3.3 Newton's Second Law of Motion

Newton's first law of motion applies to situations when the net force on an object is zero. When the net force is zero, an object that is at rest will remain at rest and an object that is in motion will continue to move in a straight line at a constant velocity. What do these cases have in common? In both cases, the acceleration is zero.

The Variables Involved in Newton's Second Law

One way to experience Newton's second law of motion is in a moving car. A light car with a powerful engine that can exert a large force will experience a large acceleration (**Figure 1(a)**). A massive truck experiences small accelerations even if the force is large (**Figure 1(b)**). A small economy car experiences smaller accelerations because the force its engine exerts is smaller (**Figure 1(c)**). The acceleration is affected by changing either the mass or the net force. To understand this relationship among acceleration, net force, and mass, we need to consider these variables more carefully.

Figure 1 (a) A race car speeds up quickly because it has a small mass and experiences a large net force. (b) A truck accelerates more slowly. Even though a large net force acts on it, the mass is large. (c) A small economy car also accelerates more slowly. It has a small mass and a small applied force.

We will first examine the relationship between the net force and the acceleration. Imagine pushing a large sled with a few small boxes on top across a nearly frictionless icy surface. If you push as hard as you can, the sled starts from rest and attains a high velocity in little time (**Figure 2(a)**). If you push with less force on the sled, the velocity will increase more gradually (**Figure 2(b)**). These two scenarios imply that acceleration increases as the net force increases. Now we will examine the effect of mass on acceleration. If you remove the boxes, the lighter sled is easier to speed up from rest (**Figure 2(c)**). If you put more boxes on the sled, the heavier sled is harder to speed up at the same rate (**Figure 2(d)**). In other words, acceleration decreases as mass increases.

WEB LINK

To learn more about how forces are applied to sports,

GO TO NELSON SCIENCE

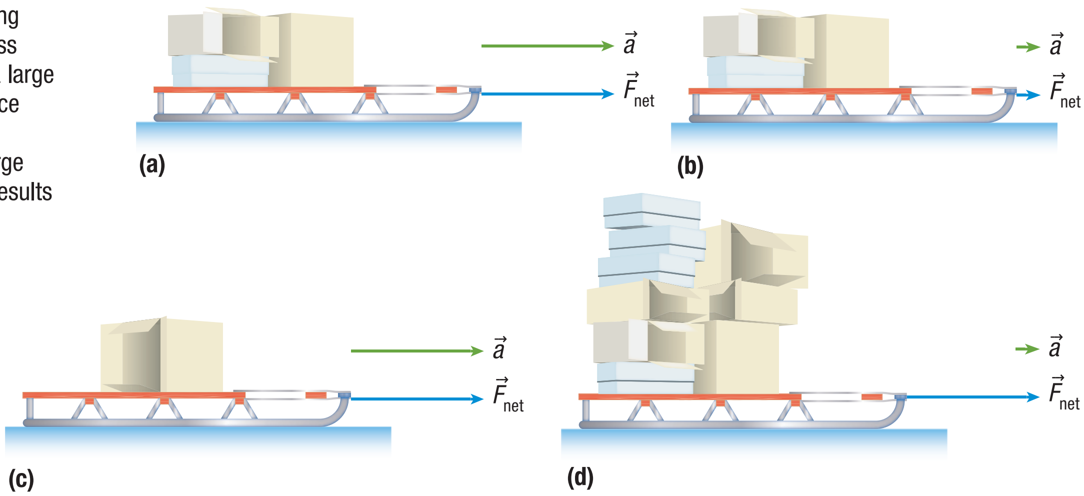


Figure 2 The relationship among acceleration, net force, and mass (a) A large net force results in a large acceleration. (b) A small net force results in a small acceleration. (c) A small mass results in a large acceleration. (d) A large mass results in a small acceleration.

Newton's **second law of motion** is summarized as follows:

second law of motion an object will accelerate in the direction of the net force; the magnitude of the acceleration is directly proportional to the magnitude of the net force and inversely proportional to the object's mass

Second Law of Motion

If the net external force on an object is not zero, the object will accelerate in the direction of this net force. The magnitude of the acceleration is directly proportional to the magnitude of the net force and inversely proportional to the mass of the object.

Representing Newton's Second Law Mathematically

From Newton's second law, we can derive an equation that connects acceleration, net force, and mass.

$$\vec{a} \propto \vec{F}_{net} \text{ when } m \text{ is constant} \quad \text{and} \quad \vec{a} \propto \frac{1}{m} \text{ when } \vec{F}_{net} \text{ is constant}$$

Combining the two proportionalities, we get

$$\vec{a} \propto \frac{\vec{F}_{net}}{m}$$

We can create an equation relating all three variables if we insert a proportionality constant k:

$$\vec{a} = k\frac{\vec{F}_{net}}{m}$$

If we define the newton as the net force required to accelerate 1 kg at 1 m/s^2 or 1 N = 1 kg·m/s^2, then $k = 1$. We can write the equation as

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

Solving for the net force, we have

$$\vec{F}_{net} = m\vec{a}$$

Consider a situation where the mass is kept constant but the net force gradually increases. From the equation $\vec{F}_{net} = m\vec{a}$, if mass is kept constant and the net force on the object increases, the acceleration also increases. If you graph net force versus acceleration, the relationship is linear (**Figure 3**).

Consider the slope of the line:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta \vec{F}_{net}}{\Delta \vec{a}}$$

The line must pass through the origin. From Newton's first law, when the acceleration is zero, the net force must also be zero. If we use the origin as the initial point on the graph, the above slope equation simplifies to

$$\text{slope} = \frac{\vec{F}_{net_2} - \vec{F}_{net_1}}{\vec{a}_2 - \vec{a}_1} = \frac{\vec{F}_{net_2} - 0}{\vec{a}_2 - 0} = \frac{\vec{F}_{net_2}}{\vec{a}_2}$$

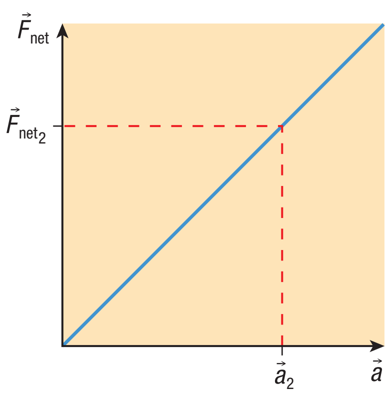


Figure 3 The graph of net force versus acceleration is linear, and the slope represents the mass. The line must pass through the origin, since when the acceleration is zero the net force must also be zero.

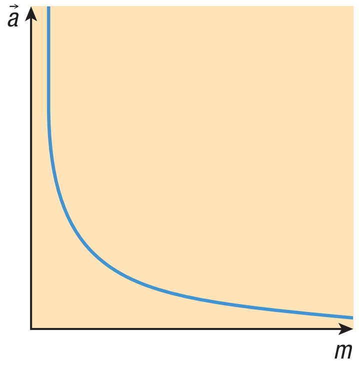


Figure 4 The graph of acceleration versus mass is a reciprocal function.

Physically, what does this slope represent? Using Newton's second law, we have

$$\vec{F}_{net} = m\vec{a}$$

$$m = \frac{\vec{F}_{net}}{\vec{a}}$$

The expressions for slope and mass can be equated, so we can conclude that the slope of the graph of net force versus acceleration is the mass of the object.

Now consider a situation where the net force is kept constant but the mass gradually increases. From the equation $\vec{a} = \frac{\vec{F}_{net}}{m}$, if the net force is kept constant and the mass of an object increases, the acceleration decreases. If you graph acceleration versus mass, the graph shows a reciprocal relationship (**Figure 4**).

Tutorial 1 Using Newton's Second Law

Newton's second law can be used to solve a variety of problems involving many different situations. In the following Sample Problems, we will demonstrate how to apply Newton's second law.

Sample Problem 1: Determining Acceleration

A net force of 36 N [forward] is applied to a volleyball of mass 0.24 kg. Determine the acceleration of the volleyball.

Given: $\vec{F}_{net} = 36$ N [forward]; $m = 0.24$ kg

Required: $\vec{a}$

Analysis: $\vec{F}_{net} = m\vec{a}$. Choose forward as positive.

Solution: $\vec{F}_{net} = m\vec{a}$

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

$$a = \frac{+36\ \text{N}}{0.24\ \text{kg}}$$

$$a = +150\ \text{m/s}^2$$

$$\vec{a} = 150\ \text{m/s}^2\ [\text{forward}]$$

Statement: The acceleration of the volleyball is 150 m/s^2 [forward].

Sample Problem 2: Calculating Net Force

A 64 kg runner starts walking at 3.0 m/s [E] and begins to speed up for 6.0 s, reaching a final velocity of 12.0 m/s [E]. Calculate the net force acting on the runner.

Given: $m = 64$ kg; $\vec{v}_1 = 3.0$ m/s [E]; $\vec{v}_2 = 12.0$ m/s [E]; $\Delta t = 6.0$ s

Required: $\vec{F}_{net}$

Analysis: $\vec{F}_{net} = m\vec{a}$, but first we have to calculate the acceleration using the kinematics equation $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$.

Choose east as positive.

Solution: $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

$$a = \frac{+12.0\ \text{m/s} - (+3.0\ \text{m/s})}{6.0\ \text{s}}$$

$$a = +1.5\ \text{m/s}^2$$

$$\vec{a} = 1.5\ \text{m/s}^2\ [\text{E}]$$

Now we can calculate the net force.

$$\vec{F}_{net} = m\vec{a}$$

$$F_{net} = (64\ \text{kg})(+1.5\ \text{m/s}^2)$$

$$F_{net} = +96\ \text{N}$$

$$\vec{F}_{net} = 96\ \text{N}\ [\text{E}]$$

Statement: The net force on the runner is 96 N [E].

Sample Problem 3: Calculating Net Force and Acceleration Using an FBD

A 9100 kg jet moving slowly on the ground fires its engines, resulting in a force of 22 000 N [E] on the jet. The force of friction on the jet is 3800 N [W].

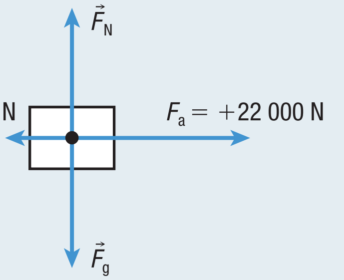
(a) Draw the FBD for the jet.

(b) Calculate the net force acting on the jet.

(c) Calculate the acceleration of the jet.

Solution

(a) Choose up and east as positive. So down and west are negative.



(b) The normal force and gravity will cancel when the jet is on horizontal ground. The net force is east since the applied force is greater than the force of friction. To find the net force, we add all the horizontal forces.

$$\vec{F}_{net} = \vec{F}_a + \vec{F}_f$$
$$F_{net} = +22\,000\text{ N} + (-3800)\text{ N}$$
$$F_{net} = +18\,200\text{ N}$$
$$\vec{F}_{net} = 18\,200\text{ N [E]}$$

The net force on the jet is 18 200 N [E].

(c) **Given:** $\vec{F}_{net} = 18\,200$ N [E]; $m = 9100$ kg

Required: $\vec{a}$

Analysis: $\vec{F}_{net} = m\vec{a}$

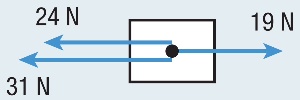
Solution: $\vec{F}_{net} = m\vec{a}$

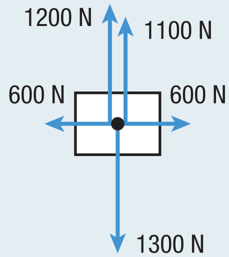
$$\vec{a} = \frac{\vec{F}_{net}}{m}$$
$$a = \frac{+18\,200\text{ N}}{9100\text{ kg}}$$
$$a = +2.0\text{ m/s}^2$$
$$\vec{a} = 2.0\text{ m/s}^2\text{ [E]}$$

Statement: The acceleration of the jet is 2.0 m/s^2 [E].

Practice

1. A net force of 126 N [S] is applied to a 70 kg sprinter. Determine the acceleration of the sprinter. T/I [ans: 1.8 m/s^2 [S]]
2. A car accelerates at 1.20 m/s^2 [forward]. The net force on the car is 1560 N [forward]. What is the mass of the car? T/I [ans: 1300 kg]
3. A cyclist starts to pedal vigorously, increasing her velocity from 6.0 m/s [E] to 14.0 m/s [E] in 6.0 s. The total mass of the cyclist and the bicycle is 58 kg. Find the net force acting on the cyclist and bicycle. T/I [ans: 77 N [E]]
4. During a road test, a driver brakes a 1420 kg car moving at 64.8 km/h [W]. The car slows down and comes to a stop after moving 729 m [W]. T/I
   (a) Calculate the net force acting on the car. [ans: 316 N [E]]
   (b) What is the force of friction acting on the car while it is slowing down? Explain your reasoning. [ans: 316 N [E]]
5. For each FBD shown below, determine the net force applied to the object and its acceleration. T/I
   (a) $m = 8.0$ kg [ans: 36 N [left], 4.5 m/s^2 [left]] (b) $m = 125$ kg [ans: 1000 N [up], 8 m/s^2 [up]]





6. In a two-person bobsled competition, athlete 1 pushes forward on the sled with 310 N and athlete 2 pushes forward with 354 N. A force of friction of 40 N [backwards] is acting on the bobsled. The mass of the bobsled is 390 kg. Calculate the acceleration of the bobsled. T/I [ans: 1.6 m/s^2 [forward]]

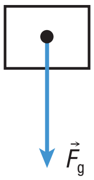


Figure 5 This FBD shows an object in free fall, with no air resistance acting on it. The net force on the object is the force of gravity. The acceleration of the object is equal to the acceleration due to gravity.

Newton's Second Law and Gravity

In Section 3.1, you learned that you can calculate the force of gravity by multiplying the mass by the acceleration due to gravity. We can now justify the equation $\vec{F}_g = m\vec{g}$ using Newton's second law.

Consider an object in free fall with no air resistance acting on it (**Figure 5**). Free fall is motion of an object toward Earth with no other forces acting on it. The only force acting on the object is the force of gravity. In this situation, the force of gravity is equal to the net force. We also know that the object accelerates down at $\vec{a} = 9.8 \text{ m/s}^2$ [down]. Combining these two facts, we have

$$\vec{F}_g = \vec{F}_{net}$$
$$= m\vec{a}$$

Then, because $\vec{a} = \vec{g}$,

$$\vec{F}_g = m\vec{g}$$

You will use the equation for the force of gravity to help you complete **Investigation 3.3.1**. To prepare for the investigation, read the following Tutorial and complete the Practice questions.

Tutorial 2 Acceleration of Falling Objects

Recall that strings can only pull on objects when they exert tension. A pulley is a small wheel that changes the direction of the tension in a rope or string without changing its magnitude. We will assume throughout the text that all pulleys are frictionless and light.

Sample Problem 1

In an investigation, students place a 0.80 kg cart on a table. They tie one end of a light string to the front of the cart, run the string over a pulley, and then tie the other end to a 0.20 kg hanging object (**Figure 6**). Assume that no friction acts on either object.

(a) Determine the magnitude of the acceleration of the cart and the hanging object.

(b) Calculate the magnitude of the tension.

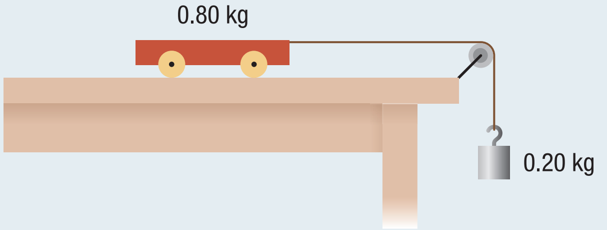
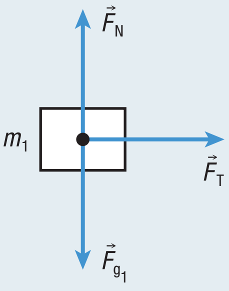


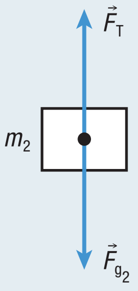
Figure 6

(a) **Step 1.** Draw the FBD for each object.

FBD of cart



FBD of hanging object



Step 2. Identify which directions are positive. Determine the net force acting on each object.

The cart is on the table, so the normal force and gravity cancel each other. The cart will accelerate to the right, so choose right as positive. The equation for the net force acting on the cart is

$F_{net} = F_T$

Because $F_{net} = m_1 a$,

$m_1 a = F_T$ (Equation 1)

The hanging object will accelerate down, so choose down as positive. The equation for the net force acting on the hanging object is

$F_{net} = F_{g_2} - F_T$

Because $F_{net} = m_2 a$ and $F_{g_2} = m_2 g$,

$m_2 a = m_2 g - F_T$ (Equation 2)

Step 3. Add the equations to solve for the acceleration.

The tension acting on the cart and on the hanging object is the same. Adding equations 1 and 2 will cancel the tension.

$$m_1 a + m_2 a = F_T + m_2 g - F_T$$
$$m_1 a + m_2 a = m_2 g$$
$$(m_1 + m_2)a = m_2 g$$
$$(0.80 \text{ kg} + 0.20 \text{ kg})a = (0.20 \text{ kg})(9.8 \text{ m/s}^2)$$
$$a = 1.96 \text{ m/s}^2$$
$$a = 2.0 \text{ m/s}^2$$

The magnitude of the acceleration of the cart is 2.0 m/s^2.

(b) To calculate the tension, substitute the acceleration into equation 1.

$m_1 a = F_T$

$F_T = (0.80 \text{ kg})(1.96 \text{ m/s}^2)$

$F_T = 0.16 \text{ N}$

The magnitude of the tension in the string is 0.16 N.

Practice

1. Calculate the acceleration of the cart in **Figure 7**, given the following assumptions. T/I
   (a) No friction is acting on the cart. [ans: 3.3 m/s^2 [right]]
   (b) A force of friction of 0.50 N acts on the cart opposite to the motion. [ans: 3.0 m/s^2 [right]]

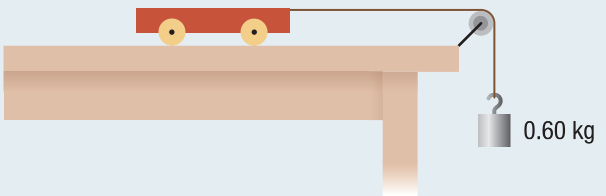


Figure 7

2. In an experiment, objects are placed on top of a cart as shown in **Figure 8**. T/I

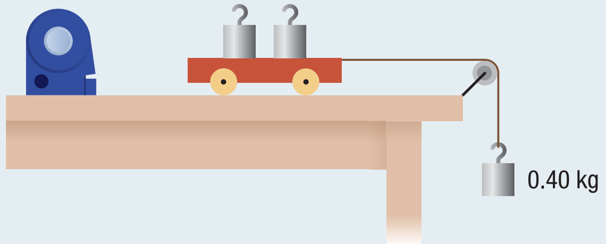


Figure 8

   (a) Calculate the acceleration of the cart. Assume that no friction acts on the cart. [ans: 1.6 m/s^2 [right]]
   (b) What will happen to the acceleration of the cart if the mass of the objects on top of the cart increases? Explain your reasoning.
   (c) What will happen to the acceleration of the cart if an object is taken from the top of the cart and tied to the hanging object? Explain your reasoning.

3.3 Summary

- According to Newton's second law, when a non-zero net force acts on an object, the object will accelerate in the direction of the net force. The acceleration is directly proportional to the net force and inversely proportional to the mass. The equation representing Newton's second law is $\vec{F}_{net} = m\vec{a}$.
- If the net force on an object increases and the mass is constant, then the acceleration increases. If the net force on an object is constant and the mass increases, then the acceleration decreases.
- When solving problems where more than one force acts on a single object, make sure you draw an FBD. You can add all the forces to calculate the net force. You can also use the equation $\vec{F}_{net} = m\vec{a}$.
- For a falling object, Newton's second law can be used to justify that $\vec{F}_g = m\vec{g}$.

Investigation 3.3.1

Investigating Newton's Second Law (p. 150)

Now that you have learned about Newton's second law, you can perform Investigation 3.3.1. You will explore how changing the net force acting on a system affects the acceleration of the system. You will also explore how changing the mass of the system affects the acceleration of the system.

3.3 Questions

1. Calculate the net force in each situation. T/I
   (a) A 72 kg rugby player accelerates at 1.6 m/s^2 [forward].
   (b) A 2.3 kg model rocket accelerates at 12 m/s^2 [up].
2. Calculate the acceleration in each situation. T/I
   (a) A cannon exerts a force of 2.4×10^4 N [E] on a 5.0 kg shell.
   (b) A hockey stick hits a 160 g puck forward with a force of 24 N.
3. Determine the mass of the object in each situation. T/I
   (a) A driver brakes and the car accelerates at 1.2 m/s^2 [backwards]. The net force on the car is 1400 N [backwards].
   (b) A woman throws a shot put with a net force of 33 N [forward] with an acceleration of 6.0 m/s^2 [forward].
4. A 54 kg skier starts from rest at the top of a snow-covered hill, reaching a velocity of 12 m/s in 5.0 s. Calculate the net force acting on the skier. T/I
5. A dynamics cart is pulled from rest by a net force of 1.2 N [forward]. The cart moves 6.6 m, reaching a velocity of 3.2 m/s [forward]. Determine the mass of the cart. T/I
6. During a parachute jump, a 58 kg person opens the parachute and the total drag force acting on the person is 720 N [up]. T/I
   (a) Calculate the net force acting on the person.
   (b) Determine the acceleration of the person.
7. A net force of magnitude 36 N gives an object of mass m_1 an acceleration of 6.0 m/s^2. The same net force gives m_1 and another object of mass m_2 fastened together an acceleration of 2.0 m/s^2. What acceleration will m_2 experience if the same net force acts on it alone? T/I
8. A 1300 kg car accelerates at 1.6 m/s^2 [E]. A frictional force of 3800 N [W] is acting on the car. T/I C
   (a) Draw the FBD of the car.
   (b) Determine the applied force acting on the car.
9. A long, heavy, metal chain is held at rest on a table with part of the chain hanging over the edge (**Figure 9**). The chain is released and it starts to accelerate. K/U

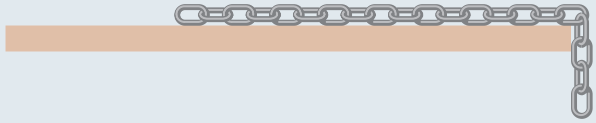


Figure 9

   (a) In which direction will the chain accelerate? What causes the acceleration? Explain your reasoning.
   (b) What will happen to the acceleration of the chain as more chain moves over the edge of the table? Explain your reasoning.
10. Three students push horizontally on a large 80 kg crate sitting on the floor. Two of them push to the left on the crate, each with a force of 170 N. The third pushes to the right on the crate with a force of 150 N. Assume that no friction acts on the crate. T/I
   (a) What is the acceleration of the crate?
   (b) What will happen to the net force and acceleration if a fourth student jumps on top of the crate? Explain your reasoning.
11. A string can hold up 12 kg without breaking. You tie the string to a 30 kg object sitting on ice and use it to pull the object horizontally for 22 m. Calculate the minimum possible time to complete the task. T/I
12. Examine the data in **Table 1**. T/I C
   (a) Copy and complete the table.
   (b) Graph $\vec{F}_a$ versus $\vec{a}$ and draw the line of best fit. What does the y-intercept represent? Explain.
   (c) Graph $\vec{F}_{net}$ versus $\vec{a}$ and draw the line of best fit. Calculate the slope of the line. What does the slope represent? Explain.

Table 1

Mass (kg)	Friction (N) [W]	Applied force (N) [E]	Net force (N) [E]	Acceleration (m/s^2) [E]
4.0	9.0	9.0		
4.0	9.0	13.0		
4.0	9.0		8.4	
4.0	9.0			3.5