Section 1.3: Acceleration

Tutorial 1 Practice, page 24

1. Given: $\vec{v}_i = 0$ m/s; $\vec{v}_f = 15.0$ m/s [S]; $\Delta t = 12.5$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Solution: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$= \frac{15.0 \text{ m/s [S]} - 0 \text{ m/s}}{12.5 \text{ s}}$$

$$= \frac{15.0 \text{ m/s [S]}}{12.5 \text{ s}}$$

$$\vec{a}_{av} = 1.20 \text{ m/s}^2 \text{ [S]}$$

Statement: The rock's average acceleration is 1.20 m/s^2 [S].

2. Given: $\vec{v}_i = 17$ m/s [N]; $\vec{v}_f = 25$ m/s [N]; $\Delta t = 12$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Solution: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$= \frac{25 \text{ m/s [N]} - 17 \text{ m/s [N]}}{12 \text{ s}}$$

$$= \frac{8 \text{ m/s [N]}}{12 \text{ s}}$$

$$\vec{a}_{av} = 0.67 \text{ m/s}^2 \text{ [N]}$$

Statement: The car's average acceleration is 0.67 m/s^2 [N].

3. Given: $\vec{v}_i = 25$ m/s [W]; $\vec{v}_f = 29$ m/s [E]; $\Delta t = 0.25$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Solution: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$= \frac{29 \text{ m/s [E]} - 25 \text{ m/s [W]}}{0.25 \text{ s}}$$

$$= \frac{29 \text{ m/s [E]} + 25 \text{ m/s [E]}}{0.25 \text{ s}}$$

$$= \frac{54 \text{ m/s [E]}}{0.25 \text{ s}}$$

$$\vec{a}_{av} = 220 \text{ m/s}^2 \text{ [E]}$$

Statement: The squash ball's average acceleration is 220 m/s^2 [E] or $2.2 \times 10^2 \text{ m/s}^2$ [E].

Tutorial 2 Practice, page 25

1. Given: $\vec{v}_i = 3.2$ m/s [W]; $\vec{v}_f = 5.8$ m/s [W]; $\vec{a}_{av} = 1.23 \text{ m/s}^2$ [W]

Required: Δt

Analysis: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$\Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}_{av}}$$

Solution: $\Delta t = \dfrac{\vec{v}_f - \vec{v}_i}{\vec{a}_{av}}$

$$= \frac{5.8 \text{ m/s [W]} - 3.2 \text{ m/s [W]}}{1.23 \text{ m/s}^2 \text{ [W]}}$$

$$= \frac{2.6 \ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}} \text{ [W]}}{1.23 \ \dfrac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}} \text{ [W]}}$$

$$\Delta t = 2.1 \text{ s}$$

Statement: The radio-controlled car's acceleration will take 2.1 s.

2. Given: $\vec{v}_f = 17$ m/s [W]; $\vec{a}_{av} = 2.4 \text{ m/s}^2$ [W]; $\Delta t = 6.2$ s

Required: $\vec{v}_i$

Analysis: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$\vec{a}_{av} \Delta t = \vec{v}_f - \vec{v}_i$$

$$\vec{v}_i = \vec{v}_f - \vec{a}_{av} \Delta t$$

Solution:

$$\vec{v}_i = \vec{v}_f - \vec{a}_{av} \Delta t$$

$$= 17 \text{ m/s [W]} - \left(2.4 \ \frac{\text{m}}{\text{s}^{\cancel{2}}} \text{ [W]}\right)(6.2 \ \cancel{\text{s}})$$

$$= 17 \text{ m/s [W]} - 14.88 \text{ m/s [W] (two extra digits carried)}$$

$$\vec{v}_i = 2.1 \text{ m/s [W]}$$

Statement: The initial velocity of the speedboat was 2.1 m/s [W].

Tutorial 3 Practice, page 26

1. (a) Given: $b = 4.0$ s; $h = 8.0$ m/s [S]

Required: $\Delta \vec{d}$

Analysis: $\Delta \vec{d} = A_{triangle}$

Solution: $\Delta\vec{d} = A_{\text{triangle}}$

$$= \frac{1}{2}bh$$

$$= \frac{1}{2}(4.0\ \cancel{\text{s}})\left(8.0\ \frac{\text{m}}{\cancel{\text{s}}}\ [\text{S}]\right)$$

$$\Delta\vec{d} = 16\text{ m [S]}$$

Statement: The object has travelled 16 m [S] after 4.0 s.

(b) Given: b = 5.0 s; h = 10.0 m/s [S]; l = 2.5 s; w = 10.0 m/s [S]

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = A_{\text{triangle}} + A_{\text{rectangle}}$

Solution:

$$\Delta\vec{d} = A_{\text{triangle}} + A_{\text{rectangle}}$$

$$= \frac{1}{2}bh + lw$$

$$= \frac{1}{2}(5.0\ \cancel{\text{s}})\left(10.0\ \frac{\text{m}}{\cancel{\text{s}}}\ [\text{S}]\right) + (2.5\ \cancel{\text{s}})\left(10.0\ \frac{\text{m}}{\cancel{\text{s}}}\ [\text{S}]\right)$$

$$= 25.0\text{ m [S]} + 25.0\text{ m [S]}$$

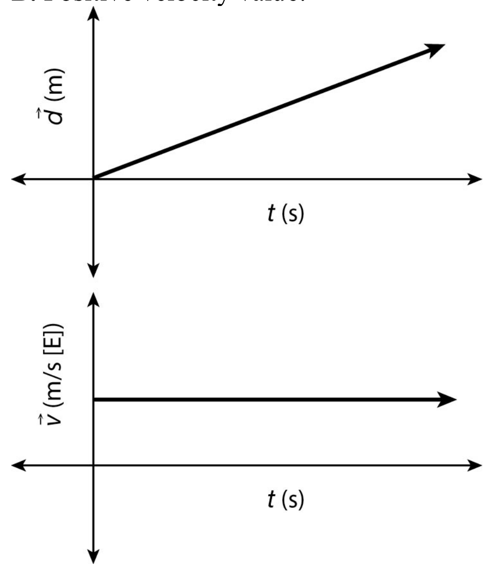
$$\Delta\vec{d} = 50\text{ m [S]}$$

Statement: The object has travelled 50 m [S] after 7.5 s.

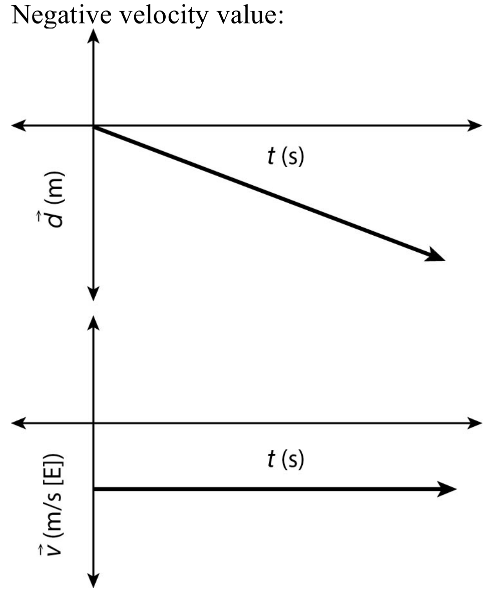
Mini Investigation: Motion Simulations, page 27

A. Positive velocity value: The position–time graph will be a straight line starting at the origin with a positive slope. The velocity–time graph will be a horizontal line with positive value.
Negative velocity value: The position–time graph will be a straight line starting at the origin with a negative slope. The velocity–time graph will be a horizontal line with negative value.
Negative initial position, positive velocity value: The position–time graph will be a straight line starting below the *x*-axis with a positive slope. The velocity–time graph will be a horizontal line with positive value.
Negative initial position, negative velocity value: The position–time graph will be a straight line starting below the *x*-axis with a negative slope. The velocity–time graph will be a horizontal line with negative value.

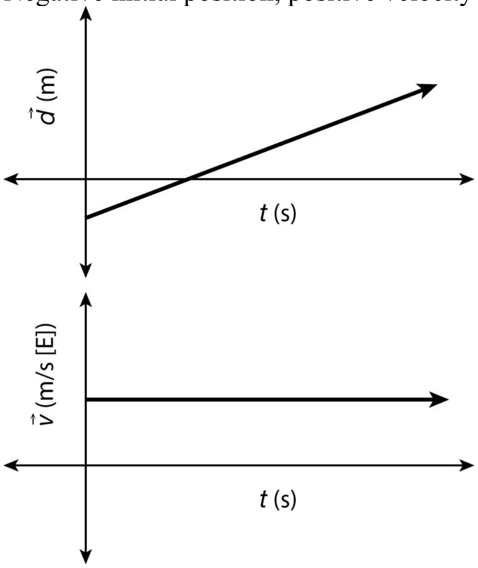
B. Positive velocity value:



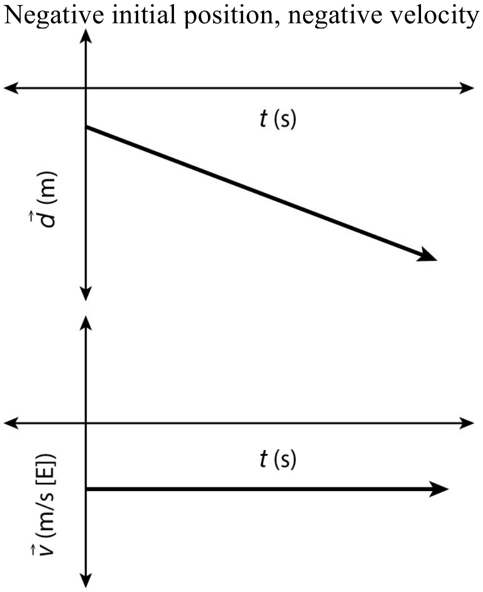
Negative velocity value:



Negative initial position, positive velocity value:



Negative initial position, negative velocity value:



C. Differences are due to the difficulty in using the mouse to mimic constant velocity.

Tutorial 4 Practice, page 29

1. (a) Given: t = 1.0 s; position–time graph

Required: $\vec{v}_{\text{inst}}$

Analysis: $\vec{v}_{\text{inst}}$ is equal to the slope, m, of the tangent to the curve at t = 1.0 s, so $m = \frac{\Delta \vec{d}}{\Delta t}$.

By placing a ruler along the curve in Figure 6 at t = 1.0 s, I can picture the tangent. The tangent has a rise of 4.0 m [E] over a run of 2.0 s.

Solution:
$$m = \frac{\Delta \vec{d}}{\Delta t}$$
$$m = \frac{4.0 \text{ m [E]}}{2.0 \text{ s}}$$
$$\vec{v}_{\text{inst}} = 2.0 \text{ m/s [E]}$$

Statement: The instantaneous velocity of the object at 1.0 s is 2.0 m/s [E].

(b) Given: t = 3.0 s; position–time graph

Required: $\vec{v}_{\text{inst}}$

Analysis: $\vec{v}_{\text{inst}}$ is equal to the slope, m, of the tangent to the curve at t = 3.0 s, so $m = \frac{\Delta \vec{d}}{\Delta t}$.

By placing a ruler along the curve in Figure 6 at t = 3.0 s, I can picture the tangent. The tangent has a rise of 12.0 m [E] over a run of 2.0 s.

Solution:
$$m = \frac{\Delta \vec{d}}{\Delta t}$$
$$m = \frac{12.0 \text{ m [E]}}{2.0 \text{ s}}$$
$$\vec{v}_{\text{inst}} = 6.0 \text{ m/s [E]}$$

Statement: The instantaneous velocity of the object at 3.0 s is 6.0 m/s [E].

(c) At t = 1.0 s, $\vec{v}_{\text{inst}}$ is 2.0 m/s [E], at t = 2.0 s, $\vec{v}_{\text{inst}}$ is 4.0 m/s [E], and at t = 3.0 s, $\vec{v}_{\text{inst}}$ is 6.0 m/s [E]. Since the increase in the instantaneous velocity is constant (2.0 m/s [E] every second), it is possible that the object is moving with constant acceleration.

2. (a) Given: t = 5.0 s; position–time graph

Required: $\vec{v}_{\text{inst}}$

Analysis: $\vec{v}_{\text{inst}}$ is equal to the slope, m, of the tangent to the curve at t = 5.0 s, so $m = \frac{\Delta \vec{d}}{\Delta t}$.

By placing a ruler along the curve in Figure 7 at t = 5.0 s, I can picture the tangent. The tangent has a rise of 150 m [E] over a run of 5.0 s.

Solution:
$$m = \frac{\Delta \vec{d}}{\Delta t}$$
$$= \frac{150 \text{ m [E]}}{5.0 \text{ s}}$$
$$\vec{v}_{\text{inst}} = 30 \text{ m/s [E]}$$

Statement: The instantaneous velocity of the object at 5.0 s is 30 m/s [E].

(b) Given: $\vec{d}_1 = 0.0$ m [E]; $\vec{d}_2 = 300.0$ m [E]; $t_1 = 0.0$ s; $t_2 = 10.0$ s

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$

$$\vec{v}_{av} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$

Solution: $\vec{v}_{av} = \dfrac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$

$$= \frac{300 \text{ m [E]} - 0.0 \text{ m [E]}}{10.0 \text{ s} - 0.0 \text{ s}}$$

$$\vec{v}_{av} = 30 \text{ m/s [E]}$$

Statement: The average velocity of the object over the time interval from 0.0 s to 10.0 s is 30 m/s [E].

(c) When an object is accelerating uniformly (constant acceleration), the average velocity over an interval of time equals the instantaneous velocity of the midpoint in that interval of time.

Section 1.3 Questions, page 30

1. Answers may vary. Sample answer:
An accelerating object may exhibit increasing velocity, such as a horse accelerating from a slow trot to a gallop.
An accelerating object may exhibit decreasing velocity, such as a cyclist who slows down while riding up a steep road.
An accelerating object may come to a complete stop, such as a car travelling east that accelerates west until it stops.

2. Answers may vary. Sample answer:
To determine the acceleration of an object from a velocity-time graph, divide the velocity by the time at a given point.

3. Answers may vary. Sample answer:
To determine the displacement of an object from a velocity–time graph, calculate the area under the graph from the initial time to the final time. The area is equal to the displacement between those two times.

4. (a) Given: $\Delta\vec{v} = 28$ m/s [E]; $\Delta t = 7.0$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Solution: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

$$= \frac{28 \text{ m/s [E]}}{7.0 \text{ s}}$$

$$\vec{a}_{av} = 4.0 \text{ m/s}^2 \text{ [E]}$$

Statement: The average acceleration described by the graph is 4.0 m/s^2 [E].

(b) Given: $\Delta\vec{v} = 24.5$ m/s [E]; $\Delta t = 7.0$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Solution: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

$$= \frac{24.5 \text{ m/s [E]}}{7.0 \text{ s}}$$

$$\vec{a}_{av} = 3.5 \text{ m/s}^2 \text{ [E]}$$

Statement: The average acceleration described by the graph is 3.5 m/s^2 [E].

(c) Given: $\Delta\vec{v} = 2.1$ m/s [E]; $\Delta t = 7.0$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Solution: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

$$= \frac{2.1 \text{ m/s [E]}}{7.0 \text{ s}}$$

$$\vec{a}_{av} = 0.30 \text{ m/s}^2 \text{ [E]}$$

Statement: The average acceleration described by the graph is 0.30 m/s^2 [E].

5. Answers may vary. Sample answer:
What you said about the constant speed of the object isn't right. Even though the speed is still the same, the direction has changed from north to south. That means that the velocity has changed, so there must have been an acceleration in the direction of south.

6. (a) In the first segment, the object accelerates from 0.0 m/s to 6.0 m/s [W] in the first 4.0 s. In the second segment, the object continues at a constant velocity of 6.0 m/s [W] for 3.0 s. In the third segment, the object accelerates east so the velocity changes from 60.0 m/s [W] to 0.0 m/s in the final 3.0 s.

(b) For the first segment:
Given: $\Delta\vec{v}$ = 6.0 m/s [W]; Δt = 4.0 s
Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Solution:
$$\begin{aligned}\vec{a}_{av} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{6.0 \text{ m/s [W]}}{4.0 \text{ s}}\\ \vec{a}_{av} &= 1.5 \text{ m/s}^2 \text{ [W]}\end{aligned}$$

Statement: The average acceleration in the first segment of the graph is 1.5 m/s^2 [W].

For the second segment:
Given: $\Delta\vec{v}$ = 0.0 m/s; Δt = 3.0 s
Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Solution:
$$\begin{aligned}\vec{a}_{av} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{0.0 \text{ m/s}}{4.0 \text{ s}}\\ \vec{a}_{av} &= 0.0 \text{ m/s}^2\end{aligned}$$

Statement: The average acceleration in the second segment of the graph is 0.0 m/s^2.

For the third segment:
Given: $\Delta\vec{v}$ = –6.0 m/s [W]; Δt = 3.0 s
Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Solution:
$$\begin{aligned}\vec{a}_{av} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{-6.0 \text{ m/s [W]}}{3.0 \text{ s}}\\ &= \frac{6.0 \text{ m/s [E]}}{3.0 \text{ s}}\\ \vec{a}_{av} &= 2.0 \text{ m/s}^2 \text{ [E]}\end{aligned}$$

Statement: The average acceleration in the third segment of the graph is 2.0 m/s^2 [E].

(c) Given: b_1 = 4.0 s; b_2 = 3.0 s; h = 6.0 m/s [W]; l = 3.0 s
Required: $\Delta\vec{d}$
Analysis: $\Delta\vec{d} = A_{\text{triangle 1}} + A_{\text{rectangle}} + A_{\text{triangle 2}}$

Solution:
$$\begin{aligned}\Delta\vec{d} &= A_{\text{triangle 1}} + A_{\text{rectangle}} + A_{\text{triangle 2}}\\ &= \frac{1}{2}b_1h + lh + \frac{1}{2}b_2h\\ &= \frac{1}{2}(4.0 \cancel{\text{s}})\left(6.0 \frac{\text{m}}{\cancel{\text{s}}} \text{ [W]}\right) + (3.0 \cancel{\text{s}})\left(6.0 \frac{\text{m}}{\cancel{\text{s}}} \text{ [W]}\right)\\ &\quad + \frac{1}{2}(3.0 \cancel{\text{s}})\left(6.0 \frac{\text{m}}{\cancel{\text{s}}} \text{ [W]}\right)\\ &= 12 \text{ m [W]} + 18 \text{ m [W]} + 9 \text{ m [W]}\\ \Delta\vec{d} &= 39 \text{ m [W]}\end{aligned}$$

Statement: The object has travelled 39 m [W] after 10.0 s.

7. Given: $\vec{v}_i$ = 2.0 m/s [W]; $\vec{v}_f$ = 4.5 m/s [W]; Δt = 1.9 s
Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Solution:
$$\begin{aligned}\vec{a}_{av} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t}\\ &= \frac{4.5 \text{ m/s [W]} - 2.0 \text{ m/s [W]}}{1.9 \text{ s}}\\ &= \frac{2.5 \text{ m/s [W]}}{1.9 \text{ s}}\\ \vec{a}_{av} &= 1.3 \text{ m/s}^2 \text{ [W]}\end{aligned}$$

Statement: The average acceleration of the car is 1.3 m/s^2 [W].

8. Given: $\vec{v}_i$ = 0.68 m/s [N]; $\vec{v}_f$ = 0.89 m/s [N]; $\vec{a}_{av}$ = 0.53 m/s^2 [N]
Required: Δt

Analysis:
$$\begin{aligned}\vec{a}_{av} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t}\\ \Delta t &= \frac{\vec{v}_f - \vec{v}_i}{\vec{a}_{av}}\end{aligned}$$

Solution:
$$\begin{aligned}\Delta t &= \frac{\vec{v}_f - \vec{v}_i}{\vec{a}_{av}}\\ &= \frac{0.89 \text{ m/s [N]} - 0.68 \text{ m/s [N]}}{0.53 \text{ m/s}^2 \text{ [N]}}\\ &= \frac{0.21 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \text{ [N]}}{0.53 \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}} \text{ [N]}}\\ \Delta t &= 0.40 \text{ s}\end{aligned}$$

Statement: It will take 0.40 s to increase the bicycle's velocity.

9. (a) Given: $\vec{v}_f = 0.0$ m/s; $\vec{a}_{av} = 2.90$ m/s^2 [S]; $\Delta t = 5.72$ s

Required: $\vec{v}_i$

Analysis:
$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$
$$\vec{a}_{av}\,\Delta t = \vec{v}_f - \vec{v}_i$$
$$\vec{v}_i = \vec{v}_f - \vec{a}_{av}\,\Delta t$$

Solution:
$$\begin{aligned}\vec{v}_i &= \vec{v}_f - \vec{a}_{av}\,\Delta t \\ &= 0.0\text{ m/s} - \left(2.90\ \frac{\text{m}}{\text{s}^{\not 2}}\ [\text{S}]\right)(5.72\ \not\text{s}) \\ &= 0.0\text{ m/s} - 16.6\text{ m/s [S]} \\ \vec{v}_i &= 16.6\text{ m/s [N]}\end{aligned}$$

Statement: The initial velocity of the car was 16.6 m/s [N].

(b) To decrease the velocity, the driver must accelerate in the opposite direction. In this example, to stop going north, the driver accelerated south.

10. Given: $\vec{v}_i = 6.0$ m/s [E]; $\vec{v}_f = 7.3$ m/s [W]; $\Delta t = 0.094$ s

Required: $\vec{a}_{av}$

Analysis:
$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Solution:
$$\begin{aligned}\vec{a}_{av} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \\ &= \frac{7.3\text{ m/s [W]} - 6.0\text{ m/s [E]}}{0.094\text{ s}} \\ &= \frac{7.3\text{ m/s [W]} + 6.0\text{ m/s [W]}}{0.094\text{ s}} \\ &= \frac{13.3\text{ m/s [W]}}{0.094\text{ s}} \\ \vec{a}_{av} &= 140\text{ m/s}^2\text{ [W]}\end{aligned}$$

Statement: The average acceleration of the tennis ball is 140 m/s^2 [W] or 1.4×10^2 m/s^2 [W].

11. (a) Given: $t = 6.0$ s; position–time graph

Required: $\vec{v}_{inst}$

Analysis: $\vec{v}_{inst}$ is equal to the slope, m, of the tangent to the curve at $t = 6.0$ s, so $m = \frac{\Delta \vec{d}}{\Delta t}$.

By placing a ruler along the curve in Figure 9 at $t = 6.0$ s, I can picture the tangent. The tangent has a rise of 120 m [E] over a run of 5.0 s.

Solution:
$$\begin{aligned}m &= \frac{\Delta \vec{d}}{\Delta t} \\ m &= \frac{120\text{ m [E]}}{5.0\text{ s}} \\ \vec{v}_{inst} &= 24\text{ m/s [E]}\end{aligned}$$

Statement: The instantaneous velocity of the object at 6.0 s is 24 m/s [E].

(b) Given: $\vec{d}_1 = 0.0$ m; $\vec{d}_2 = 200$ m; $t_1 = 0.0$ s; $t_2 = 10.0$ s

Required: $\vec{v}_{av}$

Analysis:
$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$
$$\vec{v}_{av} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$

Solution:
$$\begin{aligned}\vec{v}_{av} &= \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1} \\ &= \frac{200\text{ m [W]} - 0.0\text{ m [W]}}{10.0\text{ s} - 0.0\text{ s}} \\ \vec{v}_{av} &= 20\text{ m/s [W]}\end{aligned}$$

Statement: The average velocity of the object over the time interval from 0.0 s to 10.0 s is 20 m/s [W].