9.2 Waves at Media Boundaries

Recall from Chapter 8 that wave speed depends on some of the properties of the medium through which the wave is travelling. For example, the speed of sound in air depends on air temperature, and the speed of a wave along a rope depends on the density and tension of the rope. What happens if the medium changes? For example, what happens to a wave when it moves down a rope and encounters a different medium, such as air?

No medium is infinitely large, so all media have boundaries. The location where two media meet is called a **media boundary**. Media boundaries can take many forms. For example, they can be a change in the characteristics of a rope or the edge of a drumhead. A media boundary might also be the surface of the walls of a room where air and the wall material meet. Understanding how waves behave at media boundaries is useful. For instance, most schools have individual classrooms so that the discussion in one classroom does not interfere with the discussion next door. Musicians record their music in soundproof rooms so that only their own sound is recorded (**Figure 1**). You will learn more about the effect of sound produced in enclosed or restricted spaces—acoustics—in Chapter 10.

To explore the behaviour of waves at media boundaries, we first examine what happens in two simple cases: free-end reflections and fixed-end reflections.

media boundary the location where two or more media meet

Figure 1 The walls and shapes of recording studios are carefully designed to ensure that the sound going to the microphone is a true representation of the work of the musician. The walls contain materials that absorb sound.

Free-End Reflections

Consider two media, say medium 1 and medium 2, and a wave travelling through medium 1 into medium 2. If medium 1 is denser than medium 2, then the wave will move faster in medium 1 than in medium 2. In this case, the wave moving toward the boundary will be reflected in the same orientation as the incoming wave and with the same amplitude as the incoming wave (**Figure 2**). This is called a **free-end reflection**. You can generate a free-end reflection yourself by snapping a towel. The movement of the free end of a flag attached to a flagpole is another example of a free-end reflection (**Figure 3**). The flag flutters in the wind, but at its free end the wave encounters the atmosphere. The speed of the wave in each medium is significantly different. In free-end reflections, the medium the wave has been travelling through ends abruptly, typically into the atmosphere. Since the atmosphere is less dense than most media, the end of the medium can move freely.

free-end reflection a reflection that occurs at a media boundary where the second medium is less dense than the first medium; reflections have an amplitude with the same orientation as the original wave

Figure 3 When the waves in the flag reach the right side, they will reflect back through the flag material.

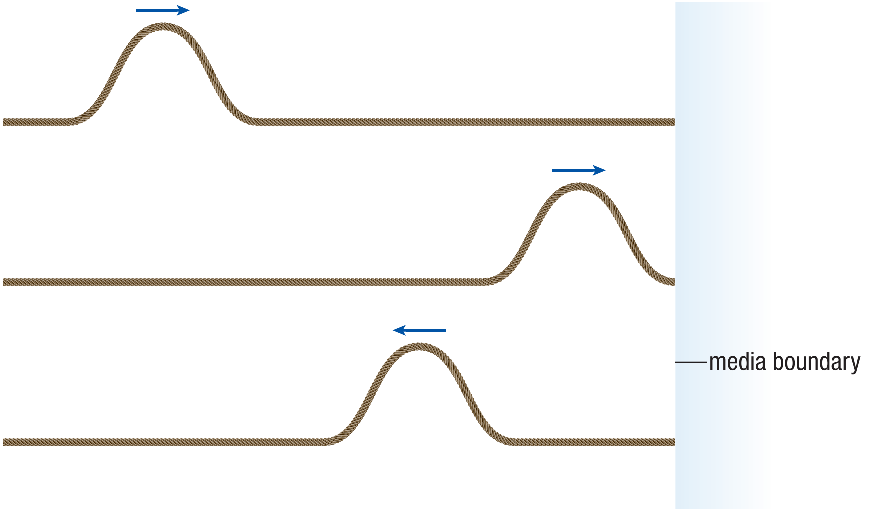


Figure 2 When a wave in one medium (for example, string) encounters a medium with a lower density (for example, air), the wave is reflected with the same orientation and amplitude as the original pulse.

As another example, suppose you are holding one end of a rope. If you send a pulse through the rope, the loose end moves freely. When the pulse encounters the end of the rope, it is reflected back to the source (Figure 2). Notice that the reflected wave from a free-end reflection is upright, and the characteristics of the pulse in the reflected waveform are identical to the characteristics of the original pulse. The wave does not continue to the right, beyond the media boundary. This is because the wave speed in the second medium is slower than the wave speed in the first medium.

Fixed-End Reflections

If a medium is fixed at one end, then when a wave reaches the media boundary a **fixed-end reflection** occurs. A fixed-end reflection also occurs when a medium is fixed at both ends, as in a harp (**Figure 4**).

fixed-end reflection a reflection that occurs at a media boundary where one end of the medium is unable to vibrate; reflections are inverted

Figure 4 A harp has strings anchored at both ends. When you pluck a harp string, you produce a wave. The wave moves back and forth along the string and encounters a fixed end at each end of the string.

Consider a pulse in a string moving toward a rigid, denser medium such as a wall (**Figure 5**). When the pulse reaches the fixed end, it is reflected. As you see in Figure 5, the reflected pulse has the same shape as the incoming pulse, but its orientation is inverted. We may explain this inversion as follows. When the pulse reaches the fixed end of the string, it exerts an upward force on the wall. In response, the wall exerts a downward force on the string in accordance with Newton's third law of motion. Therefore, the incoming upright pulse is inverted upon reflection.

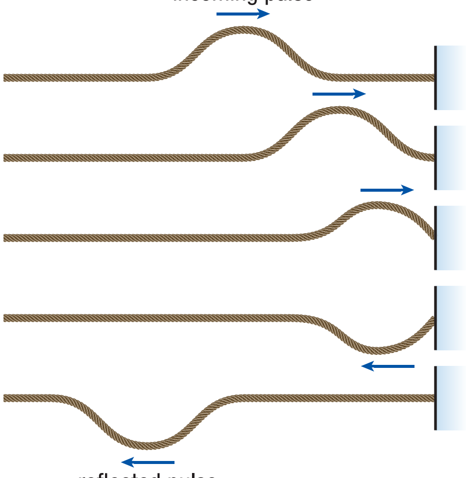


Figure 5 When a pulse in one medium meets a boundary with a denser medium, the reflected pulse is inverted.

The difference in the media as a wave reaches a media boundary may not be as dramatic as either the free-end or the fixed-end case. In nature there are media boundaries that are neither free-end nor fixed-end. For example, the boundary between water and air and the boundary between air and a tree are neither fixed-end boundaries nor free-end boundaries. If a wave travels from a medium in which its speed is faster to a medium in which its speed is slower, the wave can move more freely than it did in the faster medium.

transmission the motion of a wave through a medium, or motion of a wave from one medium to another medium

WEB LINK

To see an animation of reflection and transmission,

GO TO NELSON SCIENCE

Media Boundaries: Amplitudes

The amplitude of a wave before it encounters a media boundary is closely related to the wave's energy. The amplitude does not change if the wave's energy remains constant. When a wave encounters a media boundary that is not strictly an ideal free-end or fixed-end boundary, the wave splits into two. One wave is reflected, and the other is transmitted. The term **transmission** describes the process of a wave moving through a medium or moving from one medium into another medium. The amplitude of the original wave may not be shared equally by the reflected wave and the transmitted wave. However, the sum of the two amplitudes must equal the amplitude of the original wave (**Figure 6**).

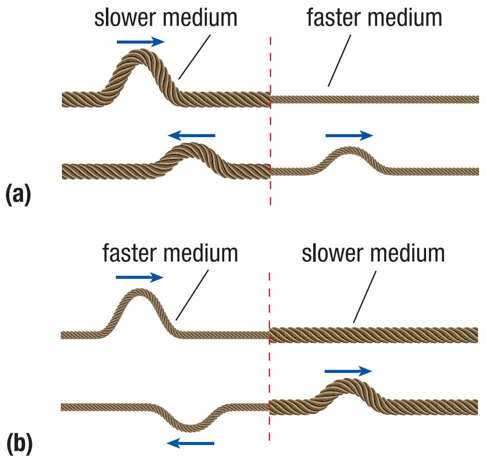


Figure 6 At a media boundary that is neither free-end nor fixed-end, the original wave splits into two waves. (a) If the wave moving along the rope encounters a medium that has a faster wave speed, then the wave splits into two, and one wave is reflected and the other is transmitted. The reflected wave is upright. (b) When a wave moves into a slower medium, then the wave splits into two, and one wave is reflected and the other is transmitted. The reflected wave is inverted.

If the difference between the wave speeds in the two media is small, transmission is preferred—the amplitude of the transmitted wave is closer to the amplitude of the original wave. As a result, the amplitude of the reflected wave is much smaller because of the conservation of energy. For cases in which the wave speed is significantly different between the two media, reflection is preferred—the amplitude of the reflected wave is closer to the amplitude of the original wave.

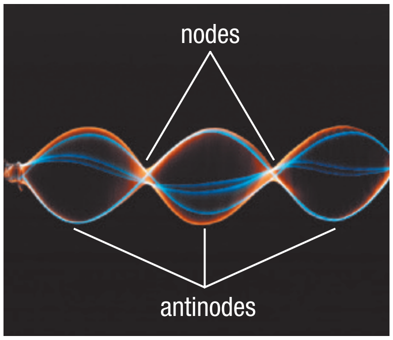


Figure 7 A standing wave pattern

standing wave an interference pattern produced when incoming and reflected waves interfere with each other; the effect is a wave pattern that appears to be stationary

node in a standing wave, the location where the particles of the medium are at rest

antinode in a standing wave, the location where the particles of the medium are moving with greatest speed; the amplitude will be twice the amplitude of the original wave

Investigation 9.2.1

Investigating Wave Speed on a String (p. 438)
In this investigation, you will investigate the wave speed of a standing wave.

Standing Waves

A special case of reflection at a media boundary is the production of standing waves. Suppose you send a series of waves at a certain frequency along a string of length L. The string is fixed at both ends. At the correct frequency, waves will reflect, and the reflected waves will superimpose on the stream of incoming waves to produce an interference pattern that makes the waves appear to be stationary. This interference pattern is called a **standing wave** and is shown in **Figure 7**.

In a standing wave there are locations where the particles of the medium do not move, called **nodes**. Standing waves also contain regions where the particles of the medium move with greatest speed. These regions are called **antinodes**.

The waves in a standing wave pattern interfere according to the principle of superposition. The waves are moving continuously. When the original wave is upright, so is the reflected wave from the previous crest. When the reflection has a trough, so does the original wave. At the antinodes, the amplitudes of the troughs and the crests are thus double that of the original wave.

The interference pattern appears to be stationary because it is produced by two otherwise identical waves travelling in opposite directions. The wave speed of a standing wave interference pattern is the difference between the wave speeds of the incoming and reflected waves. Since these are the same (direction is ignored), the wave speed of the standing wave is zero. However, remember that a standing wave is created by the interference of the incoming and reflected waves because these two waves are continuously moving.

WEB LINK

To see an animation of a standing wave,

GO TO NELSON SCIENCE

Standing Waves between Two Fixed Ends

The properties of standing waves can be predicted mathematically. Consider a string with two fixed ends—it has a standing wave with nodes at both ends, as occurs when both ends are fixed (**Table 1**). In this case, the shortest length of the string, L, is equal to one half of the wavelength, $\lambda/2$, where λ is the wavelength. The frequency of the wave that produces the simplest standing wave is called the **fundamental frequency** (f_0) or the **first harmonic**. All standing waves after this require frequencies that are whole-number multiples of the fundamental frequency. These additional standing wave frequencies are called the nth harmonic of the fundamental frequency, where $n = 1$ for the fundamental frequency. **Harmonics** consist of the fundamental frequency (or first harmonic) of a musical sound as well as the frequencies that are whole-number multiples of the fundamental frequency. When a string, such as a violin string, vibrates with more than one frequency, the resulting sounds are called **overtones**. An overtone resulting from a string is very similar to the harmonic, except that the first overtone is equal to the second harmonic.

fundamental frequency or **first harmonic (f_0)** the lowest frequency that can produce a standing wave in a given medium

harmonics whole-number multiples of the fundamental frequency

overtone a sound resulting from a string that vibrates with more than one frequency

Table 1 Producing Standing Waves in a Medium with Fixed Ends

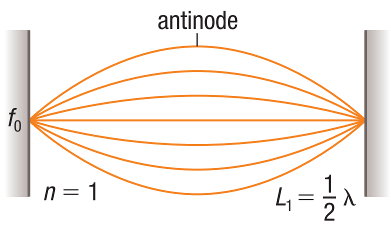
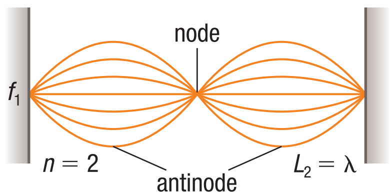
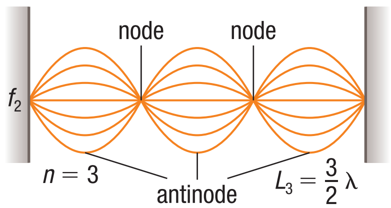
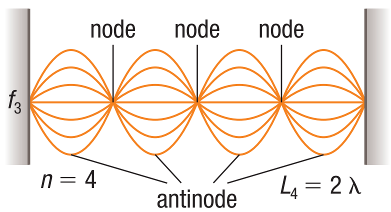
Symbol	Number of nodes between ends	Diagram	Harmonic (n)	Overtone
f_0	0		first	fundamental
f_1	1		second	first
f_2	2		third	second
f_3	3		fourth	third

Figure 10 A mute attached to the bell of a trumpet changes the sound.

Standing Waves between Free Ends and Fixed–Free Ends

Figure 8 shows standing waves in a medium with two free ends. Brass instruments, for example, have an open end for the musician to blow through and a bell at the other end for the music to come out of. **Figure 9** shows standing waves in a medium with a combination of free and fixed ends. An example of such a combination is a brass instrument using a mute, which is a device that fits over the end of the bell to change the sound the instrument makes (**Figure 10**).

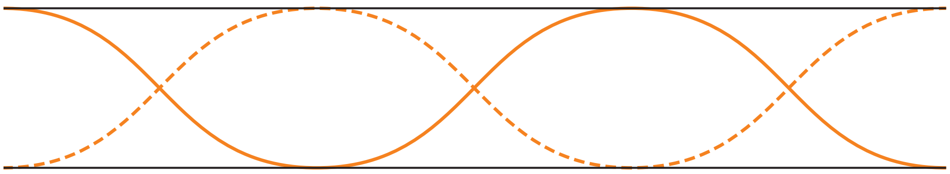


Figure 8 Standing waves can be generated in a medium with two open (free) ends. These types of standing waves are very common in brass instruments.

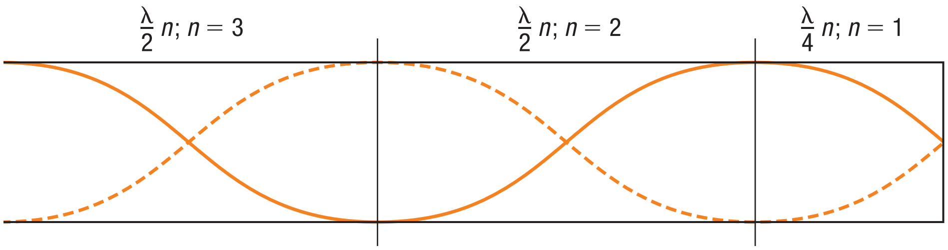


Figure 9 A standing wave may be generated by having a source on one end, which is an antinode, and a node at the other end. Such standing waves can be created by a clarinet.

Calculations with Standing Waves

A brief inspection of the standing waves in Table 1 on page 423, Figure 8, and Figure 9 generated in media with two fixed ends or two free ends shows that the standing waves have the same mathematical properties. In general, the length of the medium, L, is equal to the number of the harmonic, n, times half the standing wave's wavelength, $\frac{\lambda}{2}$:

$$L_n = \frac{n\lambda}{2} \quad \text{for } n = 1, 2, 3, \ldots; \text{ media with fixed ends or free ends}$$

For media with a combination of fixed and free ends—a node at one end and an antinode at the other—the shortest possible length to produce a standing wave is $\frac{\lambda}{4}$ (see Figure 9). The next distance that produces a standing wave is $\frac{\lambda}{2}$ more than this, which is $\frac{3\lambda}{4}$. Therefore, the sequence of possible medium lengths to produce a standing wave in media with a free end and a fixed end is $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$, and so on. The general equation for determining the length of the medium with an antinode at one end and a node at the other end is as follows:

$$L_n = \frac{(2n - 1)}{4}\lambda \quad \text{for } n = 1, 2, 3, \ldots; \text{ media with a fixed end and a free end}$$

We can use the mathematical relationships between the variables to predict what characteristics are required to produce standing waves. In Tutorial 1, we will demonstrate calculations using standing wave equations.

Tutorial 1 Standing Waves

Sample Problem 1

The speed of a wave on a string with a fixed end and a free end is 350 m/s. The frequency of the wave is 200.0 Hz. What length of string is necessary to produce a standing wave with the first harmonic?

Given: free and fixed ends; $v = 350$ m/s; $f = 200.0$ Hz; $n = 1$

Required: L_1

Analysis: We first determine the wavelength using the universal wave equation. We can then use the formula for fixed and free ends to calculate the required length.

$$\lambda = \frac{v}{f};\ L_n = \frac{(2n - 1)}{4}\lambda$$

Solution: Calculate the wavelength:

$$\lambda = \frac{v}{f}$$
$$= \frac{350\ \text{m/s}}{200.0\ \text{Hz}}$$
$$\lambda = 1.75\ \text{m (one extra digit carried)}$$

Calculate the length of string:

$$L_n = \frac{(2n - 1)}{4}\lambda$$
$$L_1 = \frac{1}{4}\lambda$$
$$= \frac{1}{4}(1.75\ \text{m})$$
$$L_1 = 0.44\ \text{m}$$

Statement: A string that is 0.44 m long will produce a standing wave with the first harmonic.

Sample Problem 2

The sixth harmonic of a 65 cm guitar string is heard. If the speed of sound in the string is 206 m/s, what is the frequency of the standing wave?

Given: two fixed ends; $L_6 = 0.65$ m; $n = 6$; $v = 206$ m/s

Required: f_6

Analysis: First, we determine the wavelength of the guitar string using the relationship for fixed ends. Then we use the universal wave equation to calculate the frequency of the standing wave.

$$L_n = \frac{n\lambda}{2};\ v = f_6\lambda$$

Solution: $L_6 = \frac{n\lambda}{2}$

$$\lambda = \frac{2L_6}{n}$$
$$= \frac{(2)(0.65\ \text{m})}{6}$$
$$\lambda = 0.2167\ \text{m (two extra digits carried)}$$
$$v = f_6\lambda$$
$$f_6 = \frac{v}{\lambda}$$
$$= \frac{206\ \text{m/s}}{0.2167\ \text{m}}$$
$$f_6 = 950\ \text{Hz}$$

Statement: The frequency of the standing wave is 950 Hz.

Practice

1. A 0.44 m length of rope has one fixed end and one free end. A wave moves along the rope at the speed 350 m/s with a frequency of 200.0 Hz at $n = 1$. T/I
   (a) What is L_1 if the frequency is doubled? [ans: $L_1 = 0.22$ m]
   (b) What is the length of the string if $n = 3$? [ans: 2.2 m]
   (c) What is L_1 if the speed of the wave on the string is reduced to 200 m/s? [ans: 0.25 m]
2. The speed of a wave travelling along a 0.65 m guitar string is 206 m/s. At $n = 6$, the frequency is 950 Hz. T/I
   (a) What is the frequency if a string with a wave speed of 150 m/s is used? [ans: 690 Hz]
   (b) What is the frequency if the string is tightened to make the wave speed 350 m/s? [ans: 1600 Hz]
3. A string fixed at both ends has a length of 1 m. With a frequency of 44 kHz (fourth overtone), standing waves are produced. Which harmonics will be audible to a human? (The frequency range for human hearing is 20 Hz to 20 kHz.) T/I [ans: the first and second harmonics]

Mini Investigation

Creating Standing Waves

Skills: Predicting, Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this activity, you will create standing waves using a skipping rope and then using a standing wave machine.

Equipment and Materials: standing wave machine; skipping rope or other rope with a diameter of approximately 1 cm and length of approximately 3 m to 4 m

Part A: Using a Skipping Rope

1. Hold one end of the skipping rope tightly, and have your partner hold the other end. Slowly oscillate the skipping rope while your partner keeps his or her hand still while still holding onto the rope.
2. Slowly increase the frequency until you reach f_0.
3. Try to double the frequency and produce f_1.

A. How did you know when you had reached f_0? K/U

B. Was the experience of moving the rope any different when the first harmonic was achieved? K/U

Part B: Using a Standing Wave Machine

4. Connect the standing wave machine to its electrical power supply or to a wave function generator.
5. Slowly increase the frequency of the function generator and generate as many harmonics as possible.

C. Using the characteristics of the standing wave machine provided by your teacher, predict f_0. T/I

D. Did the frequencies at which the standing waves were produced agree with your predictions in C? Explain. K/U

UNIT TASK BOOKMARK

As you work on the Unit Task on page 486, apply what you have learned about standing waves and musical instruments.

9.2 Summary

- The location where two different media meet is called a media boundary. At a media boundary, a wave is partly reflected and partly transmitted.
- Free-end reflections produce reflections with the same orientation as the original wave, and fixed-end reflections produce reflections that have the opposite orientation to the original wave.
- A standing wave is a special case of interference. The waves in a standing wave pattern interfere according to the principle of superposition.
- In cases where a standing wave is produced in a medium where the medium is fixed at both ends or open at both ends, the length of the medium is a whole-number multiple of $\frac{\lambda}{2}$, the first harmonic.
- In cases where a standing wave is produced in a medium where the medium is fixed at one end and open at the other end, the length of the medium is determined by $L_n = \frac{(2n - 1)}{4}\lambda$.

9.2 Questions

1. Define the following terms in your own words: K/U C
   (a) standing wave
   (b) fundamental frequency
   (c) node
   (d) harmonics
2. Identify from your own experience an example of a wave that encounters a media boundary. A
   (a) From your observations, is the amplitude of the reflected wave or transmitted wave increased?
   (b) Does the change in the medium support your answer to (a)? Explain.
3. Describe from your own experience an example of a free-end reflection and a fixed-end reflection. A
4. Describe the conditions required to form a standing wave. K/U
5. A string is 2.4 m long, and the speed of sound along this string is 450 m/s. Calculate the frequency of the wave that would produce a first harmonic. Assume the string has nodes at both ends. K/U
6. You have an open air column of length 1.2 m in air at 20 °C. Calculate the frequency of the second harmonic. (Hint: An open air column has an antinode at both ends.) T/I
7. The air temperature is 25 °C, and an air column carries a standing sound wave at a frequency of 340 Hz. What is the length of the air column, which is closed at one end, if you want to hear the third harmonic? K/U
8. Consider a standing wave that has two fixed ends. Sketch
   (a) the original wave without a reflection
   (b) the reflected wave, without the original but synchronized in time to the original
   (c) the superposition of these two waves K/U C