answer:
Thermal expansion refers to the increase in volume of a material as its temperature increases. Thermal contraction refers to the decrease in volume of a material as its temperature decreases. Thermal expansion is an important factor to consider in the design of homes. Thermal expansion of water in plumbing is often a problem in older homes. For example, when water is heated, it expands. The extra volume of water must go somewhere. When the plumbing system is closed, the extra volume has no place to go, so the pressure in a home's plumbing system can increase. Symptoms of high pressure are pressure surges, chronic dripping of the home's water heater temperature and pressure valve, dripping faucets, and leaking ballcocks (assuming plumbing seals are in good condition). Innovations in plumbing products such as pressure-reducing valves, backflow preventers, and other types of valves are now part of most home building codes.