8.4 Determining Wave Speed

In this section, you will learn about the mathematical relationships involved with wave speed, such as the universal wave equation. You will also learn what factors influence the speed of a wave; in particular, a sound wave.

The Universal Wave Equation

Imagine you are standing on a dock on a lake so that you are able to observe the passing waves. (You can assume that you have all the equipment necessary to allow you to observe and measure properties of waves, such as distance and time.) First, by timing the duration between crests passing your reference point, you can measure the period of the wave. Then you can take a picture of the waveform and measure the wavelength using the dock and other structures as distance references. From these measurements, you can calculate wave speed using the kinematics definition for average speed.

$$v = \frac{\lambda}{T}$$

Using the fact that frequency is the reciprocal of period, a substitution can be made for T in the wave speed equation:

$$f = \frac{1}{T}$$

$$v = \frac{\lambda}{\frac{1}{f}}$$

$$\boxed{v = f\lambda}$$

> **LEARNING TIP**
>
> **Reciprocal**
> A reciprocal is a number that you multiply by so that the result equals 1. For example, the reciprocal of 4 is $\frac{1}{4}$ because $4 \times \frac{1}{4} = 1$.

universal wave equation $v = f\lambda$

This important relationship is called the **universal wave equation**, and it is valid for all waves and wave types.

The universal wave equation can also be derived as follows:

$$\text{frequency } (f) = \frac{\text{cycles}}{\text{time}}$$

$$\text{wavelength } (\lambda) = \frac{\text{distance}}{\text{cycles}}$$

$$\begin{aligned}\text{frequency } (f) \times \text{ wavelength } (\lambda) &= \frac{\cancel{\text{cycles}}}{\text{time}} \times \frac{\text{distance}}{\cancel{\text{cycles}}} \\ &= \frac{\text{distance}}{\text{time}} \\ &= \text{wave speed } (v)\end{aligned}$$

Hence

$$v = f\lambda$$

> **Investigation 8.4.1**
>
> **Investigating Two-Dimensional Wave Motion (p. 403)**
> In this investigation, you will predict the relationships between frequency, speed, and wavelength.

Tutorial 1 demonstrates how wave speed can be calculated using the universal wave equation.

Tutorial 1 Using the Universal Wave Equation

Sample Problem 1: Calculating Wave Speed

A harp string supports a wave with a wavelength of 2.3 m and a frequency of 220.0 Hz. Calculate its wave speed.

Given: $\lambda = 2.3$ m; $f = 220.0$ Hz

Required: v

Analysis: In this example, both λ and f are given. So, to solve this problem, substitute for the variables and calculate the answer using the universal wave equation: $v = f\lambda$

Solution:

$$v = f\lambda$$
$$= (220.0 \text{ Hz})(2.3 \text{ m})$$
$$v = 506 \text{ m/s}$$

Statement: The wave speed on the harp string is 506 m/s.

Sample Problem 2: Calculating Wavelength

A trumpet produces a sound wave that is observed travelling at 350 m/s with a frequency of 1046.50 Hz. Calculate the wavelength of the sound wave.

Given: $v = 350$ m/s; $f = 1046.50$ Hz

Required: λ

Analysis: Rearrange the universal wave equation to solve for wavelength: $v = f\lambda$

Solution: $v = f\lambda$

$$\lambda = \frac{v}{f}$$
$$= \frac{350 \text{ m/s}}{1046.50 \text{ Hz}}$$
$$\lambda = 0.33 \text{ m}$$

Statement: The wavelength of the sound wave coming from the trumpet is 0.33 m.

Practice

1. If a wave has a frequency of 230 Hz and a wavelength of 2.3 m, what is its speed? T/I
   [ans: 530 m/s]
2. If a wave has a speed of 1500 m/s and a frequency of 11 Hz, what is its wavelength? T/I
   [ans: 140 m]
3. If a wave has a speed of 405 m/s and a wavelength of 2.0 m, what is its frequency? T/I
   [ans: 2.0×10^2 Hz]

Factors That Affect Wave Speed

The transfer of energy using waves is more efficient if the particle vibrations do not absorb much energy. For example, a more rigid object such as a soccer ball tends to bounce more effectively if it is fully inflated. If the atoms comprising an object are linked by strong intermolecular forces, the wave energy is transmitted more efficiently and thus the wave speed is faster. If these forces are not as strong, then energy transmission is less efficient and thus slower.

Temperature

In the case of gases, you might think that cooler gases are more effective at transmitting sound because they are denser. However, usually the converse is true because, with an increase in temperature, the molecules move faster and transfer their kinetic energy more efficiently (**Figure 1**).

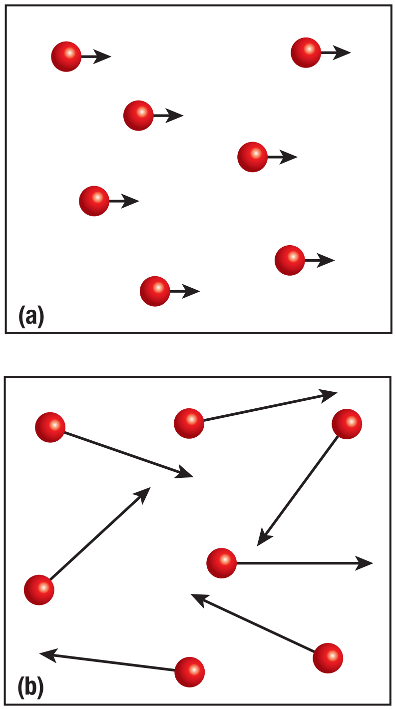

Figure 1 Comparing transmission of sound through (a) a cool gas and (b) a warm gas. The warm molecules jostle neighbouring molecules more rapidly, thus increasing the rate of sound energy transfer.

Linear Density and Tension

linear density (μ) the mass per unit distance of a string; units are kilograms per metre (kg/m)

The speed of a wave along a string, such as a violin or guitar string, is governed by the properties of the string (**Figure 2**). A string's **linear density**, or mass per unit distance, determines how much force it will take to make the string vibrate. Linear density, μ, is calculated using the equation

$$\mu = \frac{m}{L}$$

where m is the mass of the string, in kilograms, and L is its length, in metres.

Another variable affecting wave speed is tension. A loose string, for example, will quickly absorb all of the energy. A taut (tight) string, however, will transmit energy very effectively. Linear density and tension are the only variables that control the speed that waves can travel along a string. The equation for the speed of a wave along a string is

Figure 2 The diameters of the guitar strings shown here are getting progressively larger from left to right. The linear density is therefore increasing from left to right. The speed of sound is progressively slower in these strings.

$$v = \sqrt{\frac{F_T}{\mu}}$$

where F_T is the tension in the string (in newtons) and μ is the linear density (in kilograms per metre). In Tutorial 2 we will demonstrate how this equation is used to determine the properties of a string.

Tutorial 2 Determining String Properties

Sample Problem 1: Determining String Tension

On your class wave machine, you have a string of mass 350 g and length 2.3 m. You would like to send a wave along this string at a speed of 50.0 m/s. What must the tension of the string be?

Given: $m = 350$ g or 0.350 kg; $L = 2.3$ m; $v = 50.0$ m/s

Required: F_T

Analysis: First, calculate the linear density, μ. Second, rearrange the equation for the speed of a wave on a string to solve for the tension, F_T: $\mu = \frac{m}{L}$; $v = \sqrt{\frac{F_T}{\mu}}$

Solution:

$$\mu = \frac{m}{L}$$

$$= \frac{0.350 \text{ kg}}{2.3 \text{ m}}$$

$$\mu = 0.152 \text{ kg/m} \quad \text{(one extra digit carried)}$$

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$v^2 = \frac{F_T}{\mu}$$

$$F_T = v^2\mu$$

$$= (50.0 \text{ m/s})^2(0.152 \text{ kg/m})$$

$$= 380.4 \text{ N}$$

$$F_T = 380 \text{ N}$$

Statement: The required tension of the string on the wave machine is 380 N.

Practice

1. If a 2.5 m long string on the same wave machine has a tension of 240 N, and the wave speed is 300 m/s, what is the mass of the string? T/I [ans: 6.7×10^{-3} kg]
2. If a wave machine string has a linear density of 0.2 kg/m and a wave speed of 200 m/s, what tension is required? T/I [ans: 8×10^{3} N]
3. If a string on a wave machine has a linear density of 0.011 kg/m and a tension of 250 N, what is the wave speed? T/I [ans: 1.5×10^{2} m/s]

8.4 Summary

- The universal wave equation, $v = f\lambda$, relates the speed of a wave to its frequency and wavelength. The universal wave equation applies to all waves.
- More rigid intermolecular forces allow for a faster transfer of energy, and therefore a higher wave speed in a medium.
- Waves travel faster in hotter gases than in cooler gases because of the increased molecular motion caused by the higher temperature in a hotter gas.
- The speed of a wave on a string depends on the linear density of the string and the string's tension: $v = \sqrt{\frac{F_T}{\mu}}$

UNIT TASK BOOKMARK

How could you apply your understanding of the speed of sound in different materials to the Unit Task on page 486?

8.4 Questions

1. A wave has a speed of 123 m/s and a frequency of 230 Hz. What is its wavelength? T/I
2. A guitar string has a tension of 37 N. The linear density is 0.03 g/m. What is the speed of sound along this string? T/I
3. The period of a sound wave from a piano is 1.20×10^{-3} s. If the speed of the wave in the air is 3.40×10^{2} m/s, what is its wavelength? T/I
4. Earthquakes produce seismic waves, which travel through Earth. Primary waves, or P-waves, are longitudinal. They can travel through both solids and liquids. Secondary waves, or S-waves, are transverse. They can travel through solids only. P-waves travel at approximately 8.0 km/s, and S-waves travel at approximately 4.5 km/s. Following an earthquake, vibrations are recorded at seismological stations around the world. K/U T/I A
   (a) Calculate how long P-waves and S-waves take to travel from an earthquake to a seismological station that is 2.4×10^{3} km away. Express your answers in minutes.
   (b) Why do you think that transverse waves are called secondary waves?
   (c) By referring to **Figure 3**, explain how observing P-waves and S-waves helps geophysicists analyze the structure of Earth's interior.
5. Predict what happens to the wavelength of a wave on a string when the frequency is doubled. Assume that the tension in the string remains the same. Confirm your prediction mathematically. K/U T/I

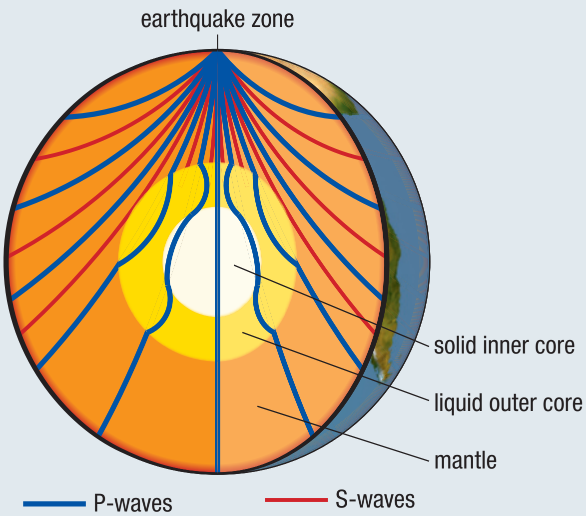

Figure 3

6. Predict what happens to the speed of a wave on a string when the frequency is doubled. Assume that the tension in the string remains the same. Confirm your prediction mathematically. K/U T/I
7. By what factor would you have to multiply the tension in a taut spring in order to double the wave speed? Confirm your answer mathematically. T/I C
8. Develop the equation for wave speed on a string. Use research if you wish. T/I C