Section 2.2: Motion in Two Dimensions—An Algebraic Approach

Tutorial 1 Practice, page 67

1. Given: $\Delta \vec{d}_1 = 27$ m [W]; $\Delta \vec{d}_2 = 35$ m [S]

Required: $\Delta \vec{d}_T$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2$

Solution: Let ϕ represent the angle $\Delta \vec{d}_T$ makes with the x-axis.

$$\begin{aligned}\Delta \vec{d}_T &= \Delta \vec{d}_1 + \Delta \vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(27\text{ m})^2 + (35\text{ m})^2}\\ \Delta d_T &= 44\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_2}{\Delta d_1}\\ \tan\phi &= \frac{35\text{ m}}{27\text{ m}}\\ \tan\phi &= 1.296\\ \phi &= \tan^{-1}(1.296)\\ \phi &= 52°\end{aligned}$$

Statement: The sum of the two vectors is 44 m [W 52° S].

2. Given: $\Delta \vec{d}_1 = 13.2$ m [S]; $\Delta \vec{d}_2 = 17.8$ m [E]

Required: $\Delta \vec{d}_T$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2$

Solution: Let ϕ represent the angle $\Delta \vec{d}_T$ makes with the y-axis.

$$\begin{aligned}\Delta \vec{d}_T &= \Delta \vec{d}_1 + \Delta \vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(13.2\text{ m})^2 + (17.8\text{ m})^2}\\ \Delta d_T &= 22.2\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_2}{\Delta d_1}\\ \tan\phi &= \frac{17.8\text{ m}}{13.2\text{ m}}\\ \tan\phi &= 1.348\\ \phi &= \tan^{-1}(1.348)\\ \phi &= 53°\end{aligned}$$

The sum of the two vectors is 22.2 m [S 53° E] or [E 37° S].

Statement: The sum of the two vectors is 22.2 m [E 37° S].

Tutorial 2 Practice, page 69

1. Given: $\Delta \vec{d}_T = 15$ m [W 35° N]

Required: $\Delta \vec{d}_x$; $\Delta \vec{d}_y$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_x + \Delta \vec{d}_y$

Solution: Since the direction of $\Delta \vec{d}_T$ is between west and north, the direction of $\Delta \vec{d}_x$ is [W] and the direction of $\Delta \vec{d}_y$ is [N].

$$\begin{aligned}\sin\theta &= \frac{\Delta d_y}{\Delta d_T}\\ \Delta d_y &= \Delta d_T \sin\theta\\ &= (15\text{ m})(\sin 35°)\\ &= 8.604\text{ m}\\ \Delta d_y &= 8.6\text{ m}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\Delta d_x}{\Delta d_T}\\ \Delta d_x &= \Delta d_T \cos\theta\\ &= (15\text{ m})(\cos 35°)\\ &= 12.29\text{ m}\\ \Delta d_x &= 12\text{ m}\end{aligned}$$

Statement: The vector has a horizontal or x-component of 12 m [W] and a vertical or y-component of 8.6 m [N].

2. Given: $\Delta \vec{d}_x = 27.2$ m [E]; $\Delta \vec{d}_y = 12.7$ m [N]

Required: $\Delta \vec{d}_T$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2$

Solution: Let ϕ represent the angle $\Delta \vec{d}_T$ makes with the x-axis.

$$\begin{aligned}\Delta \vec{d}_T &= \Delta \vec{d}_x + \Delta \vec{d}_y\\ \Delta d_T^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}}\\ &= \sqrt{(27.2\text{ m})^2 + (12.7\text{ m})^2}\\ \Delta d_T &= 30.0\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_y}{\Delta d_x}\\ \tan\phi &= \frac{12.7\text{ m}}{27.2\text{ m}}\\ \tan\phi &= 0.4669\\ \phi &= \tan^{-1}(0.4669)\\ \phi &= 25°\end{aligned}$$

Statement: The sum of the two vectors is 30.0 m [E 25° N], which is the original vector from Sample Problem 1.

Tutorial 3 Practice, page 71

1. Given: $\Delta\vec{d}_1 = 2.78$ cm [W];

$\Delta\vec{d}_2 = 6.25$ cm [S 40° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total *x*-component and *y*-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= 2.78 \text{ cm [W]} + (6.25 \text{ cm})(\sin 40°) \text{ [E]}\\ &= 2.78 \text{ cm [W]} + 4.0174 \text{ cm [E]}\\ &= -2.78 \text{ cm [E]} + 4.0174 \text{ cm [E]}\\ &= 1.2374 \text{ cm [E]}\\ \vec{d}_{Tx} &= 1.24 \text{ cm [E]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= 0 \text{ cm} + (6.25 \text{ cm})(\cos 40°) \text{ [S]}\\ &= 4.7878 \text{ cm [S]}\\ \vec{d}_{Ty} &= 4.79 \text{ cm [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2}\\ \Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}\\ &= \sqrt{(1.2374 \text{ cm})^2 + (4.7878 \text{ cm})^2}\\ &\quad \text{(two extra digits carried)}\\ \Delta d_T &= 4.95 \text{ cm}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}}\\ \tan\phi &= \frac{4.7878 \text{ }\cancel{\text{cm}}}{1.2374 \text{ }\cancel{\text{cm}}} \text{ (two extra digits carried)}\\ \tan\phi &= 3.869\\ \phi &= \tan^{-1}(3.869)\\ \phi &= 76°\end{aligned}$$

Statement: The ant's total displacement is 4.95 cm [E 76° S].

2. Given: $\Delta\vec{d}_1 = 2.64$ m [W 26° N];

$\Delta\vec{d}_2 = 3.21$ m [S 12° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total *x*-component and *y*-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (2.64 \text{ m})(\cos 26°) \text{ [W]} + (3.21 \text{ m})(\sin 12°) \text{ [E]}\\ &= 2.3728 \text{ m [W]} + 0.6674 \text{ m [E]}\\ &= 2.3728 \text{ m [W]} - 0.6674 \text{ m [W]}\\ &= 1.7054 \text{ m [W]}\\ \vec{d}_{Tx} &= 1.71 \text{ m [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (2.64 \text{ m})(\sin 26°) \text{ [N]} + (3.21 \text{ m})(\cos 12°) \text{ [S]}\\ &= 1.1573 \text{ m [N]} + 3.1399 \text{ m [S]}\\ &= -1.1573 \text{ m [S]} + 3.1399 \text{ m [S]}\\ &= 1.9826 \text{ m [S]}\\ \vec{d}_{Ty} &= 1.98 \text{ m [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2}\\ \Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}\\ &= \sqrt{(1.7054 \text{ m})^2 + (1.9826 \text{ m})^2} \text{ (two extra digits carried)}\\ \Delta d_T &= 2.62 \text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}}\\ \tan\phi &= \frac{1.9826 \text{ }\cancel{\text{m}}}{1.7054 \text{ }\cancel{\text{m}}} \text{ (two extra digits carried)}\\ \tan\phi &= 1.163\\ \phi &= \tan^{-1}(1.163)\\ \phi &= 49°\end{aligned}$$

Statement: The total displacement of the paper airplane is 2.62 m [W 49° S].

Tutorial 4 Practice, page 74

1. Answers may vary. Sample answer:
Imagine the river current is flowing south and the canoe is pointed east. As long as the boat is pointed perpendicular to the current, the current has no effect on the time it takes to cross the river. Think about the component vectors. Even though the canoe will be travelling in a direction between south and east, all its eastbound velocity is the same, no matter how fast the current is, or if there's any current at all.

2. (a) Given: Δd = 20.0 m; v = 1.3 m/s
Required: Δt

Analysis: $v = \dfrac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v}\\ &= \frac{20.0\ \cancel{\text{m}}}{1.3\ \frac{\cancel{\text{m}}}{\text{s}}}\\ &= 15.38\text{ s}\\ \Delta t &= 15\text{ s}\end{aligned}$$

Statement: It will take the swimmer 15 s to cross the river.

(b) Given: $\vec{v}_x$ = 2.7 m/s [W]

Required: $\Delta \vec{d}_x$

Analysis:
$$\begin{aligned}\vec{v}_x &= \frac{\Delta \vec{d}_x}{\Delta t}\\ \Delta \vec{d}_x &= \vec{v}_x \Delta t\end{aligned}$$

Solution:
$$\begin{aligned}\Delta \vec{d}_x &= \vec{v}_x \Delta t\\ &= \left(2.7\ \frac{\text{m}}{\cancel{\text{s}}}\ [\text{W}]\right)(15.38\ \cancel{\text{s}})\ \text{(two extra digits carried)}\\ \Delta \vec{d}_x &= 42\ \text{m [W]}\end{aligned}$$

Statement: The swimmer lands 42 m downstream from his intended location.

Section 2.2 Questions, page 75

1. (a) Given: $\Delta \vec{d}_\text{T}$ = 20 km [W 50° N]

Required: $\Delta \vec{d}_x$; $\Delta \vec{d}_y$

Analysis: $\Delta \vec{d}_\text{T} = \Delta \vec{d}_x + \Delta \vec{d}_y$

Solution: Since the direction of $\Delta \vec{d}_\text{T}$ is between west and north, the direction of $\Delta \vec{d}_x$ is [W] and the direction of $\Delta \vec{d}_y$ is [N].

$$\begin{aligned}\sin\theta &= \frac{\Delta d_y}{\Delta d_\text{T}}\\ \Delta d_y &= \Delta d_\text{T} \sin\theta\\ &= (20\text{ km})(\sin 50°)\\ \Delta d_y &= 15\text{ km}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\Delta d_x}{\Delta d_\text{T}}\\ \Delta d_x &= \Delta d_\text{T} \cos\theta\\ &= (20\text{ km})(\cos 50°)\\ \Delta d_x &= 13\text{ km}\end{aligned}$$

Statement: The vector has a vertical or y-component of 15 km [N] and a horizontal or x-component of 13 km [W].

(b) Given: $\Delta \vec{d}_\text{T}$ = 15 km [W 80° S]

Required: $\Delta \vec{d}_x$; $\Delta \vec{d}_y$

Analysis: $\Delta \vec{d}_\text{T} = \Delta \vec{d}_x + \Delta \vec{d}_y$

Solution: Since the direction of $\Delta \vec{d}_\text{T}$ is between west and south, the direction of $\Delta \vec{d}_x$ is [W] and the direction of $\Delta \vec{d}_y$ is [S].

$$\begin{aligned}\sin\theta &= \frac{\Delta d_y}{\Delta d_\text{T}}\\ \Delta d_y &= \Delta d_\text{T} \sin\theta\\ &= (15\text{ km})(\sin 80°)\\ &= 14.77\text{ km}\\ \Delta d_y &= 15\text{ km}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\Delta d_x}{\Delta d_\text{T}}\\ \Delta d_x &= \Delta d_\text{T} \cos\theta\\ &= (15\text{ km})(\cos 80°)\\ \Delta d_x &= 2.6\text{ km}\end{aligned}$$

Statement: The vector has a vertical or y-component of 15 km [S] and a horizontal or x-component of 2.6 km [W].

(c) Given: $\Delta \vec{d}_\text{T}$ = 40 km [N 65° E]

Required: $\Delta \vec{d}_x$; $\Delta \vec{d}_y$

Analysis: $\Delta \vec{d}_\text{T} = \Delta \vec{d}_x + \Delta \vec{d}_y$

Solution: Since the direction of $\Delta \vec{d}_\text{T}$ is between north and east, the direction of $\Delta \vec{d}_x$ is [E] and the direction of $\Delta \vec{d}_y$ is [N].

$$\begin{aligned}\cos\theta &= \frac{\Delta d_y}{\Delta d_\text{T}}\\ \Delta d_y &= \Delta d_\text{T} \cos\theta\\ &= (40\text{ km})(\cos 65°)\\ \Delta d_y &= 17\text{ km}\end{aligned}$$

$$\begin{aligned}\sin\theta &= \frac{\Delta d_x}{\Delta d_\text{T}}\\ \Delta d_x &= \Delta d_\text{T} \sin\theta\\ &= (40\text{ km})(\sin 65°)\\ \Delta d_x &= 36\text{ km}\end{aligned}$$

Statement: The vector has a vertical or y-component of 17 km [N] and a horizontal or x-component of 36 km [E].

2. Given: $\Delta\vec{d}_1 = 5.1$ km [E]; $\Delta\vec{d}_2 = 14$ km [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ \Delta d_T^2 &= \Delta d_1^2 + \Delta d_2^2\\ \Delta d_T &= \sqrt{\Delta d_1^2 + \Delta d_2^2}\\ &= \sqrt{(5.1\text{ km})^2 + (14\text{ km})^2}\\ \Delta d_T &= 15\text{ km}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_2}{\Delta d_1}\\ \tan\phi &= \frac{14\ \cancel{\text{km}}}{5.1\ \cancel{\text{km}}}\\ \tan\phi &= 2.745\\ \phi &= \tan^{-1}(2.745)\\ \phi &= 70°\end{aligned}$$

Statement: The sum of the two vectors is 15 km [E 70° N].

3. Given: $\Delta\vec{d}_1 = 11$ m [N 20° E]; $\Delta\vec{d}_2 = 9.0$ m [E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (11\text{ m})(\sin 20°)\ [\text{E}] + 9.0\text{ m [E]}\\ &= 3.762\text{ m [E]} + 9.0\text{ m [E]}\\ &= 12.76\text{ m [E]}\\ \vec{d}_{Tx} &= 13\text{ m [E]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (11\text{ m})(\cos 20°)\ [\text{N}] + 0\text{ m}\\ &= 10.34\text{ m [N]}\\ \vec{d}_{Ty} &= 10\text{ m [N]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^2 &= d_{Tx}^2 + d_{Ty}^2\\ \Delta d_T &= \sqrt{d_{Tx}^2 + d_{Ty}^2}\\ &= \sqrt{(12.76\text{ m})^2 + (10.34\text{ m})^2} \quad \text{(two extra digits carried)}\\ \Delta d_T &= 16\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Tx}}{\Delta d_{Ty}}\\ \tan\phi &= \frac{12.76\ \cancel{\text{m}}}{10.34\ \cancel{\text{m}}} \quad \text{(two extra digits carried)}\\ \tan\phi &= 1.234\\ \phi &= \tan^{-1}(1.234)\\ \phi &= 51°\end{aligned}$$

Statement: The total displacement of the football player is 16 m [N 51° E].

4. Given: $\Delta\vec{d}_1 = 200.0$ m [S 25° W]; $\Delta\vec{d}_2 = 150.0$ m [N 30° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (200.0\text{ m})(\sin 25°)\ [\text{W}] + (150.0\text{ m})(\sin 30°)\ [\text{E}]\\ &= 84.5236\text{ m [W]} + 75\text{ m [E]}\\ &= 84.5236\text{ m [W]} - 75\text{ m [W]}\\ &= 9.5236\text{ m [W]}\\ \vec{d}_{Tx} &= 9.524\text{ m [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (200.0\text{ m})(\cos 25°)\ [\text{S}] + (150.0\text{ m})(\cos 30°)\ [\text{N}]\\ &= 181.262\text{ m [S]} + 129.904\text{ m [N]}\\ &= 181.262\text{ m [S]} - 129.904\text{ m [S]}\\ &= 51.358\text{ m [S]}\\ \vec{d}_{Ty} &= 51.36\text{ m [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^2 &= d_{Tx}^2 + d_{Ty}^2\\ \Delta d_T &= \sqrt{d_{Tx}^2 + d_{Ty}^2}\\ &= \sqrt{(9.5236\text{ m})^2 + (51.358\text{ m})^2} \quad \text{(one extra digit carried)}\\ \Delta d_T &= 52.23\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Tx}}{\Delta d_{Ty}}\\ \tan\phi &= \frac{9.5236\ \cancel{\text{m}}}{51.358\ \cancel{\text{m}}} \quad \text{(one extra digit carried)}\end{aligned}$$

$\tan\phi = 0.1854$
$\phi = \tan^{-1}(0.1854)$
$\phi = 11°$

Statement: The total displacement of the boat is 52.23 m [S 11° W].

5. Given: $\Delta\vec{d}_1 = 25$ m [N 20° W]; $\Delta\vec{d}_2 = 35$ m [S 15° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (25\text{ m})(\sin 20°)\text{ [W]} + (35\text{ m})(\sin 15°)\text{ [E]}\\ &= 8.551\text{ m [W]} + 9.059\text{ m [E]}\\ &= -8.551\text{ m [E]} + 9.059\text{ m [E]}\\ &= 0.508\text{ m [E]}\\ \vec{d}_{Tx} &= 0.51\text{ m [E]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (25\text{ m})(\cos 20°)\text{ [N]} + (35\text{ m})(\cos 15°)\text{ [S]}\\ &= 23.49\text{ m [N]} + 33.81\text{ m [S]}\\ &= -23.49\text{ m [S]} + 33.81\text{ m [S]}\\ &= 10.32\text{ m [S]}\\ \vec{d}_{Ty} &= 1.0\times 10\text{ m [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T{}^2 &= d_{Tx}{}^2 + d_{Ty}{}^2\\ \Delta d_T &= \sqrt{d_{Tx}{}^2 + d_{Ty}{}^2}\\ &= \sqrt{(0.508\text{ m})^2 + (10.32\text{ m})^2}\ \text{(two extra digits carried)}\\ \Delta d_T &= 1.0\times 10\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\tan\phi = \frac{\Delta d_{Tx}}{\Delta d_{Ty}}$$

$$\tan\phi = \frac{0.508\ \cancel{\text{m}}}{10.32\ \cancel{\text{m}}}\ \text{(one extra digit carried)}$$

$\tan\phi = 0.0492$
$\phi = \tan^{-1}(0.0492)$
$\phi = 3°$

Statement: The total displacement of the object is 1.0 × 10 m [S 3° E].

6. (a) Given: $\Delta\vec{d}_1 = 4.3$ km [W]; $\Delta\vec{d}_2 = 8.0$ km [W 54° N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= 4.3\text{ km [W]} + (8.0\text{ km})(\cos 54°)\text{ [W]}\\ &= 4.3\text{ km [W]} + 4.702\text{ km [W]}\\ &= 9.002\text{ km [W]}\\ \vec{d}_{Tx} &= 9.0\text{ km [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= 0\text{ km} + (8.0\text{ km})(\sin 54°)\text{ [N]}\\ &= 6.472\text{ km [N]}\\ \vec{d}_{Ty} &= 6.5\text{ km [N]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T{}^2 &= d_{Tx}{}^2 + d_{Ty}{}^2\\ \Delta d_T &= \sqrt{d_{Tx}{}^2 + d_{Ty}{}^2}\\ &= \sqrt{(9.002\text{ km})^2 + (6.472\text{ km})^2}\ \text{(two extra digits carried)}\\ \Delta d_T &= 11\text{ km}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\tan\phi = \frac{\Delta d_{Ty}}{\Delta d_{Tx}}$$

$$\tan\phi = \frac{6.472\ \cancel{\text{km}}}{9.002\ \cancel{\text{km}}}\ \text{(two extra digits carried)}$$

$\tan\phi = 0.7190$
$\phi = \tan^{-1}(0.7190)$
$\phi = 36°$

Statement: The total displacement given by the two vectors is 11 km [W 36° N].

(b) Given: $\Delta\vec{d}_1 = 35$ m [E 65° N]; $\Delta\vec{d}_2 = 22$ m [E 37° S]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (35\text{ m})(\cos 65°)\text{ [E]} + (22\text{ m})(\cos 37°)\text{ [E]}\\ &= 14.79\text{ m [E]} + 17.57\text{ m [E]}\\ &= 32.36\text{ m [E]}\\ \vec{d}_{Tx} &= 32\text{ m [E]}\end{aligned}$$

$$\vec{d}_{Ty} = \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}$$
$$= (35\text{ m})(\sin 65°)\text{ [N]} + (22\text{ m})(\sin 37°)\text{ [S]}$$
$$= 31.72\text{ m [N]} + 13.24\text{ m [S]}$$
$$= 31.72\text{ m [N]} - 13.24\text{ m [N]}$$
$$= 18.48\text{ m [N]}$$
$$\vec{d}_{Ty} = 18\text{ m [N]}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\Delta d_T{}^2 = d_{Tx}{}^2 + d_{Ty}{}^2$$
$$\Delta d_T = \sqrt{d_{Tx}{}^2 + d_{Ty}{}^2}$$
$$= \sqrt{(32.36\text{ m})^2 + (18.48\text{ m})^2} \text{ (two extra digits carried)}$$
$$\Delta d_T = 37\text{ m}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\tan\phi = \frac{\Delta d_{Ty}}{\Delta d_{Tx}}$$
$$\tan\phi = \frac{18.48\text{ m}}{32.36\text{ m}} \text{ (one extra digit carried)}$$
$$\tan\phi = 0.5711$$
$$\phi = \tan^{-1}(0.5711)$$
$$\phi = 30°$$

Statement: The total displacement of the boat is 37 m [E 30° N].

7. Given: $\Delta\vec{d}_1 = 25$ m [N 30° W]; $\Delta\vec{d}_2 = 30.0$ m [N 40° E]; $\Delta\vec{d}_3 = 35$ m [S 25° W]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\vec{d}_{Tx} = \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x} + \Delta\vec{d}_{3x}$$
$$= (25\text{ m})(\sin 30°)\text{ [W]} + (30.0\text{ m})(\sin 40°)\text{ [E]} + (35\text{ m})(\sin 25°)\text{ [W]}$$
$$= 12.5\text{ m [W]} + 19.28\text{ m [E]} + 14.79\text{ m [W]}$$
$$= 12.5\text{ m [W]} - 19.28\text{ m [W]} + 14.79\text{ m [W]}$$
$$= 8.01\text{ m [W]}$$
$$\vec{d}_{Tx} = 8.0\text{ m [W]}$$

$$\vec{d}_{Ty} = \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y} + \Delta\vec{d}_{3y}$$
$$= (25\text{ m})(\cos 30°)\text{ [N]} + (30.0\text{ m})(\cos 40°)\text{ [N]} + (35\text{ m})(\cos 25°)\text{ [S]}$$
$$= 21.65\text{ m [N]} + 22.98\text{ m [N]} + 31.72\text{ m [S]}$$
$$= 21.65\text{ m [N]} + 22.98\text{ m [N]} - 31.72\text{ m [N]}$$
$$= 12.91\text{ m [N]}$$
$$\vec{d}_{Ty} = 13\text{ m [N]}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\Delta d_T{}^2 = d_{Tx}{}^2 + d_{Ty}{}^2$$
$$\Delta d_T = \sqrt{d_{Tx}{}^2 + d_{Ty}{}^2}$$
$$= \sqrt{(8.01\text{ m})^2 + (12.91\text{ m})^2} \text{ (two extra digits carried)}$$
$$\Delta d_T = 15\text{ m}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\tan\phi = \frac{\Delta d_{Tx}}{\Delta d_{Ty}}$$
$$\tan\phi = \frac{8.01\text{ m}}{12.91\text{ m}} \text{ (two extra digits carried)}$$
$$\tan\phi = 0.6204$$
$$\phi = \tan^{-1}(0.6204)$$
$$\phi = 32°$$

Statement: The total displacement of the vectors is 15 m [N 32° W].

8. (a) Given: $\Delta d = 5.1$ km; $v = 0.87$ km/h

Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{5.1\text{ km}}{0.87\ \frac{\text{km}}{\text{h}}}$$
$$= 5.862\text{ h}$$
$$\Delta t = 5.9\text{ h}$$

Statement: It will take the swimmer 5.9 h to cross the river.

(b) Given: $\vec{v}_x = 2.0$ km/h [W]

Required: $\Delta\vec{d}_x$

Analysis: $\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t}$

$$\Delta\vec{d}_x = \vec{v}_x\Delta t$$

Solution:

$$\Delta\vec{d}_x = \vec{v}_x\Delta t$$
$$= \left(2.0\ \frac{\text{km}}{\text{h}}\text{ [W]}\right)(5.862\text{ h}) \text{ (two extra digits carried)}$$
$$\Delta\vec{d}_x = 12\text{ km [W]}$$

Statement: The current has moved the swimmer 12 km downstream by the time she reaches the other side.

9. Time for the conductor to reach the other side:
Given: Δd = 4.0 m; v = 1.2 m/s
Required: Δt

Analysis: $v = \dfrac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \dfrac{\Delta d}{v}$

$$= \frac{4.0\ \cancel{\text{m}}}{1.2\ \dfrac{\cancel{\text{m}}}{\text{s}}}$$

$$\Delta t = 3.3\ \text{s}$$

Statement: It will take the conductor 3.3 s to cross the railcar.

The conductor's velocity relative to the ground:
Given: $\vec{v}_1$ = 4.0 m/s [N]; $\vec{v}_2$ = 1.2 m/s [E]
Required: $\vec{v}_T$
Analysis: $\vec{v}_T = \vec{v}_1 + \vec{v}_2$
Solution: Let ϕ represent the angle $\Delta\vec{v}_T$ makes with the y-axis.

$$\vec{v}_T = \vec{v}_1 + \vec{v}_2$$
$$v_T^2 = v_1^2 + v_2^2$$
$$v_T = \sqrt{v_1^2 + v_2^2}$$
$$= \sqrt{(4.0\ \text{m/s})^2 + (1.2\ \text{m/s})^2}$$
$$v_T = 4.2\ \text{m/s}$$

$$\tan\phi = \frac{v_2}{v_1}$$
$$\tan\phi = \frac{1.2\ \text{m/s}}{4.0\ \text{m/s}}$$
$$\phi = 17°$$

Statement: The velocity of the conductor relative to the ground is 4.2 m/s [N 17° E].

10. Answers may vary. Sample answer:
(a) I prefer the algebraic method of adding vectors. It takes more time, but I think the answers are more accurate because there is no chance of making errors measuring. I prefer to use a diagram only to double check my answer by sketching the vectors.
(b) If the navigator on a boat were working on a large map, it would probably be more useful to plot the vectors directly on the map. There would be no need for calculations and the navigator could use other map features to double check the resultant (like comparing distances and directions to landmarks on the map).