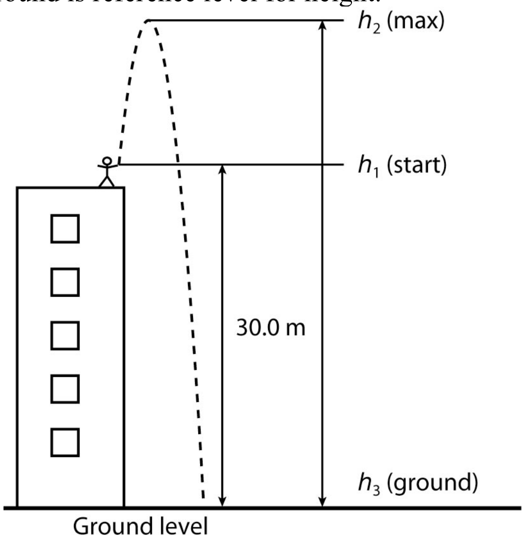
Section 5.3: Types of Energy and the Law of Conservation of Energy

Tutorial 1 Practice, page 241

1. Given: $m = 0.20$ kg; $h_1 = 30.0$ m; $v_1 = 22.0$ m/s; $h_3 = 0$ m; $g = 9.8$ N/kg;
Ground is reference level for height.



Required: E_m, total energy of the ball at start (h_1); h_2 (maximum height); v_2, velocity at h_2 (maximum height); v_3, velocity at h_3 (ground level);

Analysis: E_m = constant; $E_g = mgh$; $E_k = \frac{1}{2}mv^2$;
$E_m = E_g + E_k$

(a) The total energy of the ball at start (h_1):

Solution:

$$\begin{aligned} E_m &= E_g + E_k \\ &= mgh_1 + \frac{mv_1^2}{2} \\ &= (0.20\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)(30.0\ \text{m}) + \frac{(0.20\ \text{kg})(22.0\ \text{m/s})^2}{2} \\ &= 58.8\ \text{N•m} + 48.4\ \frac{\text{kg•m}^2}{\text{s}^2} \\ &= 58.8\ \text{J} + 48.4\ \text{J} \\ &= 107.2\ \text{J} \\ E_m &= 110\ \text{J} \end{aligned}$$

Statement: The total energy of the ball at the start was 110 J.

(b) Total energy of ball at its maximum height (h_2):

Solution: At its maximum height, the upward velocity of the ball is zero ($v_2 = 0$ m/s) because it has temporarily stopped before it begins falling to the ground. However, the total energy of the ball remains constant.

$$\begin{aligned} E_m &= E_g + E_k \\ &= mgh_2 + \frac{mv_2^2}{2} \\ E_m &= mgh_2 + 0 \\ E_m &= mgh_2 \\ h_2 &= \frac{E_m}{mg} \\ &= \frac{107.2\ \text{J}}{(0.20\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)} \\ &= 54.69\ \text{m} \\ h_2 &= 55\ \text{m} \end{aligned}$$

Statement: The upward velocity of the ball at the maximum height is 0 m/s. The maximum height of the ball occurs at the upward point where its velocity is 0 m/s. This point occurs at 55 m.

(c) Velocity of the ball when it hits the ground (h_3):

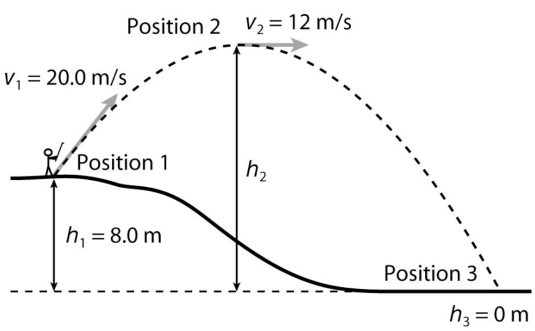
Solution: At ground level, the height of the ball is zero ($h_3 = 0$ m). At this point, its gravitational potential energy is also zero. However, the total energy of the ball remains constant.

$$\begin{aligned} E_m &= E_g + E_k \\ &= mgh_3 + \frac{mv_3^2}{2} \\ &= 0 + \frac{mv_3^2}{2} \\ E_m &= \frac{mv_3^2}{2} \\ v_3 &= \sqrt{\frac{2E_m}{m}} \\ &= \sqrt{\frac{2(107.2\ \text{J})}{0.20\ \text{kg}}} \\ &= \sqrt{1072\ \text{J/kg}} \\ &= 32.74\ \text{m/s} \\ v_3 &= 33\ \text{m/s} \end{aligned}$$

Statement: The velocity of the ball is 33 m/s as it hits the ground.

Section 5.3 Questions, page 241

1. (a) At the top of the building, the ball has gravitational energy. As it falls, the energy is converted to kinetic energy.
(b) Chemical energy is stored in the archer's arm. Elastic energy is stored in the bow and bowstring. As the arrow is released, both types of energy are converted to kinetic energy and transferred to the arrow.
(c) Chemical energy is stored in the fireworks. When the fireworks are lit, the chemical energy is converted to radiant energy (light), sound energy, and thermal energy (heat).
(d) Electrical energy (moving electrons) is transferred to the bulb with it is turned on. The electrical energy is converted to radiant energy (light) and thermal energy (heat).
(e) Chemical energy is stored in the gasoline. When the lawnmower is turned on, the chemical energy is converted to kinetic energy (moving parts), sound energy, thermal energy (heat), and electrical energy (spark plug).
2. Given: m = 45.9 g = 0.0459 kg; h_1 = 8.0 m; v_1 = 20.0 m/s; v_2 = 12 m/s; g = 9.8 N/kg; h_3 = 0 m



Required: E_m; h_2; v_3
Analysis: $E_m = E_k + E_g$; $E_k = ½\ mv^2$; $E_g = mgh$; E_m = constant
(a) The total mechanical energy at the start (at position 1):
Solution:

$$\begin{aligned}E_m &= E_k + E_g \\ &= \frac{1}{2}mv^2 + mgh \\ &= \frac{1}{2}(0.0459\text{ kg})(20.0\text{ m/s})^2 \\ &\quad + (0.0459\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)(8.0\text{ m}) \\ &= 9.18\ \frac{\text{kg}\bullet\text{m}^2}{\text{s}^2} + 3.60\text{ N}\bullet\text{m} \\ &= 12.78\text{ J} \\ E_m &= 13\text{ J}\end{aligned}$$

Statement: The total mechanical energy of the ball at the start is 13 J.
(b) The maximum height of the ball above the green (at position 2):
Solution:

$$\begin{aligned}E_m &= E_k + E_g \\ &= \frac{1}{2}mv^2 + mgh_2 \\ mgh_2 &= E_m - \frac{1}{2}mv^2 \\ h_2 &= \frac{2E_m - mv^2}{2mg} \\ &= \frac{2(12.78\text{ J}) - (0.0459\text{ kg})(12\text{ m/s})^2}{2(0.0459\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)} \\ &= 21.06\text{ m} \\ h_2 &= 21\text{ m}\end{aligned}$$

Statement: The maximum height of the ball above the green is 21 m.
(c) The speed of the ball when it strikes the green (at position 3):
Solution:

$$\begin{aligned}E_m &= E_k + E_g \\ &= \frac{1}{2}mv_3^2 + mgh_3 \\ &= \frac{1}{2}mv_3^2 + 0 \\ v_3 &= \sqrt{\frac{2E_m}{m}} \\ &= \sqrt{\frac{2(12.78\text{ J})}{0.0459\text{ kg}}} \\ &= 23.60\text{ m/s} \\ v_3 &= 24\text{ m/s}\end{aligned}$$

Statement: The speed of the ball when it strikes the green is 24 m/s.

3. Given: position 1 = top of first hill; position 2 = top of loop; $v_1 = 0$ m/s; $v_2 = 10.0$ m/s; $h_2 = 16$ m; $g = 9.8$ N/kg

Required: h_1, height of first hill

Analysis: $E_m = E_k + E_g$; $E_k = \frac{1}{2}mv^2$; $E_g = mgh$; E_m = constant

Solution:

$$E_{m1} = E_{m2}$$
$$E_{k1} + E_{g1} = E_{k2} + E_{g2}$$
$$\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gh_1 = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gh_2$$
$$v_1 = 0 \text{ m/s}$$
$$gh_1 = \frac{1}{2}v_2^2 + gh_2$$
$$2gh_1 = v_2^2 + 2gh_2$$
$$h_1 = \frac{v_2^2 + 2gh_2}{2g}$$
$$h_1 = \frac{(10.0 \text{ m/s})^2 + 2(9.8 \text{ N/kg})(16 \text{ m})}{2(9.8 \text{ N/kg})}$$
$$= 21.10 \text{ m}$$
$$h_1 = 21 \text{ m}$$

Statement: The minimum height of the first hill must be 21 m.

4. (a) Given: $v_i = 0$ m/s; $v_f = v$

Solution:

$$v_f^2 = v_i^2 + 2a\Delta d$$
$$v^2 = 0 + 2a\Delta d$$
$$v^2 = 2a\Delta d$$

(b) Given: $v^2 = 2a\Delta d$; $E_k = \frac{1}{2}mv^2$

Solution:

$$E_k = \frac{mv^2}{2}$$
$$= \frac{m(\cancel{2}a\Delta d)}{\cancel{2}}$$
$$= ma\Delta d$$
$$= F_{net}\Delta d$$
$$E_k = W_{net}$$

Statement: This result shows that the final kinetic energy of an object is equal to the net work done on the object if it starts at rest.