5.2 Energy

As you learned in Section 5.1, mechanical work is done by applying forces on objects and displacing them. How are people, machines, and Earth able to do mechanical work? The answer is energy: **energy** provides the ability to do work. Objects may possess energy whether they are moving or at rest. In this section, we will focus on a form of energy possessed only by moving objects.

energy the capacity to do work

Kinetic Energy: The Energy of Moving Objects

kinetic energy (E_k) energy possessed by moving objects

A moving object has the ability to do work because it can apply a force to another object and displace it. The energy possessed by moving objects is called **kinetic energy (E_k)** (from the Greek word kinema, meaning "motion"). For example, a moving hammer has kinetic energy because it has the ability to apply a force on a nail and push the nail into a piece of wood (**Figure 1**). The faster the hammer moves, the greater its kinetic energy, and the greater the displacement of the nail. In general, objects with kinetic energy can do work on other objects.

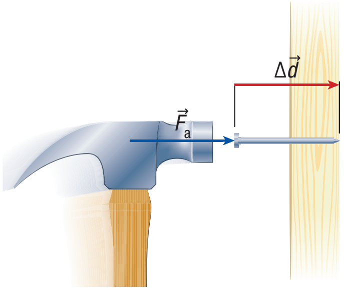

Figure 1 A moving hammer can do work on a nail because it has kinetic energy.

Quantifying Kinetic Energy

Since kinetic energy is energy possessed by moving objects, consider a motorcycle moving in a straight line on a roadway (**Figure 2**). The motorcyclist and the motorcycle have a total mass, m. When the motorcyclist twists the motorcycle's throttle, the motorcycle experiences a net force of magnitude, F_{net}, that accelerates the motorcycle from an initial speed, v_i, to a final speed, v_f. The acceleration occurs over a displacement of magnitude, Δd, in the same direction as the displacement. In this case, the engine does positive mechanical work on the motorcycle, since the displacement is in the same direction as the force. The net force is the sum of the applied force produced by the engine, the normal force, the force of gravity, the force of friction between the wheels and the road, and the force of friction between the air and the vehicle (including the motorcyclist).

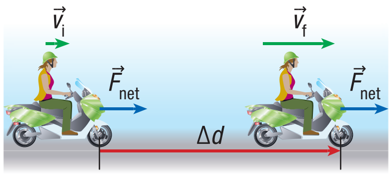

Figure 2 Displacement of a motorcycle accelerating on a flat, level roadway

The work equation may be used to describe the relationship between the total, or net, work done on the motorcycle, the magnitude of the net force and the displacement, and the angle between the applied force and the displacement:

$$W_{net} = F_{net}(\cos\theta)\Delta d$$

Since the force and displacement are in the same direction, $\cos\theta = 0°$ and $W = F_{net}\Delta d$.

Newton's second law relates the magnitude of the net force to the motorcycle's mass and the magnitude of its resulting acceleration:

$$F_{net} = ma$$

Substituting ma for F_{net} in the work equation gives

$$W_{net} = ma\Delta d$$

In Chapter 1, you learned that the following equation relates the final speed of an accelerating object to its initial speed and the magnitudes of its acceleration and displacement:

$$v_f^2 = v_i^2 + 2a\Delta d$$

Isolating $a\Delta d$ on the left side of this equation, we get

$$a\Delta d = \frac{v_f^2 - v_i^2}{2}$$

Substituting $\frac{v_f^2 - v_i^2}{2}$ for $a\Delta d$ in the equation $W_{net} = ma\Delta d$, we get

$$W_{net} = m\left(\frac{v_f^2 - v_i^2}{2}\right) \text{ or } W_{net} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$$

If we assume that the motorcycle accelerates from rest to v, then $v_i = 0$ m/s and $v_f = v$:

$$W_{net} = \frac{mv^2}{2} - 0$$

$$W_{net} = \frac{mv^2}{2}$$

The quantity $\frac{mv^2}{2}$ is equal to the net amount of work done on the motorcycle to cause it to reach a final speed of v and also equals the amount of kinetic energy the motorcycle possesses as it travels at a speed of v. Therefore, in general, an object of mass, m, travelling at a constant speed, v, has a kinetic energy, E_k, given by the equation

$$E_k = \frac{mv^2}{2}$$

Kinetic energy is a scalar quantity: it has a magnitude given by the equation above, but it does not have a direction. If mass is measured in kilograms and speed in metres per second, then E_k will have units of $\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$. The unit $\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$ may be simplified as follows:

$$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)\text{m} = \text{N}\cdot\text{m}$$

As you learned in Section 5.1,

$$1\ \text{N}\cdot\text{m} = 1\ \text{J}$$

Therefore, both kinetic energy and mechanical work may be quantified in units of joules, J. This indicates the close relationship between mechanical work and kinetic energy. We will use the kinetic energy equation in the following Tutorial.

Tutorial 1 Calculating the Kinetic Energy of a Moving Object

In this Tutorial, you will use the equation $E_k = \frac{mv^2}{2}$ to calculate the kinetic energy of an object of mass, m, moving at a constant speed, v.

Sample Problem 1

Calculate the kinetic energy of a 150 g baseball that is travelling toward home plate at a constant speed of 35 m/s.

Given: $m = 150$ g or 0.150 kg; $v = 35$ m/s

Required: E_k, kinetic energy

Analysis: $E_k = \frac{mv^2}{2}$

Solution:

$$E_k = \frac{mv^2}{2} = \frac{(0.150\ \text{kg})(35\ \text{m/s})^2}{2}$$

$$E_k = 92\ \text{J}$$

Statement: The baseball has a kinetic energy of 92 J.

Practice

1. A 70.0 kg athlete is running at 12 m/s in the 100.0 m dash. What is the kinetic energy of the athlete? T/I [ans: 5.0 kJ]
2. A dynamics cart has a kinetic energy of 4.2 J when moving across a floor at 5.0 m/s. What is the mass of the cart? T/I [ans: 0.34 kg]
3. A 150 g bird goes into a dive, reaching a kinetic energy of 30.0 J. What is the speed of the bird? T/I [ans: 20 m/s]

The Relationship between Mechanical Work and Kinetic Energy

You can observe the relationship between mechanical work and kinetic energy by analyzing the mechanical work and kinetic energy equations. Since $E_k = \frac{mv^2}{2}$, the equation $W_{net} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$ may be written as $W_{net} = E_{k_f} - E_{k_i}$ or $W_{net} = \Delta E_k$, where E_{k_f} is the final kinetic energy and E_{k_i} is the initial kinetic energy of the object. In words, this equation tells us that the total mechanical work, W, that increases the speed of an object is equal to the change in the object's kinetic energy, $E_{k_f} - E_{k_i}$. In other words, work is a change in energy. This relationship between kinetic energy and mechanical work is known as the **work–energy principle**.

work–energy principle the net amount of mechanical work done on an object equals the object's change in kinetic energy

Tutorial 2 Solving Problems Using the Work–Energy Principle

In the following Sample Problem, you will use the work–energy principle to determine the final speed of a hockey puck that is being pushed by a hockey stick.

Sample Problem 1

A 165 g hockey puck initially at rest is pushed by a hockey stick on a slippery horizontal ice surface by a constant horizontal force of magnitude 5.0 N (assume that the ice is frictionless), as shown in **Figure 3**. What is the puck's speed after it has moved 0.50 m?

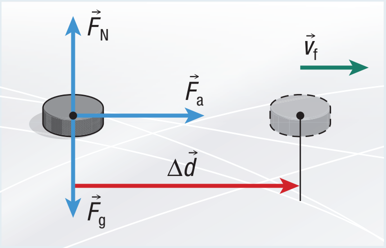

Figure 3

Consider the diagram shown in Figure 3. In this problem, the gravitational force and the normal force are equal in magnitude and opposite in direction. Therefore, they cancel each other. Since there is no friction, the magnitude of the net force on the puck is equal to the magnitude of the applied force, $\vec{F}_a$, directed horizontally. Since the net force is in the same direction as the displacement, we may use the equation $W_{net} = F_{net}\Delta d$ to calculate the total work done on the puck. We may then use the equation $W_{net} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$ to calculate the puck's final speed, v_f.

Given: $m = 0.165$ kg; $F_a = 5.0$ N; $\Delta d = 0.50$ m; $v_i = 0$ m/s

Required: v_f, the puck's final speed

Analysis: $W_{net} = F_{net}\Delta d$; $W_{net} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$

Solution:
$$\begin{aligned} W_{net} &= F_a\Delta d \\ &= (5.0\text{ N})(0.50\text{ m}) \\ &= 2.5\text{ N·m} \\ W_{net} &= 2.5\text{ J} \end{aligned}$$

Now that we have calculated the net work done on the puck, we can use this value to find the puck's final velocity.

$$\begin{aligned} W_{net} &= \frac{mv_f^2}{2} - \frac{mv_i^2}{2} \\ W_{net} &= \frac{mv_f^2}{2} - 0 \\ v_f^2 &= \frac{2W_{net}}{m} \\ v_f &= \sqrt{\frac{2W_{net}}{m}} \\ &= \sqrt{\frac{2(2.5\text{ J})}{0.165\text{ kg}}} \\ v_f &= 5.5\text{ m/s} \end{aligned}$$

Statement: The hockey puck's final speed is 5.5 m/s.

Practice

1. A 1300 kg car starts from rest at a stoplight and accelerates to a speed of 14 m/s over a displacement of 82 m. T/I
   (a) Calculate the net work done on the car. [ans: 130 kJ]
   (b) Calculate the net force acting on the car. [ans: 1.6×10^3 N]
2. A 52 kg ice hockey player moving at 11 m/s slows down and stops over a displacement of 8.0 m. T/I C
   (a) Calculate the net force on the skater. [ans: 390 N [backwards]]
   (b) Give two reasons why you can predict that the net work on the skater must be negative.

Kinetic energy is the energy possessed by moving objects. The work–energy principle tells us that the change in an object's kinetic energy is equal to the total amount of work done on that object. However, kinetic energy is not the only type of energy an object may have. Objects may possess another form of energy.

Gravitational Potential Energy: A Stored Type of Energy

In a circus stunt, one acrobat stands on a platform at the top of a tower ready to jump onto one end of a seesaw. Another acrobat stands on the other end. When the acrobat on the tower steps off his platform, the force of gravity pulls him downward, and he accelerates toward the seesaw. Just before landing, the acrobat is moving very quickly and has a lot of kinetic energy. This kinetic energy allows the acrobat to do mechanical work on his end of the seesaw, displacing it downward. This downward motion causes the other acrobat to be thrown into the air (**Figure 4**).

Figure 4 The acrobat's kinetic energy does mechanical work on the seesaw, causing the other acrobat to be displaced into the air.

When the acrobat was at the top of the tower, he was at rest and did not have kinetic energy. However, since he was positioned high above the ground, he had the ability to fall and gain kinetic energy that could do mechanical work. He was able to fall because of the force of gravity on him. The ability of an object to do work because of forces in its environment is called **potential energy**. Potential energy may be considered a stored form of energy. The force acting on the acrobat as he fell is Earth's gravitational force. Therefore, we say that the acrobat had **gravitational potential energy** when he was above the ground because of his position. The acrobat's gravitational potential energy started turning into kinetic energy as soon as he began to fall. A ski racer perched at the top of a ski hill, the water at the top of a waterfall, and a space shuttle orbiting Earth all have gravitational potential energy (**Figure 5**).

potential energy a form of energy an object possesses because of its position in relation to forces in its environment

gravitational potential energy energy possessed by an object due to its position relative to the surface of Earth

Figure 5 (a) A downhill skier, (b) the water at the top of Niagara Falls, and (c) the orbiting space shuttle all have gravitational potential energy.

Quantifying Gravitational Potential Energy

Have you ever heard the expression "the higher you go, the harder you fall"? In physics, this means that objects can do more work on other objects if they fall from higher positions above a particular **reference level**, or the level to which an object may fall. Any level close to Earth's surface may be used as a reference level. The selected reference level is assigned a gravitational potential energy value of 0 J, and levels above it have positive values (greater than 0 J). Levels below the reference level have negative values (less than 0 J).

reference level a designated level to which objects may fall; considered to have a gravitational potential energy value of 0 J

Gravitational potential energy may be calculated using the equation $E_g = mgh$ (where $g = 9.8\ m/s^2$) only near Earth's surface because the value of g changes as you move farther away from (or closer to) Earth's centre. The value of g is about 9.8 m/s^2 near Earth's surface. However, the value steadily decreases to approximately 8.5 m/s^2 at 400 km above Earth's surface (height of a space shuttle orbit) and increases to about 10.8 m/s^2 at a depth of 3000 km below the surface.

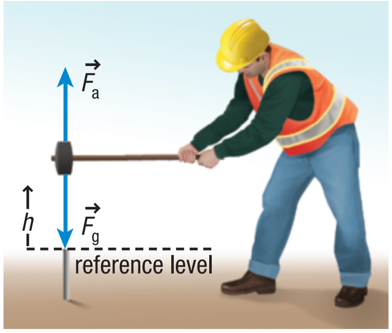

Figure 6 A construction worker uses a sledgehammer.

Assume that a construction worker needs to drive a spike into the ground with a sledgehammer (**Figure 6**). To do this, the worker must first raise the hammer and then allow it to fall onto the spike. If we assume the top of the spike to be our reference level, then the distance from the top of the spike to the hammer's head is the height, *h*. The weight of the hammer is given by the magnitude of $\vec{F}_g$ and is equal to mg, where m is the hammer's mass and g is the magnitude of the gravitational force constant, 9.8 N/kg.

If the construction worker (CW) lifts the hammer straight up at constant speed, then he applies a force $\vec{F}_{cw}$ upward that is equal in magnitude to the weight of the hammer, mg. In lifting the hammer, the worker is doing work given by $W_{cw} = F_{cw}\Delta d$ since the applied force and the displacement are in the same direction. Therefore, the work done by the construction worker on the hammer is $W_{cw} = mgh$ since $F_{cw} = mg$ and $\Delta d = h$. This mechanical work serves to transfer energy from the construction worker to the hammer, increasing the hammer's gravitational potential energy. The change in the hammer's gravitational potential energy is equal to the work done by the construction worker: $E_g = mgh$.

Note that the hammer is raised at constant speed ($v_i = v_f$). The construction worker does work on the hammer, but the force of gravity also does work on the hammer, equal to $W_g = F_g(\cos 180°)\Delta d$, or $W_g = mgh(-1)$.

In general, the gravitational potential energy of an object of mass, m, lifted to a height, h, above a reference level is given by the following equation:

$$E_g = mgh$$

The SI unit for gravitational potential energy is the newton metre, or the joule—the same SI unit used for kinetic energy and all other forms of energy. As with kinetic energy and work, gravitational potential energy is a scalar quantity.

Tutorial 3 Determining Gravitational Potiential Energy

In this Tutorial, we will determine the gravitational potential energy of a person on a drop tower amusement park ride. In a drop tower ride, people sit in gondola seats that are lifted to the top of a vertical structure and then dropped to the ground. Special brakes are used to slow the gondolas down just before they reach the ground.

Sample Problem 1

What is the gravitational potential energy of a 48 kg student at the top of a 110 m high drop tower ride relative to the ground?

In this Sample Problem, the student's mass and height above ground (the reference level) are given. Therefore, we may use the gravitational potential energy equation, $E_g = mgh$, to calculate the student's gravitational potential energy.

Given: $m = 48$ kg; $h = 110$ m; $g = 9.8$ N/kg

Required: E_g, gravitational potential energy

Analysis: $E_g = mgh$

Solution: $E_g = mgh$

$$= (48\ \cancel{\text{kg}})\left(9.8\ \frac{\text{N}}{\cancel{\text{kg}}}\right)(110\ \text{m})$$

$$= 5.2 \times 10^4\ \text{N}\cdot\text{m}$$

$$E_g = 5.2 \times 10^4\ \text{J or 52 kJ}$$

Statement: The student has 52 kJ of gravitational potential energy at the top of the ride relative to the ground.

Practice

1. A 58 kg person walks down the flight of stairs shown in **Figure 7**. Use the ground as the reference level. T/I
   (a) Calculate the person's gravitational potential energy at the top of the stairs, on the landing, and at ground level. [ans: 3400 J, 1700 J, 0 J]
   (b) What happens to gravitational potential energy as you go down a flight of stairs? What happens to gravitational potential energy as you climb a flight of stairs?

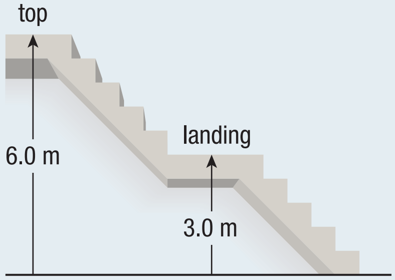

Figure 7

Mechanical Energy

Objects may possess kinetic energy only, gravitational potential energy only, or a combination of kinetic energy and gravitational potential energy relative to a particular reference level. For example, a hockey puck sliding on a flat ice surface at ground level has kinetic energy but no gravitational potential energy. An acorn hanging on a tree branch has gravitational potential energy but no kinetic energy. A parachutist descending to the ground has kinetic energy and gravitational potential energy.

The sum of an object's kinetic energy and gravitational potential energy is called **mechanical energy**. Therefore, if a sliding hockey puck has 10 J of kinetic energy and no gravitational potential energy, then it has 10 J of mechanical energy. If a descending parachutist has 5 kJ of kinetic energy and 250 kJ of gravitational potential energy, then the parachutist has 255 kJ of mechanical energy.

mechanical energy the sum of kinetic energy and gravitational potential energy

5.2 Summary

- Energy is the capacity (ability) to do work.
- Kinetic energy is energy possessed by moving objects. The kinetic energy of an object with mass, m, and speed, v, is given by $E_k = \frac{mv^2}{2}$.
- The total work, W_{net}, done on an object results in a change in the object's kinetic energy: $W_{net} = E_{k_f} - E_{k_i}$, where E_{k_f} and E_{k_i} represent the final and initial kinetic energy of the object, respectively. In other words, $W_{net} = \Delta E_k$.
- Gravitational potential energy is possessed by an object based on its position relative to a reference level, which is often the ground. $E_g = mgh$, where h is the height above the chosen reference level.
- Mechanical energy is the sum of kinetic energy and gravitational potential energy.

Investigation 5.2.1

Conservation of Energy (p. 257)
In this investigation, you will explore what happens to the gravitational potential energy, kinetic energy, and total mechanical energy of a cart as it rolls down a ramp.

5.2 Questions

1. A bobsleigh with four people on board has a total mass of 610 kg. How fast is the sleigh moving if it has a kinetic energy of 40.0 kJ? T/I
2. A 0.160 kg hockey puck starts from rest and reaches a speed of 22 m/s when a hockey stick pushes on it for 1.2 m during a shot. T/I
   (a) What is the final kinetic energy of the puck?
   (b) Determine the average net force on the puck using two different methods.
3. A large slide is shown in **Figure 8**. A person with a mass of 42 kg starts from rest on the slide at position A and then slides down to positions B, C, and D. Complete the following using the ground as the reference level: T/I

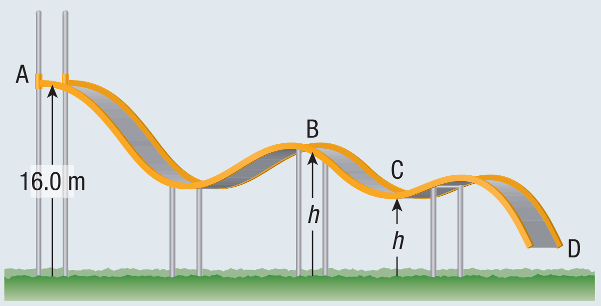

Figure 8

   (a) Calculate the gravitational potential energy of the person at position A.
   (b) The person has a gravitational potential energy of 4500 J at position B. How high above the ground is the person at position B?
   (c) The person loses 4900 J of gravitational potential energy when she moves from A to C. How high is the person at C?
   (d) What is the person's gravitational potential energy at ground level at D?
4. Forty 2.0 kg blocks 20.0 cm thick are used to make a retaining wall in a backyard. Each row of the wall will contain 10 blocks. You may assume that the first block is placed at the reference level. How much gravitational potential energy is stored in the wall when the blocks are set in place? T/I
5. You throw a basketball straight up into the air. Describe what happens to the kinetic energy and gravitational potential energy of the ball as it moves up and back down until it hits the floor. K/U C
6. Using the terms *work*, *kinetic energy*, and *gravitational potential energy*, describe what happens to people in a drop tower ride as they are slowly pulled to the top, released, and then safely slowed down at the bottom. K/U C