Section 1.5: Five Key Equations for Motion with Uniform Acceleration

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1. Given: $\vec{v}_i = 0$ m/s; $\Delta\vec{d} = 17$ m [E]; $\Delta t = 3.8$ s

Required: $\vec{v}_f$

Analysis: $\Delta\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$

$$\vec{v}_f = 2\frac{\Delta\vec{d}}{\Delta t} - \vec{v}_i$$

Solution: $\vec{v}_f = 2\dfrac{\Delta\vec{d}}{\Delta t} - \vec{v}_i$

$$= 2\left(\frac{17\text{ m [E]}}{3.8\text{ s}}\right) - 0\text{ m/s}$$

$$\vec{v}_f = 8.9\text{ m/s [E]}$$

Statement: Her final velocity is 8.9 m/s [E].

2. Given: $\vec{v}_i = 0$ m/s, $\Delta\vec{d} = 70.0$ m [downhill]; $\Delta t = 5.3$ s;

Required: $\vec{a}_{av}$

Analysis: $\Delta\vec{d} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}_{av}\Delta t^2$

$$\vec{a}_{av} = 2\frac{\Delta\vec{d} - \vec{v}_i\Delta t}{\Delta t^2}$$

Solution: $\vec{a}_{av} = 2\dfrac{\Delta\vec{d} - \vec{v}_i\Delta t}{\Delta t^2}$

$$= 2\left(\frac{70\text{ m [downhill]} - \left(0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(5.3\ \cancel{\text{s}})}{(5.3\text{ s})^2}\right)$$

$$\vec{a}_{av} = 5.0\text{ m/s}^2\text{ [downhill]}$$

Statement: The uniform acceleration experienced by the child is 5.0 m/s^2 [downhill].

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1. Given: $\vec{v}_i = 0$ m/s; $\vec{a}_{av} = 2.0$ m/s^2 [N]; $\Delta t = 15$ s

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}_{av}\Delta t^2$

Solution:

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}_{av}\Delta t^2$$

$$= \left(0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(15\ \cancel{\text{s}}) + \frac{1}{2}\left(2.0\ \frac{\text{m}}{\cancel{\text{s}^2}}\text{ [N]}\right)(15\ \cancel{\text{s}})^2$$

$$\Delta\vec{d} = 2.3\times10^2\text{ m [N]}$$

Statement: The displacement of the car is 230 m [N] or 2.3 × 10^2 m [N].

2. (a) Given: $\vec{v}_i = 20.0$ m/s [E]; $\vec{v}_f = 0$ m/s; $\Delta t = 12$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Solution: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$= \frac{0\ \frac{\text{m}}{\text{s}} - 20.0\ \frac{\text{m}}{\text{s}}\text{ [E]}}{12\text{ s}}$$

$$= \frac{0\ \frac{\text{m}}{\text{s}} + 20.0\ \frac{\text{m}}{\text{s}}\text{ [W]}}{12\text{ s}}$$

$$\vec{a}_{av} = 1.7\text{ m/s}^2\text{ [W]}$$

Statement: The uniform acceleration of the spacecraft is 1.7 m/s^2 [W].

(b) Given: $\vec{v}_i = 20.0$ m/s [E]; $\vec{v}_f = 0$ m/s; $\Delta t = 12$ s

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$

Solution: $\Delta\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$

$$= \left(\frac{0.0\ \frac{\text{m}}{\cancel{\text{s}}} + 20.0\ \frac{\text{m}}{\cancel{\text{s}}}\text{ [E]}}{2}\right)(12\ \cancel{\text{s}})$$

$$\Delta\vec{d} = 1.2\times10^2\text{ m [E]}$$

Statement: The displacement of the spacecraft is 120 m [E] or 1.2 × 10^2 m [E].

3. Given: $\vec{v}_i = 15$ m/s [W]; $\vec{a}_{av} = 7.0$ m/s^2 [E]; $\Delta t = 4.0$ s

Required: $\vec{v}_f$

Analysis: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$

Solution: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$

$$= 15\ \frac{\text{m}}{\text{s}}\text{ [W]} + \left(7.0\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\text{ [E]}\right)(4.0\ \cancel{\text{s}})$$

$$= -15\ \frac{\text{m}}{\text{s}}\text{ [E]} + 28\ \frac{\text{m}}{\text{s}}\text{ [E]}$$

$$\vec{v}_f = 13\text{ m/s [E]}$$

Statement: The final velocity of the helicopter is 13 m/s [E].

4. For go-cart A:
Given: v = 20.0 m/s; Δd = 1.0 km
Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{1.0\ \cancel{\text{km}}}{20.0\ \frac{\cancel{\text{m}}}{\text{s}}}\left(\frac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}}\right)$$

$$\Delta t = 50\ \text{s}$$

Statement: Go-cart A takes 50 s to go around the track.

For go-cart B:
Given: v_i = 0 m/s; a = 0.333 m/s^2; Δd = 1.0 km
Required: Δt

Analysis: $\Delta d = v_i \Delta t + \frac{1}{2} a_{av} \Delta t^2$

$$\Delta d = (0\ \text{m/s})\Delta t + \frac{1}{2} a_{av} \Delta t^2$$

$$\Delta t^2 = 2\frac{\Delta d}{a_{av}}$$

Solution: $\Delta t^2 = 2\frac{\Delta d}{a_{av}}$

$$= 2\frac{1.0\ \cancel{\text{km}}}{0.333\ \frac{\cancel{\text{m}}}{\text{s}^2}}\left(\frac{1000\ \cancel{\text{m}}}{1.0\ \cancel{\text{km}}}\right)$$

$$\Delta t^2 = 6000\ \text{s}^2$$

$$\Delta t = 77\ \text{s}$$

Statement: Go-cart B takes 77 s to go around the track. This time is greater than Go-cart A, which took 50 s. Go-cart A wins the race by 27 s.

5. Given: v_i = 5.0 m/s; v_f = 7.5 m/s; Δd = 50.0 m
Required: a_{av}

Analysis: $v_f^2 = v_i^2 + 2a_{av} \Delta d$

$$a_{av} = \frac{v_f^2 - v_i^2}{2\Delta d}$$

Solution: $a_{av} = \frac{v_f^2 - v_i^2}{2\Delta d}$

$$= \frac{\left(7.5\ \frac{\text{m}}{\text{s}}\right)^2 - \left(5.0\ \frac{\text{m}}{\text{s}}\right)^2}{2(50.0\ \text{m})}$$

$$= \frac{56.25\ \frac{\text{m}^{\cancel{2}}}{\text{s}^2} - 25.00\ \frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{100\ \cancel{\text{m}}}$$

$$a_{av} = 0.31\ \text{m/s}^2$$

Statement: The boat's average acceleration is 0.31 m/s^2.

6. (a) Given: $\Delta\vec{d} = 4.50 \times 10^2$ m [up]; Δt = 4.0 s; $\vec{v}_i$ = 0 m/s
Required: $\vec{a}_{av}$

Analysis: $\Delta\vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}_{av} \Delta t^2$

$$\vec{a}_{av} = 2\frac{\Delta\vec{d} - \vec{v}_i \Delta t}{\Delta t^2}$$

Solution: $\vec{a}_{av} = 2\frac{\Delta\vec{d} - \vec{v}_i \Delta t}{\Delta t^2}$

$$= 2\frac{4.5 \times 10^2\ \text{m [up]} - \left(0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(4.0\ \cancel{\text{s}})}{(4.0\ \text{s})^2}$$

$$\vec{a}_{av} = 56\ \text{m/s}^2\ \text{[up]}$$

Statement: The spacecraft's acceleration is 56 m/s² [up] ~~56 m/s^2 [E]~~.

(b) Given: $\Delta\vec{d} = 4.50 \times 10^2$ m [up]; Δt = 4.0 s; $\vec{v}_i$ = 0 m/s; $\vec{a}_{av}$ = 56 m/s^2 [up]
Required: $\vec{v}_f$

Analysis: $\vec{v}_f = \vec{v}_i + \vec{a}_{av} \Delta t$

Solution: $\vec{v}_f = \vec{v}_i + \vec{a}_{av} \Delta t$

$$= 0\ \frac{\text{m}}{\text{s}} + \left(56\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\ \text{[up]}\right)(4.0\ \cancel{\text{s}})$$

$$\vec{v}_f = 2.2 \times 10^2\ \text{m/s [up]}$$

Statement: After 4.0 s, the velocity of the spacecraft is 220 m/s [up] or 2.2 × 10^2 m/s [up].

7. Answers may vary. Sample answer:
Since Equation 4 does not include Δt, isolate Δt in Equations 1 and 2, then set them equal to each other.

Equation 1:

$$\Delta\vec{d} = \left(\frac{\vec{v}_\text{f} + \vec{v}_\text{i}}{2}\right)\Delta t$$

$$\Delta t = \frac{2\Delta\vec{d}}{\vec{v}_\text{f} + \vec{v}_\text{i}}$$

Equation 2:

$$\vec{v}_\text{f} = \vec{v}_\text{i} + \vec{a}_\text{av}\,\Delta t$$

$$\Delta t = \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\vec{a}_\text{av}}$$

$$\Delta t = \Delta t$$

$$\frac{2\Delta\vec{d}}{\vec{v}_\text{f} + \vec{v}_\text{i}} = \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\vec{a}_\text{av}}$$

$$2\vec{a}_\text{av}\,\Delta\vec{d} = \vec{v}_\text{f}^{\,2} - \vec{v}_\text{i}^{\,2}$$

$$\vec{v}_\text{f}^{\,2} = \vec{v}_\text{i}^{\,2} + 2\vec{a}_\text{av}\,\Delta\vec{d}$$

Since Equation 5 does not include $\vec{v}_\text{i}$, isolate $\vec{v}_\text{i}$ in Equations 1 and 2, then set them equal to each other.

Equation 1:

$$\Delta\vec{d} = \left(\frac{\vec{v}_\text{f} + \vec{v}_\text{i}}{2}\right)\Delta t$$

$$\vec{v}_\text{f} + \vec{v}_\text{i} = \frac{2\Delta\vec{d}}{\Delta t}$$

$$\vec{v}_\text{i} = \frac{2\Delta\vec{d}}{\Delta t} - \vec{v}_\text{f}$$

Equation 2:

$$\vec{v}_\text{f} = \vec{v}_\text{i} + \vec{a}_\text{av}\,\Delta t$$

$$\vec{v}_\text{i} = \vec{v}_\text{f} - \vec{a}_\text{av}\,\Delta t$$

$$\vec{v}_\text{i} = \vec{v}_\text{i}$$

$$\frac{2\Delta\vec{d}}{\Delta t} - \vec{v}_\text{f} = \vec{v}_\text{f} - \vec{a}_\text{av}\,\Delta t$$

$$\frac{2\Delta\vec{d}}{\Delta t} = 2\vec{v}_\text{f} - \vec{a}_\text{av}\,\Delta t$$

$$2\Delta\vec{d} = \left(2\vec{v}_\text{f} - \vec{a}_\text{av}\,\Delta t\right)\Delta t$$

$$2\Delta\vec{d} = 2\vec{v}_\text{f}\,\Delta t - \vec{a}_\text{av}\,\Delta t^2$$

$$\Delta\vec{d} = \vec{v}_\text{f}\,\Delta t - \frac{1}{2}\vec{a}_\text{av}\,\Delta t^2$$