Solving Friction Problems

4.3

Sometimes friction is desirable and we want to increase the coefficient of friction to help keep objects at rest. For example, a running shoe is typically designed to have a large coefficient of friction between the sole and the floor or ground (**Figure 1**). The design of the sole involves choosing materials with high coefficients of friction, and shaping the sole to increase friction under wet conditions, among other factors. Even the appearance is important because people want their shoes to look good.

However, sometimes we want as little friction as possible. A hockey player shooting a puck toward the net does not want the puck to slow down as it moves across the ice (**Figure 2**). In transportation technologies, you want to minimize the force of friction acting on accelerating railway cars and trailers pulled by trucks. In this section, you will explore how the force of friction affects the motion of an object, whether the object is at rest or in motion. You will also learn how to solve problems involving friction.

Figure 1 The sole of a running shoe is designed to increase friction to help runners accelerate quickly.

Static Friction Problems

Sometimes when you push on an object, it does not move. The reason is that static friction is acting on the object. If two or more objects are attached, you must overcome the combined maximum force of static friction to cause the objects to move. For example, when a train pulls several railway cars forward, it must exert enough force to overcome the combined maximum force of static friction acting on all the railway cars. If the applied force acting on the railway cars is less than this combined force of static friction, the cars will not move. In the following Tutorial, we will examine the effect of static friction when it is acting on more than one object.

Figure 2 When a puck moves across the ice, it is better to have less friction.

Tutorial 1 Static Friction Acting on Several Objects

In the following Sample Problem, we will use the coefficient of friction to determine the forces acting on sleds being pulled by an adult.

Sample Problem 1

Two sleds are tied together with a rope (**Figure 3**). The coefficient of static friction between each sled and the snow is 0.22. A small child is sitting on sled 1 (total mass of 27 kg) and a larger child sits on sled 2 (total mass of 38 kg). An adult pulls on the sleds.

Figure 3

(a) What is the greatest horizontal force that the adult can exert on sled 1 without moving either sled?

(b) Calculate the magnitude of the tension in the rope between sleds 1 and 2 when the adult exerts this greatest horizontal force.

Solution

(a) The two sleds do not move when the adult pulls on sled 1. This means that the net force acting on the sleds is zero and the applied force must be cancelled by the total maximum force of static friction acting on the two sleds. To calculate the static friction, we combine the two masses and treat the sleds as one single object.

Given: $m_T = 27\text{ kg} + 38\text{ kg} = 65\text{ kg}$; $\mu_S = 0.22$

Required: $\vec{F}_{S_{max}}$

Analysis: $\mu_S = \frac{F_{S_{max}}}{F_N}$

Solution:
$$F_{S_{max}} = \mu_S F_N$$
$$= \mu_S m_T g$$
$$= (0.22)(65 \text{ kg})(9.8 \text{ m/s}^2)$$
$$F_{S_{max}} = 140 \text{ N}$$

Statement: The greatest horizontal force that the adult can exert on sled 1 without moving either sled is 140 N [forward].

(b) Draw the FBD for sled 2 (**Figure 4**).

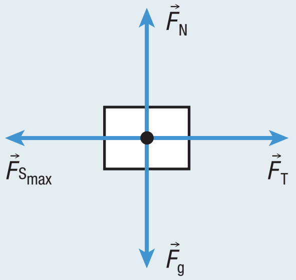


Figure 4

Keep in mind that sled 2 does not move, which means the net force is zero. In this case, the tension and the static friction acting on sled 2 will cancel.

Given: $m = 38$ kg; $\mu_S = 0.22$

Required: F_T

Analysis: Since $F_T = F_{S_{max}}$, calculate $F_{S_{max}}$ using $\mu_S = \frac{F_{S_{max}}}{F_N}$

Solution:
$$F_{S_{max}} = \mu_S F_N$$
$$= \mu_S mg$$
$$= (0.22)(38 \text{ kg})(9.8 \text{ m/s}^2)$$
$$F_{S_{max}} = 82 \text{ N}$$

Statement: The magnitude of the tension in the rope is 82 N.

Practice

1. Two trunks sit side by side on the floor. The larger trunk (52 kg) is to the left of the smaller trunk (34 kg). A person pushes on the larger trunk horizontally toward the right. The coefficient of static friction between the trunks and the floor is 0.35. T/I
   (a) Determine the magnitude of the maximum force the person can exert without moving either trunk. [ans: 290 N]
   (b) Calculate the force the larger trunk exerts on the smaller trunk. [ans: 120 N [right]]
   (c) Would either answer change if the person pushed in the opposite direction on the smaller trunk? Explain your reasoning.
2. A 4.0 kg block of wood sits on a table (**Figure 5**). A string is tied to the wood, running over a pulley and down to a hanging object. The greatest mass that can be hung from the string without moving the block of wood is 1.8 kg. Calculate the coefficient of static friction between the block of wood and the table. T/I [ans: 0.45]

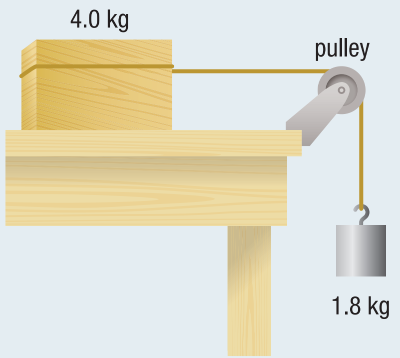


Figure 5

Figure 6 When starting a race, an athlete can achieve a greater forward acceleration by pushing backwards on starting blocks.

Static Friction and Motion

It is important to understand that static friction acts on an object when the object is at rest on a surface. This does not mean that static friction cannot be used to make an object move. For example, when you take a step forward, the bottom of your shoe is at rest with respect to the ground. The foot you are standing on is not actually moving forward with the rest of your body. The action force is you pushing backwards on the ground. According to Newton's third law, the ground pushes back on your foot with a force of equal magnitude but opposite in direction (the reaction force). This reaction force is the force of static friction, and it is this force that actually pushes you forward. Keep in mind that the force of static friction is usually greater in magnitude than the force of kinetic friction. This means that in a race from a standing start without starting blocks (**Figure 6**), you ideally want to push backwards with both feet, making sure your shoes do not slip. In the following Tutorial, we will examine the effect of static friction that causes an object to move.

Tutorial 2 Static Friction Can Cause Motion

The following Sample Problem will demonstrate how to calculate the maximum magnitude of acceleration when the coefficient of static friction is given.

Sample Problem 1

The coefficient of static friction between a person's shoe and the ground is 0.70. Determine the maximum magnitude of acceleration of the 62 kg person, if he starts running on a horizontal surface from rest.

Given: $\mu_S = 0.70$; $m = 62$ kg

Required: a

Analysis: Draw the FBD of the person (**Figure 7**).

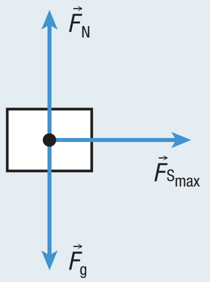


Figure 7

In this case, the maximum force of static friction is the net force since the normal force and gravity cancel. First we need to calculate $F_{S_{max}}$.

$F_{net} = F_{S_{max}}$; $F_{S_{max}} = \mu_S F_N$

Solution:
$$\begin{aligned} F_{S_{max}} &= \mu_S F_N \\ &= \mu_S mg \\ &= (0.70)(62\text{ kg})(9.8\text{ m/s}^2) \\ F_{S_{max}} &= 425.3\text{ N (two extra digits carried)} \end{aligned}$$

Now calculate the magnitude of the acceleration:

$$\begin{aligned} F_{net} &= F_{S_{max}} \\ ma &= 425.3\text{N} \\ (62\text{ kg})a &= 425.3\text{ N} \\ a &= 6.9\text{ m/s}^2 \end{aligned}$$

Statement: The maximum magnitude of the acceleration is 6.9 m/s^2.

Practice

1. Two people start running from rest. The first person has a mass of 59 kg and is wearing dress shoes with a coefficient of static friction of 0.52. The other person is wearing running shoes with a coefficient of static friction of 0.66. T/I
   (a) Calculate the maximum possible initial acceleration of the person wearing dress shoes. [ans: 5.1 m/s^2 [forward]]
   (b) Explain why we do not really need the mass of either person when finding the initial maximum possible acceleration.
   (c) Determine the ratio of the two accelerations and compare it to the ratio of the two coefficients of friction. [ans: 0.79; they are equal]
2. A skater with mass 58 kg is holding one end of a rope and standing at rest on ice. Assume no friction. Another person with mass 78 kg is standing just off the ice on level ground and is holding the other end of the rope. The person standing on the ground pulls on the rope to accelerate the skater forward. The coefficient of static friction between the ground and the off-ice person is 0.65. Calculate the maximum possible acceleration of the skater. T/I [ans: 8.6 m/s^2 [toward off-ice person]]

Kinetic Friction Problems

When an object is sliding across a surface, kinetic friction acts on the object in a direction opposite to the direction of motion. One common misconception is that kinetic friction always reduces the net force acting on an object. For example, in Sample Problem 1 in Tutorial 3, a person pushes against a sliding box to stop it from sliding any farther. In this situation, both the applied force and the kinetic friction act in the same direction (**Figure 8(a)**). So the magnitude of the net force is greater than if the person applies a force in the opposite direction (**Figure 8(b)**). In the following Tutorial, we will examine how kinetic friction causes a moving object to come to rest.

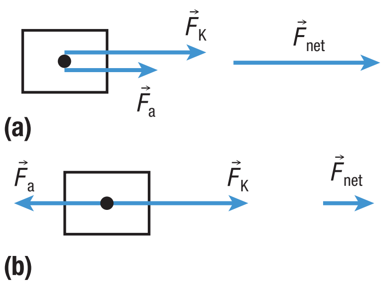


Figure 8 (a) Both the applied force and friction act in the same direction, producing a large net force. (b) The applied force and kinetic friction act in opposite directions, producing a smaller net force.

Tutorial 3 Kinetic Friction and Motion

The following Sample Problems will help to clarify the effect of kinetic friction on motion.

Sample Problem 1: Stopping a Sliding Box

A 250 kg box slides down a ramp and then across a level floor. The coefficient of kinetic friction along the floor is 0.20. A person sees the box moving at 1.0 m/s [left] and pushes on it with a horizontal force of 140 N [right].

(a) How far does the box travel before coming to rest?

(b) How will the results change if the box is moving right and the person still pushes right with the same force?

Solution

(a) **Given:** $\mu_K = 0.20$; $m = 250$ kg; $\vec{F}_a = 140$ N [right]

Required: Δd

Analysis: Draw the FBD of the box (**Figure 9**). Choose right and up as positive and left and down as negative. The kinetic friction must be opposite to the motion. We will first determine the kinetic friction. We can then use the FBD to find the acceleration. Once we have the acceleration, we can use one of the kinematics equations to calculate the distance.

$v_2^2 = v_1^2 + 2a\Delta d$; $F_K = \mu_K F_N$; $\vec{F}_{net} = \vec{F}_a + \vec{F}_K$

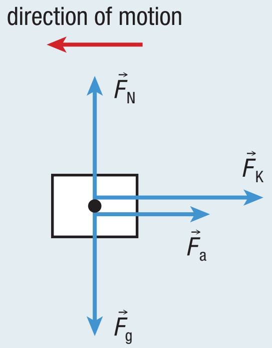


Figure 9

Solution:
$$F_K = \mu_K F_N$$
$$= \mu_K mg$$
$$= (0.20)(250\text{ kg})(9.8\text{ m/s}^2)$$
$$= 490\text{ N}$$

Now calculate the acceleration.

$$\vec{F}_{net} = \vec{F}_a + \vec{F}_K$$
$$ma = 140\text{ N} + 490\text{ N}$$
$$(250\text{ kg})a = 630\text{ N}$$
$$a = 2.52\text{ m/s}^2$$
$$\vec{a} = 2.52\text{ m/s}^2\text{ [right] (one extra digit carried)}$$

Next calculate the distance travelled. Since we do not know the time, we use the kinematics equation $v_2^2 = v_1^2 + 2a\Delta d$.

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2^2 - v_1^2 = 2a\Delta d$$
$$\Delta d = \frac{v_2^2 - v_1^2}{2a}$$
$$\Delta d = \frac{0^2 - (-1.0\text{ m/s})^2}{2(2.52\text{ m/s}^2)}$$
$$\Delta d = -0.20\text{ m}$$

Statement: The box moves 0.20 m before coming to rest.

(b) In this situation, friction is directed toward the left. Since the applied force acts in the opposite direction, the net force will be smaller. So the acceleration will now also be smaller, and the box will travel farther before coming to rest.

Sample Problem 2

Two sleds tied together are pulled across an icy surface with an applied force of 150 N [E] (**Figure 10**). The mass of sled 1 is 18.0 kg and the mass of sled 2 is 12.0 kg. The coefficient of kinetic friction for each sled is 0.20.

(a) Calculate the acceleration of the sleds.

(b) Determine the magnitude of the tension in the rope between the sleds.

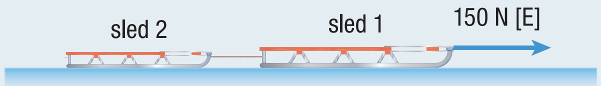


Figure 10

Solution

(a) Both sleds move together with the same acceleration so we can treat them as one large object. The total mass of the two sleds is $m_T = 18.0\text{ kg} + 12.0\text{ kg} = 30.0\text{ kg}$.

There is no need to consider the tension at this point because it is an internal force and does not contribute to the acceleration of the total mass.

Draw the FBD of the two sleds (**Figure 11**). Choose east and up as positive and west and down as negative.

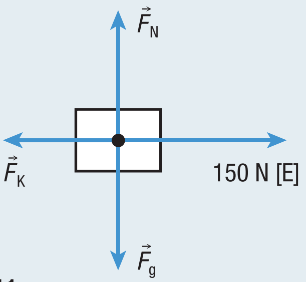


Figure 11

Given: $\mu_K = 0.20$; $m_T = 30.0$ kg; $\vec{F}_a = 150$ N [E]

Required: $\vec{a}$

Analysis: $\vec{F}_{net} = m\vec{a}$; $\vec{F}_{net} = \vec{F}_a + \vec{F}_K$

Solution: First we need to determine the force of kinetic friction.

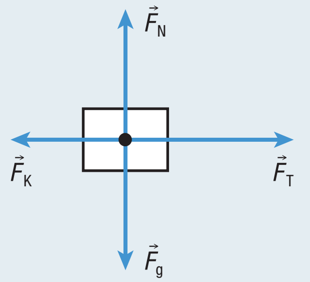
$$\begin{aligned} F_K &= \mu_K F_N \\ &= \mu_K m_T g \\ &= (0.20)(30.0\text{ kg})(9.8\text{ m/s}^2) \\ F_K &= 58.8\text{ N (one extra digit carried)} \end{aligned}$$

Now calculate the acceleration.

$$\begin{aligned} \vec{F}_{net} &= \vec{F}_a + \vec{F}_K \\ m_T a &= 150\text{ N} - 58.8\text{ N} \\ (30.0\text{ kg})a &= 91.2\text{ N} \\ a &= 3.04\text{ m/s}^2 \\ \vec{a} &= 3.0\text{ m/s}^2\text{ [E]} \end{aligned}$$

Statement: The acceleration of both sleds is 3.0 m/s^2 [E].

(b) To calculate the magnitude of the tension, we can use the FBD for sled 2.



$$\begin{aligned} \vec{F}_{net} &= \vec{F}_T + \vec{F}_{K_2} \\ m_2 a &= F_T - \mu_K m_2 g \\ (12.0\text{ kg})(3.04\text{ m/s}^2) &= F_T - (0.20)(12.0\text{ kg})(9.8\text{ m/s}^2) \\ F_T &= 60\text{ N} \end{aligned}$$

Statement: The magnitude of the tension in the rope is 60 N.

Practice

1. A 0.170 kg hockey puck is initially moving at 21.2 m/s [W] along the ice. The coefficient of kinetic friction for the puck and the ice is 0.005. T/I
   (a) What is the speed of the puck after travelling 58.5 m? [ans: 21.1 m/s]
   (b) After being played on for a while, the ice becomes rougher and the coefficient of kinetic friction increases to 0.047. How far will the puck travel if its initial and final speeds are the same as before? [ans: 6.24 m]
2. A snowmobile is used to pull two sleds across the ice. The mass of the snowmobile and the rider is 320 kg. The mass of the first sled behind the snowmobile is 120 kg and the mass of the second sled is 140 kg. The ground exerts a force of 1500 N [forward] on the snowmobile. The coefficient of kinetic friction for the sleds on ice is 0.15. Assume that no other frictional forces act on the snowmobile. Calculate the acceleration of the snowmobile and sleds. T/I C [ans: 1.9 m/s^2 [forward]]
3. A string is tied to a 3.2 kg object on a table and a 1.5 kg object hanging over a pulley (**Figure 12**). The coefficient of kinetic friction between the 3.2 kg object and the table is 0.30. T/I
   (a) Calculate the acceleration of each object. [ans: 1.1 m/s^2 [R]; 1.1 m/s^2 [down]]
   (b) Determine the magnitude of the tension in the string. [ans: 13 N]
   (c) How far will the objects move in 1.2 s if the initial velocity of the 3.2 kg object is 1.3 m/s [right]? [ans: 2.4 m]
4. An electric motor is used to pull a 125 kg box across a floor using a long cable. The tension in the cable is 350 N and the box accelerates at 1.2 m/s^2 [forward] for 5.0 s. The cable breaks and the box slows down and stops. T/I C
   (a) Calculate the coefficient of kinetic friction. [ans: 0.16]
   (b) How far does the box travel up to the moment the cable breaks? [ans: 15 m]
   (c) How far does the box travel from the moment the cable breaks until it stops? [ans: 11 m]

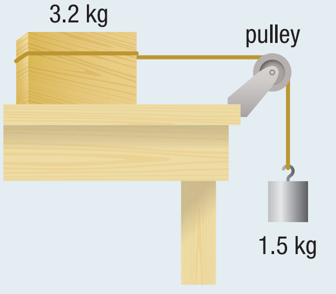


Figure 12

4.3 Summary

- Static friction exists between an object and a surface when the object is not sliding on the surface.
- Static friction can be used to move objects.
- Kinetic friction always acts in a direction that is opposite to the motion of the object.
- The kinematics equations from Unit 1 can be used to solve some problems involving friction and other forces.

Investigation 4.3.1

Predicting Motion with Friction **(p. 194)**

In this investigation, you will calculate the acceleration of a system of objects using Newton's laws. Then you will measure the actual acceleration.

4.3 Questions

1. During fundraising week at school, students decide to hold a competition of strength. Contestants pay $1 to try to move a heavy object. If they can move the object, they win $10. The object is a large box of books of mass 250 kg, with a coefficient of static friction of 0.55. One student has a mass of 64 kg and a coefficient of friction of static friction of 0.72 for his shoes on the floor. T/I
   (a) What is the maximum force of static friction acting on the student?
   (b) What is the maximum force of static friction acting on the box?
   (c) Is the competition fair? Explain your reasoning.
2. In an action movie, an actor is lying on an ice shelf and holding onto a rope. The rope hangs over a cliff to another actor who is hanging on in midair. The actor on the ice shelf has a mass of 55 kg and the actor hanging in midair has a mass of 78 kg. Neither actor can grab onto anything to help stop their motion, yet in the movie neither one is moving. T/I A
   (a) Calculate the minimum coefficient of static friction.
   (b) Is your answer to (a) reasonable considering that the surface is ice? Explain.
   (c) What could the director do to make the scene more realistic? Explain your reasoning.
3. In a physics experiment on static friction, two objects made of identical material are tied together with string. The first object has a mass of 5.0 kg and the second object has a mass of 3.0 kg. Students measure the maximum force of static friction as 31.4 N to move both objects across a horizontal surface. T/I
   (a) What is the coefficient of static friction?
   (b) What is the magnitude of the tension in the string if they pull on the first object?
   (c) A student pushes the 3.0 kg object with a force of 15.0 [down]. What are the magnitudes of the maximum force of static friction and the tension now?
   (d) Will your answers to (c) change if the student pushes down on the 5.0 kg object instead? Explain.
4. A student puts a 0.80 kg book against a vertical wall and pushes on the book toward the wall with a force of 26 N [R]. The book does not move. T/I
   (a) Calculate the minimum coefficient of static friction.
   (b) Describe two ways the student could make the book accelerate down without changing the applied force.
5. A string is tied to a 4.4 kg block and a 120 g hanging bucket (**Figure 13**). Students add 20 g washers one at a time to the bucket. The students are unaware that the coefficient of static friction for the block on the table is 0.42. T/I
   (a) What is the maximum force of static friction for the block?
   (b) How many washers can the students add to the bucket without moving the block?
   (c) Will this investigation yield an accurate result if they use it to find the coefficient of static friction? Explain your reasoning.
   (d) The coefficient of kinetic friction is 0.34. Calculate the acceleration of the block when the final washer is placed in the bucket and the objects start to move.

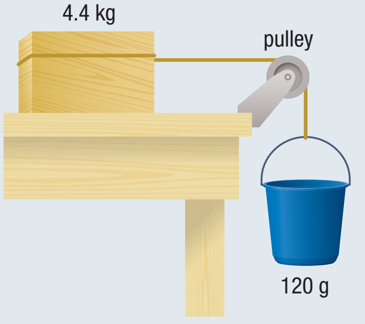


Figure 13

6. In a tug-of-war contest on a firm, horizontal sandy beach, team A has six players with an average mass of 65 kg and team B has five players with an average mass of 84 kg. Team B, pulling with a force of 3.2 kN, dislodges team A and then decreases its force to 2.9 kN to pull team A across the sand at a constant velocity. Determine team A's coefficient of
   (a) static friction (b) kinetic friction T/I
7. Two students push a 260 kg piano across the floor. Kathy pushes with 280 N [forward] while Matt pushes with 340 N [forward]. The piano accelerates at 0.30 m/s^2 [forward]. T/I
   (a) What is the coefficient of kinetic friction?
   (b) How long will it take the piano to stop moving after pushing it for 6.2 s from rest?
8. A 65 kg sprinter accelerates from rest into a strong wind that exerts a frictional force of 62 N. The ground applies a constant forward force of 250 N on the sprinter's feet. T/I
   (a) Calculate
      (i) the sprinter's acceleration
      (ii) the distance travelled in the first 2.0 s
      (iii) the minimum coefficient of friction between the sprinter's shoes and the track
   (b) Is the friction applied on the sprinter from the ground static or kinetic? Explain.
9. A homeowner accelerates a 15.0 kg lawnmower uniformly from rest to 1.2 m/s in 2.0 s. The coefficient of kinetic friction is 0.25. Calculate the horizontal applied force acting on the lawnmower. T/I
10. A 75 kg baseball player is running at 2.8 m/s [forward] when he slides into home plate for a distance of 3.8 m before coming to rest. Calculate the coefficient of kinetic friction. T/I