Section 2.3: Projectile Motion

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1. (a) Given: $\Delta d_y = -32$ m; $a_y = -9.8$ m/s^2; $v_y = 0$ m/s

Required: Δt

Analysis:
$$\begin{aligned}\Delta d_y &= v_y \Delta t + \frac{1}{2}a_y \Delta t^2 \\ \Delta d_y &= 0 + \frac{1}{2}a_y \Delta t^2 \\ \Delta t^2 &= \frac{2\Delta d_y}{a_y} \\ \Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}}\end{aligned}$$

Solution:
$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}} \\ &= \sqrt{\frac{2(-32\ \cancel{\text{m}})}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}} \\ &= 2.556\ \text{s} \\ \Delta t &= 2.6\ \text{s}\end{aligned}$$

Statement: The hockey puck is in flight for 2.6 s.

(b) Given: $\Delta t = 2.6$ s; $a_x = 0$ m/s^2; $v_x = 8.6$ m/s

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution:
$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= \left(8.6\ \frac{\text{m}}{\cancel{\text{s}}}\right)(2.556\ \cancel{\text{s}})\ \text{(two extra digits carried)} \\ \Delta d_x &= 22\ \text{m}\end{aligned}$$

Statement: The range of the hockey puck is 22 m.

2. Since the velocity of the puck does not affect its vertical motion, it would still take 2.6 s to hit the ground. The range of the puck would be half because it is travelling at half the velocity (horizontally). That means the range would be 11 m.

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1. Given: $\Delta d_y = -17$ m; $a_y = -9.8$ m/s^2; $v_i = 7.3$ m/s; $\theta = 25°$

First determine the time of flight:

Required: Δt

Analysis: $\Delta d_y = v_y \Delta t + \frac{1}{2}a_y \Delta t^2$

Solution:
$$\begin{aligned}\Delta d_y &= v_y \Delta t + \frac{1}{2}a_y \Delta t^2 \\ &= v_i(\sin\theta)\Delta t + \frac{1}{2}a_y \Delta t^2 \\ -17 &= (7.3)(\sin 25°)\Delta t + \frac{1}{2}(-9.8)\Delta t^2 \\ 0 &= 3.085\Delta t - 4.9\Delta t^2 + 17 \\ \Delta t &= \frac{-3.085 \pm \sqrt{3.085^2 - 4(-4.9)(17)}}{2(-4.9)} \\ &= \frac{-3.085 \pm 18.51}{-9.8} \\ \Delta t &= -1.574\ \text{s or } \Delta t = 2.204\ \text{s}\end{aligned}$$

The answer must be the positive value.

Statement: The superhero is in flight for 2.2 s.

Determine the range:

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution:
$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= v_i \cos\theta \Delta t \\ &= \left(7.3\ \frac{\text{m}}{\cancel{\text{s}}}\right)(\cos 25°)(2.204\ \cancel{\text{s}})\ \text{(two extra digits carried)} \\ \Delta d_x &= 15\ \text{m}\end{aligned}$$

Statement: The superhero travels 15 m horizontally before landing.

Determine the final velocity:

Required: $\vec{v}_f$

Analysis: $\vec{v}_f = \vec{v}_{fx} + \vec{v}_{fy}$

Solution:
$$\begin{aligned}\vec{v}_f &= \vec{v}_{fx} + \vec{v}_{fy} \\ &= (v_i \cos\theta) + (\vec{v}_{iy} + \vec{a}_y \Delta t) \\ &= v_i \cos\theta + v_i \sin\theta + \vec{a}_y \Delta t \\ &= \left(7.3\ \frac{\text{m}}{\text{s}}\right)(\cos 25°)\ [\text{right}] + \left(7.3\ \frac{\text{m}}{\text{s}}\right)(\sin 25°)\ [\text{up}] \\ &\quad + \left(9.8\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\ [\text{down}]\right)(2.204\ \cancel{\text{s}})\ \text{(two extra digits carried)} \\ &= 6.616\ \text{m/s [right]} + 3.085\ \text{[up]} + 21.60\ \text{m/s [down]} \\ &= 6.616\ \text{m/s [right]} - 3.085\ \text{[down]} + 21.60\ \text{m/s [down]} \\ \vec{v}_f &= 6.616\ \text{m/s [right]} + 18.52\ \text{m/s [down]}\end{aligned}$$

Use the Pythagorean theorem:
$$\begin{aligned}{v_f}^2 &= {v_{fx}}^2 + {v_{fy}}^2 \\ v_f &= \sqrt{{v_{fx}}^2 + {v_{fy}}^2} \\ &= \sqrt{(6.616\ \text{m/s})^2 + (18.52\ \text{m/s})^2}\ \text{(two extra digits carried)} \\ v_f &= 2.0\ \times 10\ \text{m/s}\end{aligned}$$

Let ϕ represent the angle $\vec{v}_f$ makes with the x-axis.

$$\tan\phi = \frac{v_{fy}}{v_{fx}}$$
$$= \frac{18.52\ \frac{\cancel{m}}{\cancel{s}}}{6.616\ \frac{\cancel{m}}{\cancel{s}}} \text{ (two extra digits carried)}$$
$$= 2.799$$
$$\phi = \tan^{-1}(2.799)$$
$$\phi = 70°$$

Statement: The superhero's final velocity is 2.0×10 m/s [right 70° down].

2. The ball thrown at an angle above the horizontal will take longer to reach the ground. It has an initial velocity with a vertical component, so it will take more time for it to reach the ground due to gravity. The difference in the times to reach the ground depends on the initial height and the initial velocity of the first ball.

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1. The horizontal and vertical motions of a projectile take the same amount of time.

2. Given: $\Delta d_x = 20.0$ m; $a_y = -9.8$ m/s^2; $v_x = 10.0$ m/s; $v_y = 0$ m/s

Determine the time of flight first:

Required: Δt

Analysis: $v_x = \frac{\Delta d_x}{\Delta t}$

$$\Delta t = \frac{\Delta d_x}{v_x}$$

Solution: $\Delta t = \frac{\Delta d_x}{v_x}$

$$= \frac{20.0\ \cancel{m}}{10.0\ \frac{\cancel{m}}{s}}$$
$$\Delta t = 2.00 \text{ s}$$

Statement: It takes the tennis ball 2.00 s to reach the ground.

Determine the vertical displacement:

Required: Δd_y

Analysis: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

Solution: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

$$= 0 + \frac{1}{2}\left(9.8\ \frac{m}{\cancel{s^2}}\right)(2.00\ \cancel{s})^2$$
$$\Delta d_y = 2.0 \times 10 \text{ m}$$

Statement: The water tower is 2.0×10 m high.

3. (a) To give a projectile the greatest time of flight, launch it at 90° from the ground because this angle maximizes the vertical component of the velocity. At 90° from the ground, all the velocity is straight up instead of some of the velocity going into the horizontal component.

(b) To give a projectile the greatest time of flight, launch it at 45°. If the angle is less than 45°, the flight will be too short to travel any farther. If the angle is greater than 45°, the horizontal component of the velocity is too short to travel any farther.

4. (a) The ball experiences no horizontal acceleration. The ball accelerates 9.8 m/s^2 down due to gravity.

(b) The ball experiences no horizontal acceleration. The ball accelerates 9.8 m/s^2 down due to gravity.

(c) The ball experiences no horizontal acceleration. The ball accelerates 9.8 m/s^2 down due to gravity.

5. The arrow strikes the ground before reaching the target. It would take the arrow more than 1 s to travel to 60 m to the target when travelling at 55 m/s. But in 1 s, the arrow would fall more than 1.5 m due to gravity. For her next shot, the archer should increase the initial velocity, aim higher, or a combination of the two.

6. (a) Given: $v_i = 26$ m/s; $\theta = 60°$; $a_y = -9.8$ m/s^2; $v_{fy} = 0$ m/s

Required: Δt

Analysis: $v_{fy} = v_{iy} + a_y \Delta t$

$$\Delta t = \frac{v_{fy} - v_{iy}}{a_y}$$

Solution: $\Delta t = \frac{v_{fy} - v_{iy}}{a_y}$

$$= \frac{0 - (26 \text{ m/s})(\sin 60°)}{-9.8 \text{ m/s}^2}$$
$$= \frac{22.52\ \frac{\cancel{m}}{\cancel{s}}}{9.8\ \frac{\cancel{m}}{s^{\cancel{2}}}}$$
$$= 2.298 \text{ s}$$
$$\Delta t = 2.3 \text{ s}$$

Statement: It takes the acrobat 2.3 s to reach his maximum height.

(b) Given: $v_i = 26$ m/s; $\theta = 60°$; $a_y = -9.8$ m/s^2; $v_{fy} = 0$ m/s

Determine the maximum height:

Required: Δd_y

Analysis: $\Delta d_y = \frac{v_{fy} + v_{iy}}{2} \Delta t$

Solution:

$$\Delta d_y = \frac{v_{fy} + v_{iy}}{2}\Delta t$$
$$= \frac{0 - 22.52\ \frac{\text{m}}{\cancel{\text{s}}}}{2}(2.298\ \cancel{\text{s}})\ \text{(two extra digits carried)}$$
$$= 25.88\ \text{m}$$
$$\Delta d_y = 26\ \text{m}$$

Statement: The maximum height of the acrobat is 26 m.

Determine the time to reach half the maximum height (13 m) the second time:

Required: Δt

Analysis: $\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$

Solution: $\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$

$$13 = (22.52)\Delta t + \frac{1}{2}(-9.8)\Delta t^2$$
$$0 = 22.52\Delta t - 4.9\Delta t^2 - 13$$
$$\Delta t = \frac{-22.52 \pm \sqrt{22.52^2 - 4(-4.9)(-13)}}{2(-4.9)}$$
$$= \frac{-22.52 \pm 15.89}{-9.8}$$
$$\Delta t = 0.68\ \text{s or } \Delta t = 3.9\ \text{s}$$

The first time is on his way up, so the correct time is 3.9 s.

Statement: It takes the acrobat 3.9 s to reach a point halfway back down to the ground.

7. Given: $v_i = 20$ m/s; $\theta = 45°$; $a_y = -9.8$ m/s^2; $v_{fy} = 0$ m/s

Determine the time of flight:

Required: Δt

Analysis: $\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$

Solution:

$$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$0 = v_i(\sin 45°)\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$0 = v_i(\sin 45°) + \frac{1}{2}a_y\Delta t,\ \ \Delta t \neq 0$$
$$\frac{1}{2}a_y\Delta t = -v_i(\sin 45°)$$
$$\Delta t = \frac{-2v_i(\sin 45°)}{a_y}$$
$$= \frac{-2\left(20\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)(\sin 45°)}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}\right)}$$
$$= 2.886\ \text{s}$$
$$\Delta t = 2.9\ \text{s}$$

Statement: The time of flight of the golf ball is 2.9 s.

Determine the horizontal distance:

Required: Δd_x

Analysis: $\Delta d_x = v_{ix}\Delta t$

Solution: $\Delta d_x = v_{ix}\Delta t$

$$= v_i(\cos 45°)\Delta t$$
$$= \left(20\ \frac{\text{m}}{\cancel{\text{s}}}\right)(\sin 45°)(2.9\ \cancel{\text{s}})$$
$$\Delta d_x = 41\ \text{m}$$

Statement: The golfer was 41 m from the hole when he hit the ball.

Determine the maximum height:

Required: Δd_y

Analysis: $v_{fy}^{\ 2} = v_{iy}^{\ 2} + 2a_y\Delta d_y$

$$\Delta d_y = \frac{v_{fy}^{\ 2} - v_{iy}^{\ 2}}{2a_y}$$

Solution: $\Delta d_y = \frac{v_{fy}^{\ 2} - v_{iy}^{\ 2}}{2a_y}$

$$= \frac{0 - (v_i \sin 45°)^2}{2(-9.8\ \text{m/s}^2)}$$
$$= \frac{0 - [(20\ \text{m/s})(\sin 45°)]^2}{-19.6\ \text{m/s}^2}$$
$$= \frac{200\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{19.6\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$
$$\Delta d_y = 1.0 \times 10\ \text{m}$$

Statement: The golf ball reached a maximum height of 1.0 × 10 m.

8. Given: $\Delta d_y = -12$ m; $a_y = -9.8$ m/s^2; $v_i = 4.5$ m/s; $\theta = 25°$

First determine the time of flight:

Required: Δt

Analysis: $\Delta d_y = v_y\Delta t + \frac{1}{2}a_y\Delta t^2$

Solution: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

$$= v_{\mathrm{i}}(\sin\theta)\Delta t + \frac{1}{2} a_y \Delta t^2$$

$$-12 = (4.5)(\sin 25°)\Delta t + \frac{1}{2}(-9.8)\Delta t^2$$

$$0 = 1.902\,\Delta t - 4.9\,\Delta t^2 + 12$$

$$\Delta t = \frac{-1.902 \pm \sqrt{1.902^2 - 4(-4.9)(12)}}{2(-4.9)}$$

$$= \frac{-1.902 \pm 15.45}{-9.8}$$

$$\Delta t = -1.382 \text{ s or } \Delta t = 1.771 \text{ s}$$

The answer must be the positive value.
Statement: The beanbag is in flight for 1.8 s.

Determine the range:
Required: Δd_x
Analysis: $\Delta d_x = v_x \Delta t$
Solution:

$$\Delta d_x = v_x \Delta t$$

$$= v_{\mathrm{i}} \cos\theta \,\Delta t$$

$$= \left(4.5 \frac{\text{m}}{\cancel{\text{s}}}\right)(\cos 25°)(1.771\,\cancel{\text{s}}) \text{ (two extra digits carried)}$$

$$\Delta d_x = 7.2 \text{ m}$$

Statement: The student's friend must stand 7.2 m from the building to catch the beanbag.