Section 4.3: Solving Friction Problems

Tutorial 1 Practice, page 174

1. (a) Given: m_T = 52 kg + 34 kg = 86 kg; μ_S = 0.35

Required: $F_{S_{max}}$

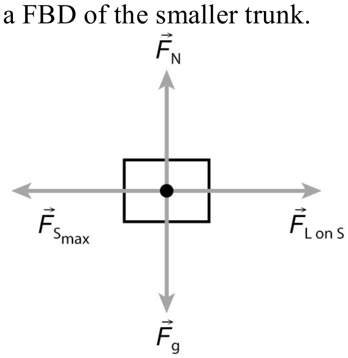
Analysis: $\mu_S = \frac{F_{S_{max}}}{F_N}$

Solution:

$$\begin{aligned} F_{S_{max}} &= \mu_S F_N \\ &= \mu_S mg \\ &= (0.35)(86\text{ kg})(9.8\text{ m/s}^2) \\ F_{S_{max}} &= 290\text{ N} \end{aligned}$$

Statement: The magnitude of the maximum force the person can exert without moving either trunk is 290 N.

(b) Draw a FBD of the smaller trunk.



Given: m = 34 kg; μ_S = 0.35

Required: $\vec{F}_{L\text{ on }S}$

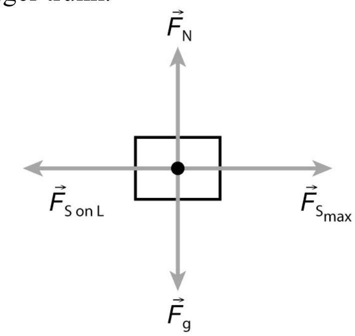
Analysis: Since the smaller trunk does not move, the magnitude of the force that the larger trunk exerts on the smaller trunk equals the magnitude of the static friction acting on the smaller trunk. Use the equation $\mu_S = \frac{F_{S_{max}}}{F_N}$ to calculate $F_{S_{max}}$. Choose right as positive. So left is negative.

Solution:

$$\begin{aligned} F_{S_{max}} &= \mu_S F_N \\ &= \mu_S mg \\ &= (0.35)(34\text{ kg})(9.8\text{ m/s}^2) \\ &= +120\text{ N} \\ \vec{F}_{S_{max}} &= 120\text{ N [right]} \end{aligned}$$

Statement: The force that the larger trunk exerts on the smaller trunk is 120 N [right].

(c) Since we can combine the two trunks and treat them as one single object, when the person pushed in the opposite direction on the smaller trunk, the answer to part (a) remains the same. However, the answer to part (b) would change. Look at this FBD of the larger trunk.



When the direction of the pushing force is in the opposite direction, the force exerted by the larger trunk on the smaller trunk will still be to the right. According to the FBD of the larger trunk above, the force that the smaller trunk exerts on the larger trunk is:

$$\begin{aligned} F_{S_{max}} &= \mu_S F_N \\ &= \mu_S mg \\ &= (0.35)(52\text{ kg})(9.8\text{ m/s}^2) \\ &= +180\text{ N} \\ F_{S_{max}} &= 180\text{ N [right]} \end{aligned}$$

So, the force that the larger trunk exerts on the smaller trunk is 180 N [right].

2. Given: m_1 = 4.0 kg; m_2 = 1.8 kg

Required: μ_S

Analysis: The tension is the same throughout the string. First calculate the tension using the equation $F_T = m_2 g$ for the hanging object. As the wooden block is stationary, the tension and the static friction will cancel. So F_T equals $F_{S_{max}}$. Then use the equation $\mu_S = \frac{F_{S_{max}}}{F_N}$ to calculate μ_S.

Solution:

$$\begin{aligned} F_T &= m_2 g \\ &= (1.8\text{ kg})(9.8\text{ m/s}^2) \\ F_T &= 17.64\text{ N (two extra digits carried)} \end{aligned}$$

$$\begin{aligned} \mu_S &= \frac{F_{S_{max}}}{F_N} \\ &= \frac{F_T}{m_1 g} \\ &= \frac{17.64\text{ N}}{(4.0\text{ kg})(9.8\text{ m/s}^2)} \\ \mu_S &= 0.45 \end{aligned}$$

Statement: The coefficient of static friction between the wooden block and the table is 0.45.

Tutorial 2 Practice, page 175

1. (a) Given: $m = 59$ kg; $\mu_S = 0.52$

Required: a

Analysis: First calculate the maximum force of static friction using $F_{S_{max}} = \mu_S F_N$. When the person starts to run, $F_{net} = F_{S_{max}}$. Then use $F_{net} = ma$ to calculate the acceleration.

Solution:

$$\begin{aligned} F_{S_{max}} &= \mu_S F_N \\ &= \mu_S mg \\ &= (0.52)(59 \text{ kg})(9.8 \text{ m/s}^2) \\ F_{S_{max}} &= 300 \text{ N} \end{aligned}$$

Calculate the magnitude of the acceleration.

$$\begin{aligned} F_{net} &= F_{S_{max}} \\ ma &= 300 \text{ N} \\ (59 \text{ kg})a &= 300 \text{ N} \\ a &= 5.1 \text{ m/s}^2 \end{aligned}$$

Statement: The maximum possible initial acceleration of the person wearing dress shoes is 5.1 m/s^2 [forwards].

(b) When we substitute $F_{S_{max}} = \mu_S F_N$ into the equation $F_{net} = F_{S_{max}}$ and simplify, the mass of the person cancels out.

$$\begin{aligned} F_{net} &= F_{S_{max}} \\ ma &= \mu_S F_N \\ ma &= \mu_S mg \\ a &= \mu_S g \end{aligned}$$

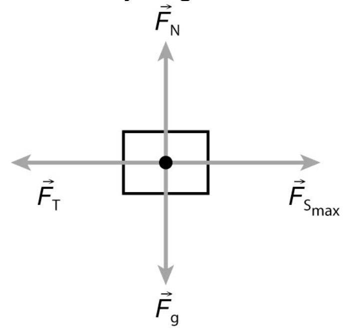
So we do not need the mass of either person when finding the maximum possible initial acceleration.

(c) The ratio of the two accelerations is:

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{\mu_{S1} g}{\mu_{S2} g} \\ &= \frac{\mu_{S1}}{\mu_{S2}} \\ &= \frac{0.52}{0.66} \\ \frac{a_1}{a_2} &= 0.79 \end{aligned}$$

From the above calculation, the ratio of the two accelerations is equal to the ratio of the two coefficients of friction.

2. Draw a free body diagram of the off-ice person.



As the off-ice person overcomes the force of static friction in order to move ahead, the tension in the rope equals in magnitude to the static friction acting on the person.

Given: $m_1 = 78$ kg; $\mu_S = 0.65$; $m_2 = 58$ kg

Required: a

Analysis: First calculate the tension in the rope using $F_T = F_{S_{max}} = \mu_S F_N$. When the skater starts to accelerate, $F_{net} = F_T$. Then use $F_{net} = m_2 a$ to calculate the acceleration.

Solution:

$$\begin{aligned} F_T &= F_{S_{max}} \\ &= \mu_S m_1 g \\ &= (0.65)(78 \text{ kg})(9.8 \text{ m/s}^2) \\ F_T &= 497 \text{ N (one extra digit carried)} \end{aligned}$$

Calculate the magnitude of the acceleration.

$$\begin{aligned} F_{net} &= F_T \\ m_2 a &= 497 \text{ N} \\ (58 \text{ kg})a &= 497 \text{ N} \\ a &= 8.6 \text{ m/s}^2 \end{aligned}$$

Statement: The maximum possible acceleration of the skater is 8.6 m/s^2 [towards off-ice person].

Tutorial 3 Practice, page 177

1. (a) Given: $m = 0.170$ kg; $\vec{v}_1 = 21.2$ m/s [W]; $\mu_K = 0.005$; $\Delta d = 58.5$ m

Required: v_2

Analysis: Consider the forces acting on the puck. The magnitude of the net force on the puck equals the force of kinetic friction. First calculate the acceleration using $F_{net} = ma$. Then use the equation $v_2^2 = v_1^2 + 2a\Delta d$ to calculate the final speed of the puck.

Solution:

$$F_{net} = F_K$$
$$ma = \mu_K F_N$$
$$ma = \mu_K mg$$
$$a = \mu_K g$$
$$= (0.005)(9.8 \text{ m/s}^2)$$
$$a = 0.049 \text{ m/s}^2$$

The acceleration of the puck is 0.049 m/s^2.

Next calculate the final speed of the puck.

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2 = \sqrt{v_1^2 + 2a\Delta d}$$
$$= \sqrt{(-21.2 \text{ m/s})^2 + 2(-0.049 \text{ m/s}^2)(58.5 \text{ m})}$$
$$v_2 = 21.1 \text{ m/s}$$

Statement: The speed of the puck after travelling 58.5 m is 21.1 m/s.

(b) Given: m = 0.170 kg; μ_K = 0.047;
$\vec{v}_1$ = 21.2 m/s [W];
$\vec{v}_2$ = 21.06 m/s [W] (one extra digit carried)

Required: Δd

Analysis: First calculate the acceleration as done in part (a). Then use the equation $v_2^2 = v_1^2 + 2a\Delta d$ to calculate the distance travelled.

Solution:

$$F_{net} = F_K$$
$$ma = \mu_K F_N$$
$$ma = \mu_K mg$$
$$a = \mu_K g$$
$$= (0.047)(9.8 \text{ m/s}^2)$$
$$a = 0.461 \text{ m/s}^2 \quad \text{(one extra digit carried)}$$

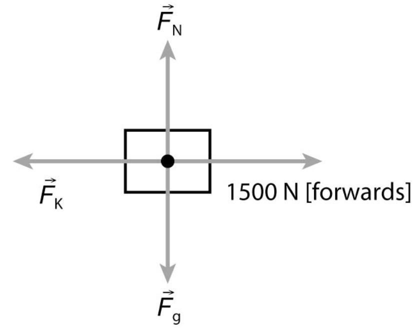
The magnitude of the acceleration of the puck is 0.461 m/s^2.

Next calculate the distance travelled.

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2^2 - v_1^2 = 2a\Delta d$$
$$\Delta d = \frac{v_2^2 - v_1^2}{2a}$$
$$= \frac{(-21.06 \text{ m/s})^2 - (-21.2 \text{ m/s})^2}{2(-0.461 \text{ m/s}^2)}$$
$$\Delta d = 6.42 \text{ m}$$

Statement: The puck will travel 6.42 m for the same initial and final speeds.

2. Draw a FBD of the snowmobile.



Given: m_T = 320 kg + 120 kg + 140 kg = 580 kg; m_1 = 120 kg; m_2 = 140 kg; μ_K = 0.15; F_a = 1500 N [forwards]

Required: a

Analysis: First calculate the force of kinetic friction for the sleds using the equation $F_K = \mu_K F_N$. Then use the equation $F_{net} = m_T a$ to calculate the acceleration. Choose forwards as positive. So backwards is negative.

Solution:

$$F_K = \mu_K F_N$$
$$= \mu_K (m_1 + m_2)g$$
$$= (0.15)(120 \text{ kg} + 140 \text{ kg})((9.8 \text{ m/s}^2)$$
$$F_K = 382 \text{ N (one extra digit carried)}$$

From the FBD of the snowmobile,

$$\vec{F}_{net} = \vec{F}_a + \vec{F}_K$$
$$m_T a = +1500 \text{ N} + (-382 \text{ N})$$
$$(580 \text{ kg})a = +1118 \text{ N}$$
$$a = +1.9 \text{ m/s}^2$$
$$\vec{a} = 1.9 \text{ m/s}^2 \text{ [forwards]}$$

Statement: The acceleration of the snowmobile and the sleds is 1.9 m/s^2 [forwards].

3. (a) Given: m_1 = 3.2 kg; m_2 = 1.5 kg; μ_K = 0.30

Required: a

Analysis: First calculate the kinetic friction acting on the object on the table using $F_K = \mu_K F_N$. Then consider the magnitudes of the forces acting on each object to determine the acceleration.

Solution:

$$F_K = \mu_K F_N$$
$$= \mu_K m_1 g$$
$$= (0.30)(3.2 \text{ kg})((9.8 \text{ m/s}^2)$$
$$F_K = 9.41 \text{ N (one extra digit carried)}$$

For the object on the table, the force of kinetic friction is in the opposite direction of motion.

$F_{net} = F_T - F_K$

$m_1 a = F_T - 9.41 \text{ N}$ (Equation 1)

For the hanging object, the tension acting is in the opposite direction of motion.

$F_{net} = F_g - F_T$

$m_2 a = m_2 g - F_T$ (Equation 2)

Add the equations to solve for a.

$$\begin{aligned}(m_1 + m_2)a &= m_2 g - 9.41 \text{ N} \\ (4.7 \text{ kg})a &= (1.5 \text{ kg})(9.8 \text{ m/s}^2) - 9.41 \text{ N} \\ a &= 1.13 \text{ m/s}^2 \\ a &= 1.1 \text{ m/s}^2\end{aligned}$$

Statement: The acceleration of the object on the table is 1.1 m/s^2 [right] and the acceleration of the hanging object is 1.1 m/s^2 [down].

(b) From equation 1,

$$\begin{aligned}m_1 a &= F_T - 9.41 \text{ N} \\ F_T &= m_1 a + 9.41 \text{ N} \\ &= (3.2 \text{ kg})(1.13 \text{ m/s}^2) + 9.41 \text{ N} \\ F_T &= 13 \text{ N}\end{aligned}$$

The magnitude of the tension in the string is 13 N.

(c) Given: m_1 = 3.2 kg; a = 1.13 m/s^2 [right]; Δt = 1.2 s; $\vec{v}_1$ = 1.3 m/s [right]

Required: Δd

Analysis: Use the equation $\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$ to calculate the distance travelled.

Solution:

$$\begin{aligned}\Delta d &= v_i \Delta t + \frac{1}{2} a \Delta t^2 \\ &= (+1.3 \text{ m/s})(1.2 \text{ s}) + \frac{1}{2}(+1.13 \text{ m/s}^2)(1.2 \text{ s})^2 \\ \Delta d &= 2.4 \text{ m}\end{aligned}$$

Statement: The objects will move 2.4 m in 1.2 s.

4. (a) Given: m = 125 kg; F_T = 350 N; a = 1.2 m/s^2 [forwards]

Required: μ_K

Analysis: First calculate the force of kinetic friction for the box using the equation $F_{net} = F_T + F_K$. Then use the equation $\mu_K = \frac{F_K}{F_N}$ to calculate μ_K. Choose forwards as positive. So backwards is negative.

Solution:

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_T + \vec{F}_K \\ ma &= +350 \text{ N} + F_K \\ (125 \text{ kg})(+1.2 \text{ m/s}^2) &= +350 \text{ N} + F_K \\ F_K &= -200 \text{ N} \\ \vec{F}_K &= 200 \text{ [backwards]}\end{aligned}$$

Use the magnitude of the kinetic friction to calculate μ_K.

$$\begin{aligned}\mu_S &= \frac{F_K}{F_N} \\ &= \frac{F_T}{mg} \\ &= \frac{200 \text{ N}}{(125 \text{ kg})(9.8 \text{ m/s}^2)} \\ \mu_S &= 0.16\end{aligned}$$

Statement: The coefficient of kinetic friction is 0.16.

(b) Given: a = 1.2 m/s^2 [forwards]; Δt = 5.0 s

Required: Δd

Analysis: Use the equation $\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$ to calculate the distance travelled.

Solution: Choose forwards as positive. So backwards is negative. Since the box starts its motion from rest, v_i = 0 m/s.

$$\begin{aligned}\Delta d &= \frac{1}{2} a \Delta t^2 \\ &= \frac{1}{2}(+1.2 \text{ m/s}^2)(5.0 \text{ s})^2 \\ \Delta d &= 15 \text{ m}\end{aligned}$$

Statement: The box travels 15 m up to the moment the cable breaks.

(c) Given: a_1 = 1.2 m/s^2 [forwards]; Δt = 5.0 s

Required: Δd

Analysis: First use the equation $v_2 = v_1 + a\Delta t$ to calculate the velocity v_2 of the box just before the cable breaks. For the second part of the motion, the tension in the cable is zero. Use $F_{net} = F_K$ to calculate the acceleration of the box. Then use the equation $v_f^{\,2} = v_i^{\,2} + 2a\Delta d$ to calculate the distance travelled when the motion of the box stops. Choose forwards as positive. So backwards is negative.

Solution: For the first part of the motion, the initial velocity v_1 of the box is 0 m/s.

$$\begin{aligned}\vec{v}_2 &= \vec{a}_1\Delta t \\ &= (+1.2\ \text{m/s}^2)(5.0\ \text{s}) \\ &= +6.0\ \text{m/s} \\ \vec{v}_2 &= 6.0\ \text{m/s [forwards]}\end{aligned}$$

For the second part of the motion,

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_{\text{K}} \\ ma_2 &= -200\ \text{N} \\ (125\ \text{kg})a_2 &= -200\ \text{N} \\ a_2 &= \frac{-200\ \text{N}}{125\ \text{kg}} \\ &= -1.6\ \text{m/s}^2 \\ \vec{a}_2 &= 1.6\ \text{m/s}^2\ \text{[backwards]}\end{aligned}$$

Now calculate the distance travelled. The initial velocity $\vec{v}_1$ is 6.0 m/s [forwards] and the final velocity $\vec{v}_{\text{f}}$ is 0 m/s.

$$\begin{aligned}v_{\text{f}}^2 &= v_{\text{i}}^2 + 2a\Delta d \\ 0 &= v_{\text{i}}^2 + 2a\Delta d \\ v_{\text{i}}^2 &= -2a\Delta d \\ \Delta d &= \frac{v_{\text{i}}^2}{-2a} \\ &= \frac{(+6.0\ \text{m/s})^2}{-2(-1.6\ \text{m/s}^2)} \\ \Delta d &= 11\ \text{m}\end{aligned}$$

Statement: The box travels 11 m from the moment the cable breaks until it stops.

Section 4.3 Questions, page 178

1. (a) Given: m = 64 kg; μ_{S} = 0.72

Required: $F_{\text{S}_{\text{max}}}$

Analysis: $\mu_{\text{S}} = \dfrac{F_{\text{S}_{\text{max}}}}{F_{\text{N}}}$

Solution:

$$\begin{aligned}F_{\text{S}_{\text{max}}} &= \mu_{\text{S}}F_{\text{N}} \\ &= \mu_{\text{S}}mg \\ &= (0.72)(64\ \text{kg})(9.8\ \text{m/s}^2) \\ F_{\text{S}_{\text{max}}} &= 450\ \text{N}\end{aligned}$$

Statement: The maximum force of static friction acting on the student is 450 N.

(b) Given: m = 250 kg; μ_{S} = 0.55

Required: $F_{\text{S}_{\text{max}}}$

Analysis: $\mu_{\text{S}} = \dfrac{F_{\text{S}_{\text{max}}}}{F_{\text{N}}}$

Solution:

$$\begin{aligned}F_{\text{S}_{\text{max}}} &= \mu_{\text{S}}F_{\text{N}} \\ &= \mu_{\text{S}}mg \\ &= (0.55)(250\ \text{kg})(9.8\ \text{m/s}^2) \\ F_{\text{S}_{\text{max}}} &= 1300\ \text{N}\end{aligned}$$

Statement: The maximum force of static friction acting on the box is 1300 N.

(c) Answers may vary. Sample answer:
The competition is unfair. We know that the magnitude of the coefficient of friction is always less than one. The mass of the box of books is more than three times the average mass of a student. To provide a force large enough to move the box, you need a large coefficient of static friction. However, it is unlikely that the coefficient for the student's shoes on the floor to be more than three times that for the box.

2. (a) Given: m_1 = 55 kg; m_2 = 78 kg

Required: μ_{S}

Analysis: First calculate the maximum magnitude of the maximum force of static friction for the actor on ice. Since neither actor is moving, the net force on each is zero. For the hanging actor, $F_{\text{T}} = F_{\text{g}} = m_2 g$. For the actor on ice, $F_{\text{T}} = F_{\text{S}_{\text{max}}}$.

Then use $\mu_{\text{S}} = \dfrac{F_{\text{S}_{\text{max}}}}{F_{\text{N}}}$ to calculate μ_{S}.

Solution:

$$\begin{aligned}\mu_{\text{S}} &= \frac{F_{\text{S}_{\text{max}}}}{F_{\text{N}}} \\ &= \frac{m_2 g}{m_1 g} \\ &= \frac{m_2}{m_1} \\ &= \frac{78\ \cancel{\text{kg}}}{55\ \cancel{\text{kg}}} \\ \mu_{\text{S}} &= 1.4\end{aligned}$$

Statement: The minimum coefficient of static friction is 1.4.

(b) Answers may vary. Sample answer:
The answer is not reasonable since the coefficient of static friction is usually less than one. For an ice surface, the force of static friction is very low and so will be the coefficient of static friction (often around 0.1).

(c) Answers may vary. Sample answer:
To make the scene more realistic, make the value of μ_S less than one by switching the two actors. Change the ice shelf to a shelf with a rough surface that will give a greater value of static friction to stop the heavier actor from sliding.

3. (a) Given: m_T = 5.0 kg + 3.0 kg = 8.0 kg; $F_{S_{max}}$ = 31.4 N

Required: μ_S

Analysis: $\mu_S = \frac{F_{S_{max}}}{F_N}$

Solution:

$$\begin{aligned}\mu_S &= \frac{F_{S_{max}}}{F_N} \\ &= \frac{F_{S_{max}}}{m_T g} \\ &= \frac{31.4 \text{ N}}{(8.0 \text{ kg})(9.8 \text{ m/s}^2)} \\ \mu_S &= 0.40\end{aligned}$$

Statement: The coefficient of static friction is 0.40.

(b) Consider forces acting on the second object. Since the net force is zero, the tension in the string equals the magnitude of the force of static friction.

$$\begin{aligned}F_T &= F_{S_{max}} \\ &= \mu_S F_N \\ &= \mu_S mg \\ &= (0.40)(3.0 \text{ kg})(9.8 \text{ m/s}^2) \\ F_T &= 12 \text{ N}\end{aligned}$$

If the students pull on the first object, the magnitude of the tension in the string is 12 N.

(c) The maximum force of static friction is given by the equation $F_{S_{max}} = \mu_S F_N$. If the students push on the second object with 15.0 N [down], the total normal force becomes: $F_N = m_T g + 15.0 \text{ N}$

$$\begin{aligned}F_{S_{max}} &= \mu_S F_N \\ &= \mu_S (m_T g + 15.0 \text{ N}) \\ &= (0.40)[(8.0 \text{ kg})(9.8 \text{ m/s}^2) + 15 \text{ N}] \\ F_{S_{max}} &= 37 \text{ N}\end{aligned}$$

So the magnitude of the maximum force of static friction is 37 N.

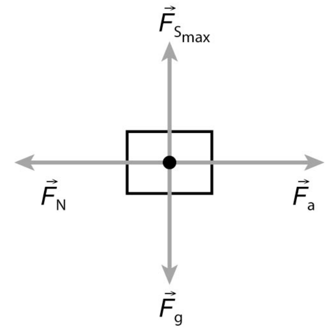
Since the net force is zero, the magnitude of the tension in the string equals the magnitude of the force of static friction on the second object.
In this case, $F_N = mg + 15.0 \text{ N}$.

$$\begin{aligned}F_T &= F_{S_{max}} \\ &= \mu_S F_N \\ &= \mu_S mg \\ &= (0.40)[(3.0 \text{ kg})(9.8 \text{ m/s}^2) + 15.0 \text{ N}] \\ F_T &= 18 \text{ N}\end{aligned}$$

So the magnitude of the tension is 18 N.

(d) If the student pushes down on the 5.0 kg object in part (c), the total normal force is the same so the answer will not change. However, the normal force on the second object will not be the same. So the answer for the magnitude of the tension in the string will change.

4. (a) Draw a FDB of the book.



Since the book does not move, the net force on the book is zero.

Given: m_1 = 0.80 kg; F_N = 26 N

Required: μ_S

Analysis: Use the equation $\mu_S = \frac{F_{S_{max}}}{F_N}$ to calculate μ_S. In this case, $F_{S_{max}} = F_g = mg$

Solution:

$$\begin{aligned}\mu_S &= \frac{F_{S_{max}}}{F_N} \\ &= \frac{mg}{F_N} \\ &= \frac{(0.80 \text{ kg})(9.8 \text{ m/s}^2)}{26 \text{ N}} \\ \mu_S &= 0.30\end{aligned}$$

Statement: The coefficient of static friction is 0.30.

(b) Answers may vary. Sample answer:
The student could add an object on top of the book or tie an object to the bottom of the book so that the magnitude of F_g is greater than the magnitude of $F_{S_{max}}$ to make the net force non-zero.

5. (a) Given: m_1 = 4.4 kg; μ_S = 0.42
Required: $F_{S_{max}}$
Analysis: $\mu_S = \dfrac{F_{S_{max}}}{F_N}$
Solution:
$$\begin{aligned} F_{S_{max}} &= \mu_S F_N \\ &= \mu_S m_1 g \\ &= (0.42)(4.4\text{ kg})(9.8\text{ m/s}^2) \\ &= 18.1\text{ N} \\ F_{S_{max}} &= 18\text{ N} \end{aligned}$$
Statement: The maximum force of static friction for the block is 18 N.
(b) Given: $F_{S_{max}}$ = 18.1 N; m_b = 0.12 kg; m_w = 0.02 kg
Required: n
Analysis: Since the block is not moving, the net force is zero. So $F_T = F_{S_{max}}$. For the bucket, $F_T = F_g = m_T g$. Use the equation $F_{S_{max}} = m_T g$ to calculate the total mass, m_T, of bucket and washers added. Then use the equation $m_T = m_b + nm_w$ to find n.
Solution:
$$\begin{aligned} F_{S_{max}} &= m_T g \\ m_T &= \frac{F_{S_{max}}}{g} \\ &= \frac{18.1\text{ N}}{9.8\text{ m/s}^2} \\ m_T &= 1.85\text{ kg} \;\; \text{(one extra digit carried)} \end{aligned}$$

Calculate the maximum number of washers added.
$$\begin{aligned} m_T &= m_b + nm_w \\ nm_w &= m_T - m_b \\ n &= \frac{m_T - m_b}{m_w} \\ &= \frac{1.85\text{ kg} - 0.12\text{ kg}}{0.02\text{ kg}} \\ n &= 86.5 \end{aligned}$$
Statement: The students can add 86 washers to the bucket without moving the block.
(c) Answers may vary. Sample answer:
This investigation may not yield accurate results if the students use it to find the coefficient of static friction. The number of washers added is a discrete quantity so the total mass m_T found could differ by a quantity of 0.02 kg. This difference will affect the accuracy of $F_{S_{max}}$ used to find the coefficient of static friction.

(d) Given: m_1 = 4.4 kg; μ_K = 0.34; m_T = 0.12 kg + 87(0.02 kg) = 1.86 kg;
Required: $\vec{a}$
Analysis: First calculate the kinetic friction acting on the block using $F_K = \mu_K F_N$. Then consider the magnitudes of the forces acting on the block and the bucket containing the washers to determine the acceleration.
Solution: First calculate the kinetic friction for the block.
$$\begin{aligned} F_K &= \mu_K F_N \\ &= \mu_K m_1 g \\ &= (0.34)(4.4\text{ kg})((9.8\text{ m/s}^2) \\ F_K &= 14.66\text{ N} \;\; \text{(two extra digits carried)} \end{aligned}$$

The force of kinetic friction is in the opposite direction of motion. For the block,
$$\begin{aligned} F_{net} &= F_T - F_K \\ m_1 a &= F_T - 14.66\text{ N} \;\; \text{(Equation 1)} \end{aligned}$$

For the bucket with the washers, the tension acting is in the opposite direction of motion.
$$\begin{aligned} F_{net} &= F_g - F_T \\ m_T a &= m_T g - F_T \;\; \text{(Equation 2)} \end{aligned}$$

Add the equations to solve for a.
$$\begin{aligned} m_1 a + m_T a &= m_T g - 14.66\text{ N} \\ (m_1 + m_T)a &= m_T g - 14.66\text{ N} \\ (4.4\text{ kg} + 1.86\text{ kg})a &= (1.86\text{ kg})(9.8\text{ m/s}^2) - 14.66\text{ N} \\ a &= 0.57\text{ m/s}^2 \end{aligned}$$
Statement: The acceleration of the block when the 87th washer is added is 0.57 m/s^2 [right].
6. (a) Given: m_A = 6(65 kg) = 390 kg; $F_{S_{max}}$ = 3200 N
Required: μ_S
Analysis: $\mu_S = \dfrac{F_{S_{max}}}{F_N}$
Solution:
$$\begin{aligned} \mu_S &= \frac{F_{S_{max}}}{F_N} \\ &= \frac{F_{S_{max}}}{m_A g} \\ &= \frac{3200\text{ N}}{(390\text{ kg})(9.8\text{ m/s}^2)} \\ \mu_S &= 0.84 \end{aligned}$$
Statement: Team A's coefficient of static friction is 0.84.

(b) Given: m_A = 390 kg; F_K = 2900 N
Required: μ_K
Analysis: $\mu_K = \dfrac{F_K}{F_N}$
Solution:

$$\begin{aligned}\mu_K &= \frac{F_K}{F_N}\\ &= \frac{F_K}{m_A g}\\ &= \frac{2900 \text{ N}}{(390 \text{ kg})(9.8 \text{ m/s}^2)}\\ \mu_K &= 0.76\end{aligned}$$

Statement: Team A's coefficient of kinetic friction is 0.76.
7. (a) Given: m = 260 kg;
a = 0.30 m/s^2 [forwards];
F_a = (280 N + 340 N) [forwards]
= 620 N [forwards]
Required: μ_K
Analysis: First find the force of kinetic friction for the piano using the equation $F_{net} = F_a + F_K$. Then use the equation $\mu_K = \dfrac{F_K}{F_N}$ to calculate μ_K. Choose forwards as positive. So backwards is negative.
Solution:

$$\begin{aligned}F_{net} &= F_a + F_K\\ ma &= +620 \text{ N} + F_K\\ (260 \text{ kg})((+0.30 \text{ m/s}^2) &= +620 \text{ N} + F_K\\ F_K &= -542 \text{ N}\end{aligned}$$

Use the magnitude of the force of kinetic friction to calculate the coefficient of kinetic friction.

$$\begin{aligned}\mu_K &= \frac{F_K}{F_N}\\ &= \frac{F_K}{mg}\\ &= \frac{542 \text{ N}}{(260 \text{ kg})(9.8 \text{ m/s}^2)}\\ \mu_K &= 0.21\end{aligned}$$

Statement: The coefficient of kinetic friction is 0.21.
(b) Given: m = 260 kg; $\vec{a}_1$ = 0.30 m/s^2 [forwards];
Δt_1 = 6.2 s
Required: Δt_2

Analysis: First use the equation $v_2 = v_1 + a\Delta t$ to calculate the velocity v_2 of the piano just before the students stop pushing. Use $F_{net} = F_K$ to calculate the new acceleration of the piano. Then use the equation $v_f = v_i + a\Delta t$ to calculate the time it takes the piano to stop moving. Choose forwards as positive. So backwards is negative.
Solution: When the students are pushing, the initial velocity v_1 of the box is 0 m/s.

$$\begin{aligned}\vec{v}_2 &= \vec{a}_1\Delta t_1\\ v_2 &= \left(+0.30\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(6.2\ \cancel{\text{s}})\\ v_2 &= +1.86 \text{ m/s}\\ \vec{v}_2 &= 1.86 \text{ m/s [forwards]}\end{aligned}$$

When the students stop pushing,

$$\begin{aligned}F_{net} &= F_K\\ ma_2 &= -542 \text{ N}\\ (260 \text{ kg})a_2 &= -542 \text{ N}\\ a_2 &= \frac{-542 \text{ N}}{260 \text{ kg}}\\ a_2 &= -2.08 \text{ m/s}^2 \text{ (one extra digit carried)}\end{aligned}$$

Now use the new acceleration to calculate the time it takes the piano to stop moving. The initial velocity v_i is 1.86 m/s [forwards] and the final velocity v_f is 0 m/s.

$$\begin{aligned}v_f &= v_i + a_2\Delta t_2\\ 0 &= +1.86 \text{ m/s} + (-2.08 \text{ m/s}^2)\Delta t_2\\ \Delta t_2 &= 0.89 \text{ s}\end{aligned}$$

Statement: It will take the piano 0.89 s to stop moving.
8. (a) (i) Given: m = 65 kg; $\vec{F}_a$ = 250 N [forwards]; $\vec{F}_K$ = 62 N [backwards]
Required: $\vec{a}$
Analysis: Use the equation $\vec{F}_{net} = \vec{F}_a + \vec{F}_K$ to find the net force on the sprinter and use the equation $\vec{F}_{net} = m\vec{a}$ to calculate $\vec{a}$. Choose forwards as positive. So backwards is negative.
Solution:

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_a + \vec{F}_K\\ ma &= +250 \text{ N} + (-62 \text{ N})\\ (65 \text{ kg})a &= +188 \text{ N}\\ a &= +2.89 \text{ m/s}^2\\ \vec{a} &= 2.9 \text{ m/s}^2 \text{ [forwards]}\end{aligned}$$

Statement: The acceleration of the sprinter is 2.9 m/s^2 [forwards].

(ii) Given: $\vec{a} = 2.89\text{ m/s}^2$ [forwards]; $\Delta t = 2.0$ s
Required: Δd

Analysis: Use the equation $\Delta d = v_i \Delta t + \frac{1}{2}a\Delta t^2$ to calculate the distance travelled. Since the sprinter starts from rest, $v_i = 0$ m/s.

Solution:

$$\begin{aligned}\Delta d &= \frac{1}{2}a\Delta t^2 \\ &= \frac{1}{2}(+2.89\text{ m/s}^2)(2.0\text{ s})^2 \\ \Delta d &= 5.8\text{ m}\end{aligned}$$

Statement: The distance travelled is 5.8 m.

(iii) Given: $m = 65$ kg; $F_{S_{max}} = 250$ N

Required: μ_S

Analysis: $\mu_S = \frac{F_{S_{max}}}{F_N}$

Solution:

$$\begin{aligned}\mu_K &= \frac{F_{S_{max}}}{F_N} \\ &= \frac{F_{S_{max}}}{mg} \\ &= \frac{250\text{ N}}{(65\text{ kg})(9.8\text{ m/s}^2)} \\ \mu_K &= 0.39\end{aligned}$$

Statement: The coefficient of friction between the sprinter's shoes and the track is 0.39.

(b) The friction applied on the sprinter from the ground is static friction. When the sprinter's shoes push backwards on the ground, the ground pushes back on the sprinter's feet with a reaction force equal in magnitude to the force that pushes the sprinter forwards. This force is the static friction that will start the sprinter moving from rest. Every time the sprinter pushes backwards on the ground, the ground provides this force to keep the sprinter moving. So this applied force is static friction.

9. Given: $m = 15.0$ kg; $\vec{v}_f = 1.2$ m/s [forwards]; $\Delta t = 2.0$ s; $\mu_K = 0.25$

Required: $\vec{F}_a$

Analysis: First use the equation $a = \frac{v_f - v_i}{\Delta t}$ to calculate the acceleration of the lawnmower and use the equation $F_K = \mu_K F_N$ to calculate the force of kinetic friction acting on it. Then use $F_{net} = ma$ and $F_{net} = F_a + F_K$ to calculate the applied force. Choose forwards as positive. So backwards is negative. Since the homeowner starts from rest, $v_i = 0$ m/s.

Solution:

$$\begin{aligned}a &= \frac{v_f - v_i}{\Delta t} \\ &= \frac{v_f}{\Delta t} \\ &= \frac{+1.2\text{ m/s}}{2.0\text{ s}} \\ a &= +0.60\text{ m/s}^2\end{aligned}$$

Calculate the kinetic friction that acts in the opposite direction of motion.

$$\begin{aligned}F_K &= \mu_K F_N \\ &= \mu_K mg \\ &= (0.25)(15.0\text{ kg})(9.8\text{ m/s}^2) \\ F_K &= 36.75\text{ N}\end{aligned}$$

Now calculate the applied force.

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_a + \vec{F}_K \\ ma &= F_a + (-36.75\text{ N}) \\ (15.0\text{ kg})(+0.60\text{ m/s}^2) &= F_a - 36.75\text{ N} \\ F_a &= +46\text{ N} \\ \vec{F}_a &= 46\text{ N [forwards]}\end{aligned}$$

Statement: The horizontal applied force acting on the lawnmower is 46 N [forwards].

10. Given: $m = 75$ kg; $v_i = 2.8$ m/s [forwards]; $\Delta d = 3.8$ m

Required: μ_K

Analysis: First use the equation $v_f^2 = v_i^2 + 2a\Delta d$ to calculate the acceleration of the baseball player. Then use $F_{net} = F_a + F_K$ to calculate the force of kinetic friction and use $\mu_K = \frac{F_K}{F_N}$ to calculate μ_K.

Choose forwards as positive. So backwards is negative. Since the player slides to come to rest, $v_f = 0$ m/s.

Solution:

$$0 = v_i^2 + 2a\Delta d$$

$$a = \frac{v_i^2}{-2\Delta d}$$

$$= \frac{(+2.8\ \text{m/s})^2}{-2(3.8\ \text{m})}$$

$$a = -1.03\ \text{m/s}^2 \quad \text{(one extra digit carried)}$$

Since there is no applied force on the player,

$$F_{net} = F_K$$

$$ma = F_K$$

Now calculate μ_K.

$$\mu_K = \frac{F_K}{F_N}$$

$$= \frac{ma}{mg}$$

$$= \frac{a}{g}$$

$$= \frac{1.03\ \cancel{\text{m/s}^2}}{9.8\ \cancel{\text{m/s}^2}}$$

$$\mu_K = 0.11$$

Statement: The coefficient of kinetic friction is 0.11.