Section 5.5: Power

Tutorial 1 Practice, page 251

1. Given: $P = 0.50$ kW $= 5.0 \times 10^2$ W; $W = 1200$ J
Required: Δt

Analysis: $P = \dfrac{W}{\Delta t}$

Solution:
$$P = \frac{W}{\Delta t}$$
$$\Delta t = \frac{W}{P} = \frac{1200\text{ J}}{500\text{ W}} = 2.4\text{ J/W}$$
$$\Delta t = 2.4\text{ s}$$

Statement: It would take the motor 2.4 s to do 1200 J of work.

2. Given: $m = 55$ kg; $h_{\text{start}} = 850$ m; $h_{\text{finish}} = 2400$ m; $g = 9.8$ N/kg
$$\Delta t = 3\text{ h} = 3\cancel{\text{h}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\text{ s}}{1\ \cancel{\text{min}}}$$
$$\Delta t = 1.08 \times 10^4\text{ s (one extra digit carried)}$$
Assume heights and time are accurate to two significant digits.

Required: P

Analysis:
$$E = mgh$$
$$P = \frac{\Delta E}{\Delta t} = \frac{E_{\text{g finish}} - E_{\text{g start}}}{\Delta t} = \frac{mgh_{\text{finish}} - mgh_{\text{start}}}{\Delta t}$$

Solution:
$$P = \frac{mgh_{\text{finish}} - mgh_{\text{start}}}{\Delta t}$$
$$= \frac{(55\ \cancel{\text{kg}})\left(9.8\dfrac{\text{N}}{\cancel{\text{kg}}}\right)(2400\text{ m}) - (55\ \cancel{\text{kg}})\left(9.8\dfrac{\text{N}}{\cancel{\text{kg}}}\right)(850\text{ m})}{1.08 \times 10^4\text{ s}}$$
$$= 77.36\ \frac{\text{N}\bullet\text{m}}{\text{s}}$$
$$= 77.36\text{ W}$$
$$P = 77\text{ W}$$

Statement: The average power of the climber is 77 W.

3. Given: $m = 60.0$ kg; $v_{\text{i}} = 0$ m/s; $v_{\text{f}} = 12$ m/s; $\Delta t = 6.0$ s
Required: P

Analysis: $E_{\text{k}} = \dfrac{mv^2}{2}$; $P = \dfrac{\Delta E}{\Delta t}$

Solution:
$$P = \frac{\Delta E}{\Delta t} = \frac{E_{\text{k (finish)}} - E_{\text{k (start)}}}{\Delta t} = \frac{\dfrac{mv_{\text{f}}^2}{2} - \dfrac{mv_{\text{i}}^2}{2}}{\Delta t} = \frac{mv_f^{\,2}}{2\Delta t} - 0$$
$$= \frac{(60.0\text{ kg})(12\text{ m/s})^2}{2(6.0\text{ s})} = 720\text{ J/s}$$
$$P = 720\text{ W}$$

Statement: The person's power is 720 W.

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A. Answers may vary. Sample answer:
Given: $m = 63$ kg; $\Delta d = 2.00$ m; $\Delta t = 9.0$ s; $g = 9.8$ m/s^2
Required: P

Analysis: $P = \dfrac{\Delta E}{\Delta t}$; $F_{\text{g}} = mgh$

Solution:
$$\Delta E = F_{\text{f}} - F_{\text{i}} = mgh - 0$$
$$\Delta E = mgh$$

$$P = \frac{\Delta E}{\Delta t} = \frac{mgh}{\Delta t} = \frac{mg\Delta d}{\Delta t} = \frac{(63\text{ kg})(9.8\text{ m/s})(2.0\text{ m})}{9.0\text{ s}} = 1.4 \times 10^2\text{ W}$$

Statement: The average power produced by walking slowly is 1.4×10^2 W.

B. Given: m = 63 kg; Δd = 2.00 m; Δt = 5.5 s; $g = 9.8\ \text{m/s}^2$

Required: P

Analysis: $P = \dfrac{\Delta E}{\Delta t}$; $F_g = mgh$

Solution: From A:

$$\begin{aligned} P &= \frac{mg\Delta d}{\Delta t} \\ &= \frac{(63\ \text{kg})(9.8\ \text{m/s})(2.0\ \text{m})}{5.5\ \text{s}} \\ &= 2.2 \times 10^2\ \text{W} \end{aligned}$$

Statement: The average power produced by walking slowly is 2.2×10^2 W.

C. My partner's power was greater when walking up the stairs quickly because he had to exert more energy to walk more quickly.

D. Answers may vary. Sample answer:
If you assume the 100 W bulb is on for approximately 5 s before it gets hot, the power it uses is 1.1×10^2 W. Walking quickly produces the most power because you needs lots of energy to for a quick speed. You need energy to power your muscles. In a bulb, you need power (electricity) to heat a very small, thin metal filament for light. This amount of power you would need is very small relative to your leg muscles.

Tutorial 2 Practice, page 252

1. Given: P = 3100 MW = 3.1×10^9 W;

$$\begin{aligned} \Delta t &= 1\ \text{day} \\ &= 1\ \cancel{\text{day}} \times \frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}} \\ \Delta t &= 86\ 400\ \text{s} \end{aligned}$$

Required: ΔE

Analysis: $P = \dfrac{\Delta E}{\Delta t}$

Solution:

$$\begin{aligned} P &= \frac{\Delta E}{\Delta t} \\ \Delta E &= P\Delta t \\ &= (3.1 \times 10^9\ \text{W})(86\ 400\ \text{s}) \\ &= 2.678 \times 10^{14}\ \text{W}\bullet\text{s} \\ &= 2.678 \times 10^{14}\ \text{J} \\ &= 2.678 \times 10^{8}\ \text{MJ} \\ \Delta E &= 2.7 \times 10^{8}\ \text{MJ} \end{aligned}$$

Statement: The generating station can produce 2.7×10^8 MJ of energy in one day.

Tutorial 3 Practice, page 253

1. Given:

$$\begin{aligned} P_{\text{incand}} &= \frac{100\ \text{W}}{\cancel{\text{bulb}}} \times 20\ \cancel{\text{bulbs}} \\ &= 2000\ \text{W} \\ P_{\text{incand}} &= 2.0\ \text{kW} \end{aligned}$$

P_{CFL} = 23 W/bulb; rate = 6.0¢/kWh;

$$\Delta t = 1\ \cancel{\text{year}} \times \frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}} \times \frac{12\ \text{h}}{1\ \cancel{\text{day}}}$$

$$\Delta t = 4380\ \text{h}$$

Assume two significant digits in power rating.

Required: ΔE, total energy; total cost; savings using CFLs

Analysis: $P = \dfrac{\Delta E}{\Delta t}$; cost = $\Delta E \times$ rate

(a) The total amount of energy used by all the bulbs in the year:

Solution:

$$\begin{aligned} P &= \frac{\Delta E}{\Delta t} \\ \Delta E &= P\Delta t \\ &= (2.0\ \text{kW})(4380\ \text{h}) \\ \Delta E &= 8760\ \text{kWh} \end{aligned}$$

Statement: The total amount of energy used by all the bulbs is 8800 kWh.

(b) Cost of lighting the store for the year:

Solution:

$$\begin{aligned} \text{cost} &= \Delta E \times \text{rate} \\ &= (8760\ \cancel{\text{kWh}})\left(6.0\frac{¢}{\cancel{\text{kWh}}}\right) \\ &= 52\ 560¢ \\ &= \$525.60 \\ \text{cost} &= \$530 \end{aligned}$$

Statement: It costs \$530 to light the store for the year.

(c) Money saved by using CFLs instead of incandescent bulbs:

Solution: This can be solved by using a simple ratio to compare the costs based on the power of the two types of bulbs. The difference in costs is the savings.

$$\frac{\text{cost}_{\text{CFL}}}{\text{cost}_{\text{incand}}} = \frac{P_{\text{CFL}}}{P_{\text{incand}}}$$

$$\begin{aligned} \text{cost}_{\text{CFL}} &= \text{cost}_{\text{incand}} \times \frac{P_{\text{CFL}}}{P_{\text{incand}}} \\ &= \$525.60 \times \frac{23\ \text{W}}{100\ \text{W}} \\ \text{cost}_{\text{CFL}} &= \$120.89 \end{aligned}$$

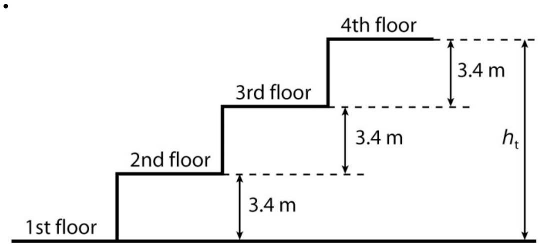
$$\begin{aligned}\text{savings} &= \text{cost}_{\text{incand}} - \text{cost}_{\text{CFL}} \\ &= \$525.60 - \$120.89 \\ &= \$404.71\end{aligned}$$

$$\text{savings} = \$400$$

Statement: Using CFLs creates a total savings of $400 (correct to two significant digits) for the year.

Section 5.5 Questions, page 254

1.



Given: m = 54 kg; h_{floor} = 3.4 m; g = 9.8 N/kg; h_{ground} = 0 m; Δt = 32 s

Required: E_g; P

Analysis: $E_g = mgh$; $P = \dfrac{W}{\Delta t}$; $W = \Delta E_g$;

$\Delta E_g = E_{\text{g at top}} - E_{\text{g at ground}}$; $h_{\text{Total}} = 3 \times h_{\text{floor}}$

(a) Gravitational potential energy at the top of the climb:

Solution:

$$\begin{aligned}\Delta E_g &= E_{\text{g at top}} - E_{\text{g at ground}} \\ &= mgh_{\text{Total}} - mgh_{\text{at ground}} \\ &= mg(3h_{\text{floor}}) - 0 \\ &= (54\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)(3)(3.4\ \text{m}) \\ &= 5397.8\ \text{N}\bullet\text{m}\end{aligned}$$

$$\Delta E_g = 5400\ \text{J}$$

Statement: The person's gravitational energy at the top is 5400 J.

(b) The power of the person for the climb:

Solution:

$$\begin{aligned}P &= \frac{W}{\Delta t} \\ &= \frac{E_g}{\Delta t} \\ &= \frac{5397.8\ \text{J}}{32\ \text{s}} \\ &= 168.7\ \text{J/s}\end{aligned}$$

$$P = 170\ \text{W}$$

Statement: The person's power output is 170 W.

(c) If a lighter person climbed the stairs in the same amount of time, the power output would be lower. Since the person's mass is lower, the amount of gravitational energy acquired would be lower (gravitational energy is proportional to mass). Then less work would be done in the same amount of time and a lower power output would result.

2. Given: m = 65 kg; g = 9.8 N/kg; h_{top} = 5.0 m; h_{ground} = 0 m; v = 1.4 m/s

Required: Δt; P

Analysis: $v = \dfrac{\Delta d}{\Delta t}$; $P = \dfrac{W}{\Delta t}$; $W = \Delta E_g$;

$\Delta E_g = E_{\text{g at top}} - E_{\text{g at ground}}$; $E_g = mgh$

(a) The time it takes the student to climb the rope and the student's power:

Solution:

$$v = \frac{\Delta d}{\Delta t}$$

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{5.0\ \cancel{\text{m}}}{1.4\ \cancel{\text{m}}/\text{s}} \\ &= 3.57\ \text{s}\end{aligned}$$

$$\Delta t = 3.6\ \text{s}$$

$$\begin{aligned}\Delta E_g &= E_{\text{g at top}} - E_{\text{g at ground}} \\ &= mgh_{\text{Total}} - mgh_{\text{ground}} \\ &= (65\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)(5.0\ \text{m}) - 0 \\ &= 3185\ \text{N}\bullet\text{m}\end{aligned}$$

$$\Delta E_g = 3185\ \text{J}$$

$$\begin{aligned}P &= \frac{W_{\text{net}}}{\Delta t} \\ &= \frac{\Delta E_g}{\Delta t} \\ &= \frac{3185\ \text{J}}{3.57\ \text{s}} \\ &= 892.2\ \text{J/s} \\ &= 892.2\ \text{W}\end{aligned}$$

$$P = 890\ \text{W}$$

Statement: It takes 3.6 s for the student to climb the rope and the student's power is 890 W.

(b) The student's power without finding the time:
Solution:

$$\begin{aligned}
P &= \frac{W_{\text{net}}}{\Delta t} \\
&= \frac{\Delta E_{\text{g}}}{\Delta t} \\
&= \frac{mgh}{\Delta t}\ (h = \Delta d) \\
&= \frac{mg\Delta d}{\Delta t}\left(v = \frac{\Delta d}{\Delta t}\right) \\
&= mgv \\
&= (65\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)\left(1.4\ \frac{\text{m}}{\text{s}}\right) \\
&= 891.8\ \frac{\text{N•m}}{\text{s}} \\
&= 891.8\ \text{J/s} \\
P &= 890\ \text{W}
\end{aligned}$$

Statement: The student's power is 890 W. Since the height climbed is the distance travelled at a constant speed, the simple kinematics formula can be substituted into the equation, as shown above.

3. (a) In order to calculate power, the amount of work done and the time taken is needed. In this case, you could measure the time taken (in seconds) for the mass to be lifted at a constant speed. Knowing the mass of the object (in kilograms), you could calculate the change in gravitational energy of the object using $E_g = mgh$, which is equivalent to the work done on the object.

Then using the power formula, $P = \frac{W_{\text{net}}}{\Delta t}$, the power could be calculated. (In this case, since the speed is constant, there is no change in kinetic energy so no corresponding work done to change it.)

(b) In this case, since the speed is no longer constant, there is a change in kinetic energy and some corresponding work done to change the speed. As the object is being pulled up, both its gravitational and kinetic energy increase. In order to calculate the student's power in this case, the change in gravitational energy and the change in kinetic energy must be taken into account. The change in gravitational energy is calculated in the same way as in part (a). In order to calculate the change in kinetic energy, we must find the object's final speed and use the equation $E_k = \frac{1}{2} mv^2$ to calculate the additional work done. The power can then be calculated by finding the total work done (add ΔE_g and ΔE_k) and the total time taken using $P = \frac{W_{\text{net}}}{\Delta t}$.

4. (a) The amount of solar energy transformed into electrical energy each day:
Given: $P_{\text{panel}} = 600$ W; number of panels = 10; $\Delta t = 4.5$ h
Required: E_{Total}
Analysis: $P_{\text{Total}} = \frac{\Delta E_{\text{Total}}}{\Delta t}$ P_{Total}; $P_{\text{Total}} = 10\ P_{\text{panel}}$
Solution:

$$\begin{aligned}
P_{\text{Total}} &= \frac{\Delta E_{\text{Total}}}{\Delta t} \\
\Delta E_{\text{Total}} &= P_{\text{Total}}\,\Delta t \\
&= (10P_{\text{panel}})\Delta t \\
&= (10)(600\ \text{W})(4.5\ \text{h}) \\
&= 27\ 000\ \text{Wh} \\
\Delta E_{\text{Total}} &= 27\ \text{kWh (in one day)}
\end{aligned}$$

Statement: The solar panels will produce 27 kWh of energy in one day.

(b) The amount of money saved in a year on the electrical energy bill:
Given: $E_{\text{day}} = 27$ kWh; $\Delta t = 365$ days; rate = 5.5¢/kWh
Required: cost savings
Analysis: $E_{\text{year}} = E_{\text{day}} \times 365$ days/year; cost savings = rate $\times E_{\text{year}}$
Solution:

$$\begin{aligned}
E_{\text{year}} &= E_{\text{day}} \times \frac{365\ \text{days}}{\text{year}} \\
&= \frac{27\ \text{kWh}}{\cancel{\text{day}}} \times \frac{365\ \cancel{\text{days}}}{\text{year}} \\
E_{\text{year}} &= 9855\ \text{kWh/year}
\end{aligned}$$

$$\begin{aligned}
\text{cost savings} &= \text{rate} \times \text{energy} \\
&= \left(5.5\ \frac{¢}{\cancel{\text{kWh}}}\right)\left(9855\ \frac{\cancel{\text{kWh}}}{\text{year}}\right) \\
&= 54\ 202.5\ ¢/\text{year} \\
&= \$542.03/\text{year} \\
\text{cost savings} &= \$540\ /\ \text{year}
\end{aligned}$$

Statement: This family will save $540 per year in electrical energy costs.

(c) Given: number of persons, $n = 5$; consumption, $E = 2$ kWh/person/day
Required: energy consumption of family in one day, E_{family}
Analysis: $E_{\text{family}} = E \times n$
Solution: In one day,

$$\begin{aligned}E_{\text{family}} &= E \times n \\ &= \left(2\right)\frac{\text{kWh}}{\cancel{\text{person}}} \times 5\ \cancel{\text{persons}} \\ &= 10\ \text{kWh}\end{aligned}$$

The solar panels produce 27 kWh per day, which is more than enough to supply the family's needs.
Statement: The family should not have to buy additional electricity. They could sell the excess energy back to their energy supplier on full-production days. (However, on cloudy days, when the solar panels are not creating electrical energy, they would still need to purchase energy from the supplier.)

5. Given: 1 kW
Required: Show 1 kW = 3.6 MJ.
Analysis: Covert hours to seconds. Convert watts to joules. Convert kilowatt hours to Joules. Convert joules to megajoules.
Solution:

$$\begin{aligned}1\ \text{kW} &= 1000\ \text{W} \\ 1\ \text{kWh} &= 1000\ \text{Wh} \\ &= 1000\text{W}\cancel{\text{h}} \times \frac{3600\ \text{s}}{1\ \cancel{\text{h}}} \\ &= 3\ 600\ 000\ \text{Ws}\end{aligned}$$

$$\begin{aligned}1\ \text{W} &= 1\ \text{J/s} \\ 1\ \text{kWh} &= 3\ 600\ 000\ \frac{\text{J}}{\cancel{\text{s}}}\cancel{\text{s}} \\ &= 3\ 600\ 000\ \text{J}\end{aligned}$$

$$\begin{aligned}1\ 000\ 000\ \text{J} &= 1\ \text{MJ} \\ 1\ \text{kWh} &= 3.6\ \text{MJ}\end{aligned}$$

Statement: This shows 1 kW = 3.6 MJ.