7.3 Half-Life

Radioactive decay reactions have applications in a wide range of fields. For example, in 1996 the remains of a prehistoric man, shown in **Figure 1**, were found in Kennewick, Washington. Scientists used the properties of radioactive decay to determine that the remains belonged to a man who lived over 9000 years ago! This discovery has led to further information about North American ancestry and the evolution of humans as a species. We will explore the techniques that were used to date this fascinating artifact in this section.


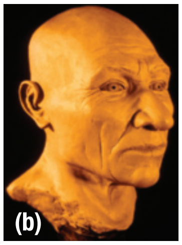

Figure 1 (a) Skull of Kennewick man (b) Reconstruction of Kennewick man

Measuring the Rate of Radioactive Decay Processes: Half-Life

Radioactive decay reactions are spontaneous. There is no way to predict exactly when a particular unstable nucleus will disintegrate. However, it is possible to predict the decay rate for a large sample of an isotope. Radioactive materials decay at different rates, which can vary significantly. The average length of time it takes a radioactive material to decay to half its original mass is called the **half-life**.

half-life the average length of time it takes radioactive material to decay to half of its original mass

The half-life of any given isotope is actually an average time for a particular parent atom to decay to its daughter atom. Cobalt-60, for example, has a half-life of 5.27 years. This does not mean that every atom of this isotope decays to its daughter atom after 5.27 years. Some atoms decay sooner, and some later. On average, however, it takes 5.27 years for an atom of cobalt-60 to decay. The larger the sample size, the more accurately a material decays according to its half-life.

Mini **Investigation**

Analyzing Half-Life

Skills: Predicting, Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A6.5

When a radioactive material decays, the amount of the parent decreases, while the amount of the daughter increases. How can these relationships be represented graphically?

Equipment and Materials: periodic table; graph paper or graphing technology; half-life simulation applet (optional)

1. Carbon-15 with a half-life of 2.5 s decays to nitrogen-15. Suppose a sample of carbon-15 has an initial mass of 256 mg. Copy and complete **Table 1**. Assume that mass is conserved.

Table 1

Time (s)	Mass of C-15 (mg)	Mass of N-15 (mg)	Total mass (mg)
0	256	0	256
2.5	128	128	256
5			
7.5			

2. Predict the shape of the graph of mass of carbon-15 versus time. Explain your reasoning. Plot a graph of mass of carbon-15 versus time, with time on the horizontal axis. Use a smooth curve to join the points.
3. Repeat Step 2 for the graph of the mass of nitrogen-15 versus time. Plot both graphs on the same grid.

A. What type of radioactive decay is this reaction? Explain how you know. K/U C

B. Write the nuclear reaction equation. T/I C

C. Discuss the rates of change for carbon-15 and nitrogen-15. Explain why the two graphs have the shapes that they do. T/I C

D. Interpret the point of intersection of the two graphs. Explain what each coordinate of this point represents. T/I C

Mathematical Models Using Half-Life

Radioactive decay is an example of an exponential relationship—as time increases, the mass of a radioactive isotope remaining in a sample decreases at an exponential rate. The rate of decay is greater in the initial stages of the process because there are more atoms to decay. The rate of decay continuously decreases as the sample gets smaller and smaller. The mass, A, of a radioactive material with an initial sample mass of A_0 is related to time, t, and half-life, h. This can be represented by the following equation:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

When using this equation it is important to measure the masses A and A_0 using the same units. The same is true for t and h. In the following Tutorial, you will apply this equation to solve problems involving the half-life of radioactive isotopes.

LEARNING TIP

The Half-Life Equation

The exponent $\frac{t}{h}$ is time divided by half-life. This quotient represents the number of half-lives, which is the number of times the initial amount is reduced by one-half.

Tutorial 1 Calculations Involving Half-Life

Sample Problem 1

Neon-19 has a half-life of 17.22 s. What mass of neon-19 will remain from a 100 mg initial sample after 30 s?

Given: $A_0 = 100$ mg; $h = 17.22$ s; $t = 30$ s

Required: A

Analysis: $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

Solution: $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$$= (100 \text{ mg})\left(\frac{1}{2}\right)^{\frac{30 \cancel{s}}{17.22 \cancel{s}}}$$

$$= (100 \text{ mg})\left(\frac{1}{2}\right)^{1.7422}$$

$$A = 30 \text{ mg}$$

Statement: There will be 30 mg of neon-19 remaining after 30 s.

Sample Problem 2

A 100 mg sample of magnesium-27 decays by 7 % of its previous mass every minute. Determine its half-life and state the half-life decay equation.

Step 1. The decay of magnesium-27 can be modelled using a table or graph. If 7 % decays during each minute, then 93 % remains. Create a table similar to **Table 2** to determine the mass remaining after each minute.

Table 2 Mass of Magnesium-27 Remaining

Time (min)	Initial mass (mg)	Final mass (mg)
0	100	0.93(100) = 93
1	93	0.93(93) = 86.49
2	86.49	0.93(86.49) = 80.44
3	80.44	0.93(80.44) = 74.81
4	74.81	69.57
5	69.57	64.70
6	64.70	60.17
7	60.17	55.96
8	55.96	52.04
9	52.04	48.40
10	48.40	45.01

Step 2. Use the data in Table 2 to create a graph of mass remaining versus time (**Figure 2**).

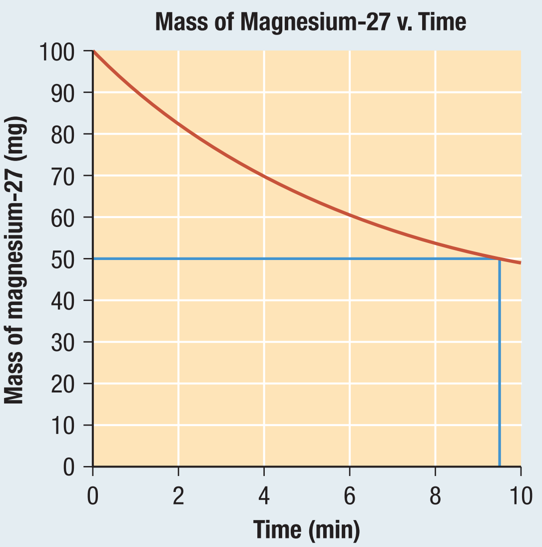

Figure 2

Step 3. Use the data in Table 2 and the graph in Figure 2 to determine the half-life of magnesium-27. Half of the initial mass of magnesium-27 has decayed approximately halfway between 9 min and 10 min. So, the half-life of magnesium-27 is about 9.5 min.

Step 4. Model the decay algebraically by substituting the half-life of 9.5 min into the half-life decay equation (note that time is measured in minutes, not seconds or hours):

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{9.5}}$$

Practice

1. Aluminum-30 has a half-life of 3.6 s. T/I C
   (a) What percent of an initial sample will remain after 10 s? [ans: 15 %]
   (b) What percent of an initial sample will remain after 10 min? [ans: 6.7×10^{-49} %]
2. After 10 years, a 100 mg sample of argon-42 has decayed to 81 mg. Estimate the half-life of argon-42. T/I C [ans: 33 years]

Applications of Half-Life: Carbon Dating

The half-life of carbon-14 is 5730 years. It decays into nitrogen-14 according to the following nuclear reaction equation:

$${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}$$

The half-life of C-14 makes it a useful material for measuring the age of organisms that once lived long ago. When plants absorb carbon dioxide through the process of photosynthesis, the carbon is typically a mixture of the common C-12 and the relatively rare C-14 isotopes. Herbivores ingest C-14 when they eat plants, and carnivores do so when they feed on herbivores. The ratio of C-14 to C-12 is generally constant and equal in all living things. When an organism dies, however, it no longer consumes food, and therefore no longer ingests carbon. As the carbon-14 decays, the C-14 to C-12 ratio in the dead organism is reduced by half every 5730 years. Scientists can use this known rate of decrease to determine when the organism died. **Figure 3** shows the percentage of C-14 remaining from an initial mass present as a function of time after an organism dies.

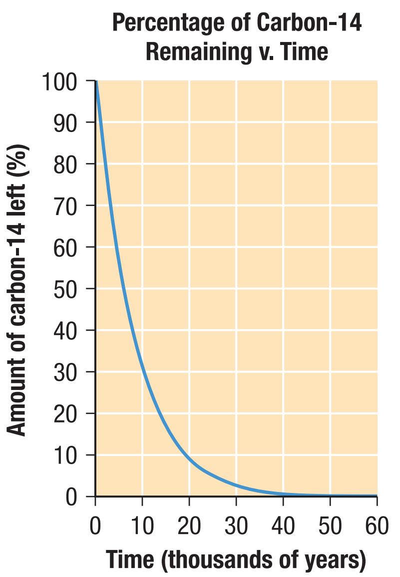

Figure 3 Decay curve for C-14

Other isotopes are also useful for dating objects and organisms. Aluminum-26, for example, has been used to measure the ages of interstellar rocks. Al-26 decays into magnesium-26 with a half-life of approximately 720 000 years. Scientists can approximate the age of a sample by comparing the relative masses of Al-26 and Mg-26, in a similar way as with carbon dating.

WEB **LINK**

To learn more about carbon dating,

GO TO NELSON SCIENCE

7.3 Summary

- The half-life of a radioactive isotope is the amount of time required for it to decay to one-half of its original mass.
- Half-lives can vary from a tiny fraction of a second to millions of years.
- The decay of a radioactive isotope can be mathematically modelled using a table, a graph, or an equation.
- Some isotopes like carbon-14 and aluminum-26 have useful applications due in part to their particular half-lives.
- Carbon-14 is a useful isotope for dating fossils and other archaeological objects.

7.3 Questions

1. Chlorine-38, which undergoes beta-negative decay, has a half-life of 37.24 min. K/U T/I C
   (a) Construct a table that compares the mass of Cl-38 remaining after *t* minutes for several values of *t*.
   (b) Draw a graph that illustrates this relationship.
   (c) What isotope does Cl-38 decay into?
2. Gold-198, with a half-life of 2.6 days, is used to diagnose and treat liver disease. T/I C
   (a) Write a half-life decay equation that relates the mass of Au-198 remaining to time in days.
   (b) What percentage of a sample of Au-198 would remain after
      (i) 1 day?
      (ii) 1 week?
3. Cobalt-60, with a half-life of 5.3 years, has a number of applications, including medical therapy and the sterilization of medical tools. Determine the mass of a 50 g sample that would remain after
   (a) 6 months
   (b) 5 years T/I C
4. What type of radioactive decay is involved in carbon dating? Explain the process of carbon dating. K/U C
5. A fossil contains 70 % of the carbon-14 it once had as a living creature. Use the half-life decay equation to determine when the creature died. T/I C

Aluminum-26, which decays into magnesium-26, has a half-life of approximately 720 000 years. Use this information to answer Questions 6 and 7.

6. (a) What type of decay does Al-26 undergo?
   (b) Does Al-26 decay in the same way as C-14? Explain. K/U C
7. A moon rock has 3 % of its original Al-26 mass. K/U T/I C
   (a) Determine the age of the moon rock.
   (b) Discuss any assumptions that must be made when using this method of dating.
8. Take a regular sheet of paper. Measure its length and width and determine the area. Fold the paper neatly in half. Determine the new area. Repeat until you cannot fold the paper any longer. Explain how this model can be used to describe half-life. K/U C