Figure 1 (a) Waxing skis helps to reduce friction. Different types of wax are used for different snow temperatures. (b) A typical flexible joint in the human body. Like oil in a car engine or wax on a ski, synovial fluid helps reduce friction between the layers of cartilage lining a flexible joint.


Figure 2 A force sensor is used to pull an object horizontally. Static friction keeps the object at rest.
static friction $\left(\vec{F}_{\mathrm{s}}\right)$ a force of friction that prevents the sliding of two surfaces relative to one another
kinetic friction $\left(\vec{F}_{\mathbf{K}}\right)$ the force exerted on a moving object by a surface opposite to the direction of motion of the object

## Friction

Sometimes friction is a problem and we try to minimize its effects. For example, by adding wax to skis (Figure $\mathbf{1}(\mathbf{a})$ ), a skier can reduce friction between the skis and the snow. The type of wax used depends on the snow temperature. In the flexible joints of the human body (Figure 1(b)), synovial fluid helps reduce friction between the moving bones. When there is too much friction, these joints can become very painful.


Other times, friction is necessary. When a car is starting to move or a person is walking or running, friction is needed. Without friction, cars could not slow down or turn corners. In this section, you will learn about the factors that affect the force of friction acting on objects and ways to control friction.

## The Difference between Static and Kinetic Friction

To help clarify the difference between static and kinetic friction, consider the following experiment. We use a force sensor to pull an object along a horizontal surface. At first the sensor exerts no force on the object, but we gradually pull with more force (Figure 2). Initially the object does not move due to static friction. Static friction $\left(\vec{F}_{\mathrm{s}}\right)$ is the force exerted on a stationary object by a surface that prevents the object from starting to move. In this case, the object remains at rest because the static friction is equal in magnitude and opposite in direction to the applied force.

Eventually the applied force becomes large enough to start moving the object. This means a maximum amount of static friction $\left(\vec{F}_{s_{\max }}\right)$ must be overcome to cause a stationary object to begin to move.

Once the object starts moving, kinetic friction, not static friction, acts on the object. Kinetic friction $\left(\vec{F}_{\mathbf{K}}\right)$ is the force exerted on a moving object by a surface, and acts opposite to the direction of motion of the object. As the applied force continues to increase, the object begins to accelerate. If the applied force decreases and the object starts moving at a constant velocity, the applied force must be equal in magnitude to the kinetic friction. Figure 3 shows the graph of the force of friction versus the applied force during this experiment. Notice that during the time that static friction acts on the object, $F_{\mathrm{S}}$ equals $F_{\mathrm{a}}$. So, during that time the graph is a straight line starting from the origin with a slope of 1 .


Figure 3 A graph of the magnitude of friction versus the magnitude of the applied force. Once the object starts to move, the friction drops suddenly.

Different types of kinetic friction apply to different situations. If an object is scraping or sliding across a surface, we call it sliding friction. If the object is round and it rolls across a surface, it is called rolling friction. Fluid friction or air resistance (also known as drag) are involved when a boat goes through water or a plane moves through the air. In this chapter, we will deal with sliding friction for most of the problems. However, we will use the generic term "kinetic friction" under most circumstances.

## Coefficients of Friction

Many factors affect the force of friction acting on an object. The magnitude of friction acting on an object may depend on the mass of the object, the type of material the object is made of, and the type of surface the object is in contact with. When dealing with air resistance, the speed of the object and the shape of the object also have an effect. In this section, we will deal only with friction acting on an object in contact with horizontal surfaces. The only applied forces acting on the object will be horizontal.

Many experiments have been done to measure the magnitude of kinetic friction and the maximum force of static friction. Kinetic friction and static friction depend on the types of materials in the two surfaces that are in contact. However, for a particular pair of surfaces, the ratio of the frictional force (kinetic or static) to the normal force is a constant. This has led to the definition of a quantity called the coefficient of friction. The coefficient of friction is the ratio of the magnitude of the force of friction, $F_{f}$, acting on an object to the magnitude of the normal force, $F_{\mathrm{N}}$, acting on the object (Figure 4). The Greek letter $\mu$ is used to represent the coefficient of friction. We define the coefficient of friction mathematically as

$$
\mu=\frac{F_{f}}{F_{\mathrm{N}}}
$$

where $F_{\mathrm{f}}$ is the magnitude of the force of friction acting on an object in newtons, $F_{\mathrm{N}}$ is the magnitude of the normal force acting on the object in newtons, and $\mu$ is the coefficient of friction. The coefficient of friction is just a number, with no direction or units.

To calculate the coefficient of static friction for an object on a surface, we need to determine the maximum force of static friction in that particular situation. For almost all situations, the force required to start an object moving is greater than the kinetic resistance acting on the object when it is moving. This means that $F_{\mathrm{S}_{\max }}$ is usually slightly greater than $F_{\mathrm{K}}$. Since there are two types of friction (static and kinetic), there are two coefficients of friction. One is the coefficient of static friction, which represents the ratio of $F_{\mathrm{S}_{\max }}$ to the normal force. The coefficient of static friction is represented by the symbol $\mu_{\mathrm{S}}$. The other is the coefficient of kinetic friction, which represents the ratio of $F_{\mathrm{K}}$ to the normal force. The coefficient of kinetic friction is represented by the symbol $\mu_{\mathrm{K}}$. Since the maximum force of static friction is usually greater than the kinetic friction, the coefficient of static friction is usually greater than the coefficient of kinetic friction. The corresponding equations are

$$
\mu_{\mathrm{S}}=\frac{F_{\mathrm{S}_{\mathrm{max}}}}{F_{\mathrm{N}}} \quad \text { and } \quad \mu_{\mathrm{K}}=\frac{F_{\mathrm{K}}}{F_{\mathrm{N}}}
$$

The coefficient of friction between an object and a surface depends only on the type of materials. These coefficients of friction can only be determined experimentally. The results of such experiments are often inconsistent, even when performed carefully. Results can be affected by the condition of the surface, including the cleanliness of the surface, whether the surface is wet or dry, and the roughness of the surface. This means one scientist may obtain a different coefficient of friction than another scientist even when neither one has made any mistakes. So, in many cases, a range of values is given for coefficients of friction. Table 1 on the next page gives several coefficients of kinetic and static friction for some common materials.
coefficient of friction $(\boldsymbol{\mu})$ the ratio of the force of friction to the normal force


Figure 4 An FBD of an object pulled by a horizontal force. The magnitudes of the force of friction and the normal force are used to find the coefficient of friction.
coefficient of static friction ( $\mu_{\mathrm{s}}$ ) the ratio of the maximum force of static friction to the normal force
coefficient of kinetic friction ( $\mu_{\mathrm{K}}$ ) the ratio of kinetic friction to the normal force

Factors That Affect Friction (p. 192) In this investigation, you will explore some of the factors that affect $F_{\mathrm{S}_{\text {max }}}$ and $F_{\mathrm{K}}$ acting on an object. You will design your own procedure and take an average of several measurements before forming a conclusion about a specific factor.

Table 1 Approximate Coefficients of Kinetic and Static Friction

Coefficients of Friction (p. 193) In this investigation, you will explore how the coefficient of static friction compares to the coefficient of kinetic friction. You will also determine some of the factors that affect these values.

| Material | $\mu_{\mathbf{S}}$ | $\mu_{\mathbf{K}}$ |
| :--- | :---: | :---: |
| rubber on concrete (dry) |  | $0.6-0.85$ |
| rubber on concrete (wet) |  | $0.45-0.75$ |
| rubber on asphalt (dry) | 0.78 | $0.5-0.80$ |
| rubber on asphalt (wet) | $0.05-0.11$ | $0.25-0.75$ |
| steel on steel (dry) | 0.61 | 0.42 |
| steel on steel (greasy) | 0.1 | $0.029-0.12$ |
| leather on oak | 0.1 | 0.52 |
| ice on ice |  | 0.03 |
| steel on ice | 0.22 | 0.01 |
| rubber on ice | 0.14 | 0.005 |
| wood on dry snow | 0.04 | 0.18 |
| wood on wet snow |  | 0.10 |
| Teflon on Teflon |  | 0.04 |
| near-frictionless carbon |  | 0.001 |
| synovial joints in humans |  | 0.003 |

In the following Tutorial, we will apply the equations for the coefficient of friction to determine the force of friction acting on a wooden block.

## Tutorial 1 Determining the Forces of Friction

The following Sample Problem will help to clarify how the normal force and the force of friction are related to the coefficients of friction.

## Sample Problem 1

A 3.0 kg block of wood sits on a horizontal wooden floor. The largest horizontal force that can be applied to the block before it will start moving is 14.7 N . Once the block starts moving, it only takes 8.8 N to keep it moving at a constant velocity.
(a) Calculate the coefficient of static friction for the block and the floor.
(b) Determine the force of friction acting on the block if a horizontal force of $6.8 \mathrm{~N}[\mathrm{E}]$ acts on the block.
(c) Calculate the maximum magnitude of static friction acting on the block if a 2.1 kg object is placed on top of it.
(d) Determine the coefficient of kinetic friction.

## Solution

(a) Given: $m=3.0 \mathrm{~kg} ; F_{\mathrm{a}}=14.7 \mathrm{~N}$

Required: $\mu_{\mathrm{S}}$
Analysis: Since the block is not moving, the net force on the block is zero. This means that $\vec{F}_{\mathrm{S}_{\text {max }}}$ on the block must also be 14.7 N acting in the opposite direction to keep the block at rest. To calculate the coefficient of static friction, we can use the equation $\mu_{\mathrm{S}}=\frac{F_{\mathrm{S}_{\text {max }}}}{F_{\mathrm{N}}}$. Also, the force of gravity must be equal to the normal force.

Solution: $\mu_{\mathrm{S}}=\frac{F_{\mathrm{s}_{\text {max }}}}{F_{\mathrm{N}}}=\frac{F_{\mathrm{s}_{\text {max }}}}{m g}$

$$
\begin{aligned}
& =\frac{14.7 \mathrm{~N}}{(3.0 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
\mu_{\mathrm{S}} & =0.50
\end{aligned}
$$

Notice that $1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}=1 \mathrm{~N}$, so the units cancel.
Statement: The coefficient of static friction is 0.50 .
(b) The horizontal applied force ( $6.8 \mathrm{~N}[\mathrm{E}]$ ) is less than the maximum force of static friction ( 14.7 N ), so the block will remain at rest. This means the net force on the block is still zero and the applied force must be cancelled by the static friction. The static friction on the block must be 6.8 N [W].
(c) Given: $m_{\mathrm{T}}=3.0 \mathrm{~kg}+2.1 \mathrm{~kg}=5.1 \mathrm{~kg}$

Required: $F_{\mathrm{S}_{\text {max }}}$
Analysis: Since the same materials are still in contact, $\mu_{\mathrm{S}}=0.50$. To calculate the maximum force of static friction, we can use the equation $\mu_{\mathrm{S}}=\frac{F_{\mathrm{S}_{\text {max }}}}{F_{\mathrm{N}}}$. Since the total mass is now 5.1 kg , we can calculate the normal force using the equation $F_{\mathrm{N}}=m_{\mathrm{T}} g$.

Solution: $\mu_{\mathrm{s}}=\frac{F_{\mathrm{s}_{\text {max }}}}{F_{\mathrm{N}}}$

$$
\begin{aligned}
F_{\mathrm{S}_{\max }} & =\mu_{\mathrm{S}} F_{\mathrm{N}}=\mu_{\mathrm{S}} m_{\mathrm{T}} g \\
& =(0.50)(5.1 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) \\
F_{\mathrm{S}_{\max }} & =25 \mathrm{~N}
\end{aligned}
$$

Statement: The maximum magnitude of static friction acting on the block is 25 N .
(d) Given: $m=3.0 \mathrm{~kg} ; F_{\mathrm{a}}=8.8 \mathrm{~N}$

Required: $\mu_{\mathrm{K}}$
Analysis: The block is moving with a constant velocity, so the net force on the block is zero. This means the kinetic friction, $F_{\mathrm{k}}$, on the block must also be 8.8 N acting in the opposite direction. To find the coefficient of kinetic friction we can use the equation $\mu_{\mathrm{K}}=\frac{F_{\mathrm{K}}}{F_{\mathrm{N}}}$.

## Solution:

$$
\begin{aligned}
\mu_{\mathrm{K}} & =\frac{F_{\mathrm{K}}}{F_{\mathrm{N}}}=\frac{F_{\mathrm{K}}}{m g} \\
& =\frac{8.8 \mathrm{~N}}{(3.0 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
\mu_{\mathrm{K}} & =0.30
\end{aligned}
$$

Statement: The coefficient of kinetic friction is 0.30 .

## Practice

1. Determine the coefficient of friction for each situation. Tir
(a) It takes a horizontal force of 62 N to get a 22 kg box to just start moving across the floor. [ans: 0.29 ]
(b) It only takes 58 N of horizontal force to move the same box at a constant velocity. [ans: 0.27]
2. A 75 kg hockey player glides across the ice on his skates with steel blades. What is the magnitude of the force of friction acting on the skater? Use Table 1 to help you. Troll [ans: 7.4 N ]
3. A 1300 kg car skids across an asphalt road. Use Table 1 to calculate the magnitude of the force of friction acting on the car due to the road if the road is
(a) dry [ans: 6000 N to 10000 N ]
(b) wet [ans: 3200 N to 9600 N ]
(c) covered with ice TTII [ans: 64 N$]$

### 4.2 Summary

- Static friction $\left(\vec{F}_{\mathrm{S}}\right)$ is the force of friction that prevents two surfaces in contact from sliding relative to one another.
- The maximum force of static friction $\left(\vec{F}_{S_{\max }}\right)$ is the amount of force that must be overcome to start a stationary object moving.
- Kinetic friction $\left(\vec{F}_{\mathrm{K}}\right)$ is the force exerted on a moving object by a surface. Kinetic friction acts in the opposite direction to the motion of the object.
- The coefficient of friction is the ratio of the force of static or kinetic friction to the normal force. The equations are $\mu_{\mathrm{S}}=\frac{F_{\mathrm{S}_{\text {max }}}}{F_{\mathrm{N}}}$ and $\mu_{\mathrm{K}}=\frac{F_{\mathrm{K}}}{F_{\mathrm{N}}}$.
- Coefficients of friction are determined experimentally and depend only on the types of materials in contact.

1. For each situation, determine if friction is helpful, makes the action more difficult, or both. Explain your reasoning.
(a) turning a doorknob
(b) pushing a heavy box across a rough surface
(c) gliding across smooth ice to demonstrate uniform motion
(d) tying a knot
2. A typical bicycle braking system involves a lever that you pull on the handlebars and a brake pad near the rim of the wheel (Figure 5). Describe how the braking system works using the concepts of normal force and friction.


Figure 5
3. A 1.4 kg block on a horizontal surface is pulled by a horizontal applied force. It takes 5.5 N to start the block moving and 4.1 N to keep it moving at a constant velocity.
(a) Calculate the coefficients of friction.
(b) Which changes below will affect the coefficients of friction? Explain.
(i) turning the block onto another side
(ii) changing the surface
(iii) putting an object on top of the block
(c) What effect will each situation have on the static and kinetic friction acting on each object? Explain.
(i) putting an object on the block
(ii) applying an upward force on the block
(iii) putting slippery grease on the surface
4. Examine the coefficients of friction in Table 1 on page 170 to answer the following.
(a) Roads in Canada are typically made out of asphalt or concrete. Is one material significantly safer than the other? Explain your reasoning.
(b) Explain why drivers should reduce speed on wet roads.
(c) Why do we salt roads in the winter, especially when there is freezing rain?
5. You are dragging a 110 kg trunk across a floor at a constant velocity with a horizontal force of 380 N .
(a) Calculate the coefficient of kinetic friction.
(b) A friend decides to help by pulling on the trunk with a force of 150 N [up]. Will this help? Calculate the force required to pull the trunk at a constant velocity to help you decide.
(c) Instead of pulling on the trunk, your 55 kg friend just sits on it. What force is required keep the trunk moving at a constant velocity?
6. A 26 kg desk is at rest on the floor. The coefficient of static friction is 0.25 . One person pulls on the desk with a force of $52 \mathrm{~N}[\mathrm{E}]$ and another pulls with a force of 110 N [W]. Will the desk move? Explain your reasoning.
7. A 12000 kg bin is sitting in a parking lot. The coefficient of static friction for the bin is 0.50 and the coefficient of kinetic friction is 0.40 . A truck pushes on the bin and it starts to move. Determine the minimum force exerted by the truck to
(a) start the bin moving
(b) keep the bin moving at a constant velocity
8. A gradually increasing horizontal force is applied to an object initially at rest on a horizontal surface. Draw a graph of the force of friction versus the applied force in each situation.
(a) the coefficient of static friction is slightly greater than the coefficient of kinetic friction
(b) the coefficient of static friction is equal to the coefficient of kinetic friction
9. A doorstop keeps a door open when it is wedged underneath the door. Use the concepts from this section to explain how a doorstop works.
10. Describe how to determine each quantity experimentally. TIN $c^{-}$
(a) the coefficient of static friction
(b) the coefficient of kinetic friction
11. Explain why the manufacturer of a running shoe might be more concerned about having a high coefficient of friction than the manufacturer of a dress shoe. $\qquad$

