2.2 Motion in Two Dimensions—An Algebraic Approach

In Section 2.1 you learned how to solve motion problems in two dimensions by using vector scale diagrams. This method has some limitations. First, the method is not very precise. Second, scale diagrams can become very cumbersome when you need to add more than two vectors. The map in **Figure 1** shows several different legs of a trip. Each leg represents an individual displacement. Without the map and the scale, adding these displacements by scale diagram would be quite challenging. In many situations an algebraic approach is a better way to add vectors. To use this method, we will revisit some of the mathematics from Grade 10—the Pythagorean theorem and trigonometry.

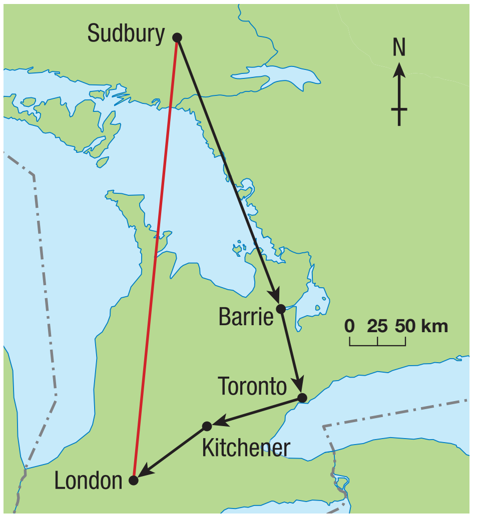

Figure 1 How would you determine the total displacement from Sudbury to London in this problem?

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Adding Displacements in Two Dimensions

GPS technology, which surveyors use to precisely locate positions, depends on computing the resultant vector when displacement vectors are added together. Tutorials 1 to 3 introduce the algebraic method of adding vectors. If two displacements are perpendicular to each other, we can add them relatively easily. Adding non-perpendicular displacements algebraically involves breaking them down into perpendicular parts.

Tutorial 1 Adding Two Perpendicular Displacements Using Algebra

In this Tutorial, we will use an algebraic approach to add two perpendicular displacements.

Sample Problem 1: Adding Two Perpendicular Vectors

A jogger runs 200.0 m [E], turns at an intersection, and continues for an additional displacement of 300.0 m [N]. What is the jogger's total displacement?

Given: $\Delta\vec{d}_1 = 200.0\text{ m [E]}; \Delta\vec{d}_2 = 300.0\text{ m [N]}$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 200.0\text{ m [E]} + 300.0\text{ m [N]}$$

At this point, we can draw a diagram showing these two vectors joined tip to tail (**Figure 2**). Notice that this is only a sketch—*it is not a scale diagram*.

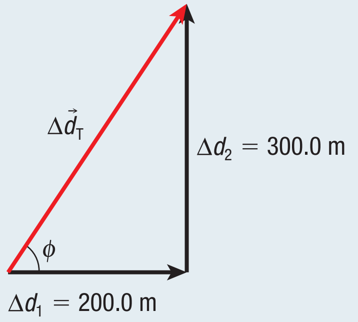


Figure 2 The two given displacement vectors, joined tip to tail

Notice that vector arrows are not shown above Δd_1 and Δd_2 in Figure 2. This is because only the magnitudes of these two displacements are shown in the diagram. The directions are represented by the direction of each vector as drawn in the diagram. The Greek symbol ϕ (phi) is used to represent the angle of the resultant vector with respect to the *x*-axis.

Now we need to determine the magnitude and direction of $\Delta\vec{d}_T$. Since this is a right triangle, we can use the Pythagorean theorem to solve for the magnitude of this vector:

$$\Delta d_T^2 = \Delta d_1^2 + \Delta d_2^2$$

$$\Delta d_T = \sqrt{\Delta d_1^2 + \Delta d_2^2}$$

$$= \sqrt{(200.0\text{ m})^2 + (300.0\text{ m})^2}$$

$$\Delta d_T = 360.6\text{ m}$$

To determine the direction, we need to calculate the magnitude of the angle using the tangent ratio:

$$\tan\phi = \frac{\Delta d_2}{\Delta d_1}$$

$$\tan\phi = \frac{300.0\text{ m}}{200.0\text{ m}}$$

$$\tan\phi = 1.5$$

$$\phi = \tan^{-1}(1.5)$$

$$\phi = 56°$$

Stating the magnitude and direction gives $\Delta\vec{d}_T = 360.6$ m [E 56° N].

Statement: The jogger's total displacement is 360.6 m [E 56° N].

Practice

1. Add the following perpendicular displacement vectors algebraically: T/I

   $\Delta\vec{d}_1 = 27\text{ m [W]}, \Delta\vec{d}_2 = 35\text{ m [S]}$ [ans: 44 m [W 52° S]]

2. What is the vector sum of the displacements $\Delta\vec{d}_1 = 13.2\text{ m [S]}$ and $\Delta\vec{d}_2 = 17.8\text{ m [E]}$? T/I [ans: 22.2 m [E 37° S]]

The graphic organizer in **Figure 3** summarizes the method for adding two perpendicular vectors algebraically.

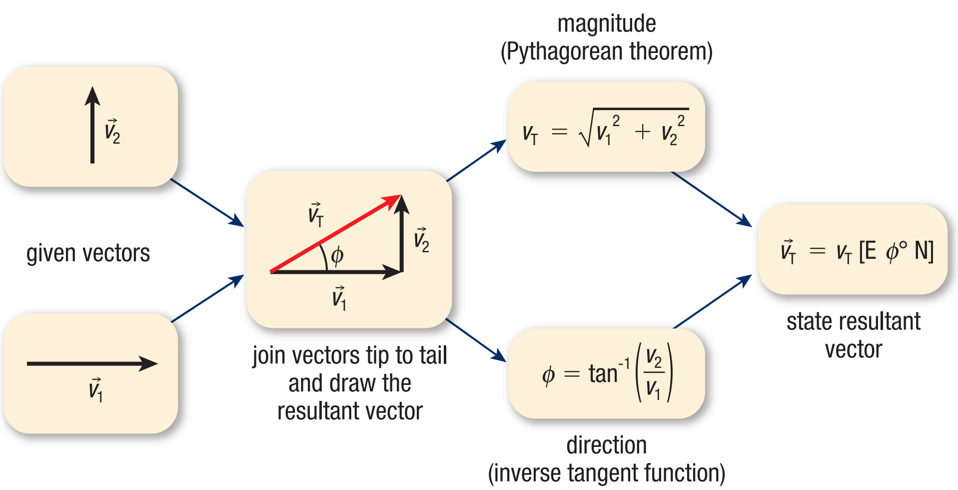


Figure 3 Adding two perpendicular vectors algebraically

Sample Problem 1 in Tutorial 1 is a very important example. Our goal from this point on, when solving more complex problems, is to turn every vector addition problem into a problem similar to Sample Problem 1. That is, we will turn every problem into a situation where we are adding two perpendicular vectors. The method for doing this is shown in **Figure 4**.

Figure 4 builds on the methods introduced in Figure 3. In Figure 4, each given vector is broken down into x (horizontal) and y (vertical) **component vectors**. All of the x-component vectors in this example have the same direction, and we can add them together (just as we did in Chapter 1) to get an overall x-vector. Similarly, all of the y-component vectors are added together to get an overall y-vector. These two overall vectors are perpendicular to each other and can be added together as we did in Sample Problem 1 of Tutorial 1. This will be our procedure in Tutorial 2 and Tutorial 3, but how do we take a vector and break it down into two perpendicular components? We will use trigonometry.

component vectors vectors which when added together give the original vector from which they were derived; one component is parallel to the x-axis and the other is parallel to the y-axis

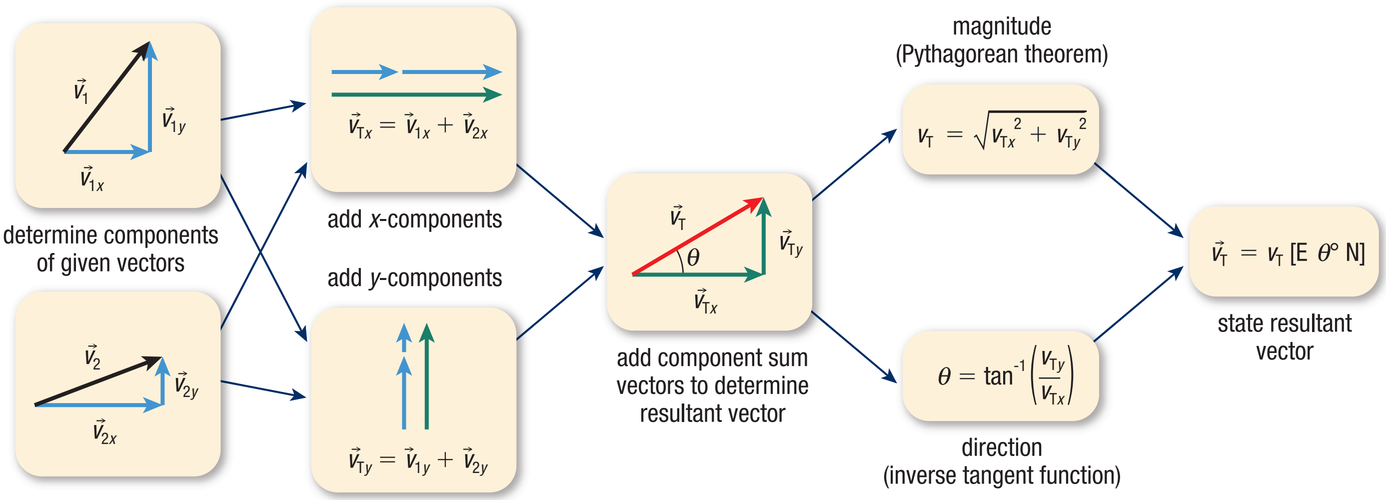


Figure 4 Adding two non-perpendicular vectors algebraically

Tutorial 2 Breaking Down Vectors into Two Perpendicular Components

In this Tutorial, we will go through the process required to break down a vector into perpendicular components.

Sample Problem 1: Breaking Down Vectors into Component Vectors

Break the displacement vector 30.0 m [E 25° N] down into two perpendicular component vectors.

Given: $\Delta\vec{d}_T = 30.0$ m [E 25° N]

Required: $\Delta\vec{d}_x; \Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

In this case, we need to work backwards from $\Delta\vec{d}_T$ to determine the horizontal and vertical component vectors.

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

$$\Delta\vec{d}_T = 30.0 \text{ m [E 25° N]} = \Delta\vec{d}_x + \Delta\vec{d}_y$$

In **Figure 5** the two component vectors $\Delta\vec{d}_x$ and $\Delta\vec{d}_y$ are drawn and joined tip to tail. These two vectors are joined such that the x-component vector ($\Delta\vec{d}_x$) is along the x-axis. As we will see, this is a good habit to develop because it will help to minimize your chances of making an error when solving problems involving vector components.

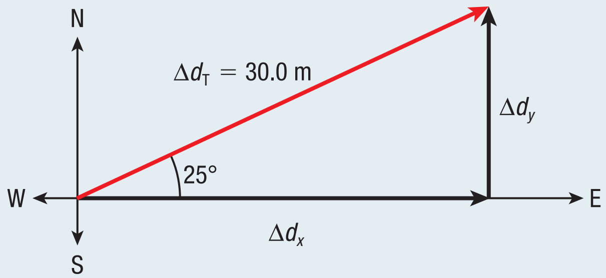


Figure 5 A displacement vector in the northeast quadrant of a Cartesian coordinate system

The direction of each component vector is clear from the diagram. $\Delta\vec{d}_x$ points due east, and $\Delta\vec{d}_y$ points due north. To determine the magnitude of each vector, you need to recall some trigonometry from Grade 10 math, specifically the sine and cosine functions.

Recall that

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

In this case,

$$\sin\theta = \frac{\Delta d_y}{\Delta d_T}$$

Solving for Δd_y, we get

$$\Delta d_y = \Delta d_T \sin\theta$$
$$= (30.0\text{ m})(\sin 25°)$$
$$\Delta d_y = 12.68\text{ m (two extra digits carried)}$$

The *y*-component of this vector has magnitude 12.68 m (and direction [N]).

To determine the *x*-component of the given vector, we will use a similar method. In this case, however, we will use the cosine function. Recall that

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

In this case,

$$\cos\theta = \frac{\Delta d_x}{\Delta d_T}$$

Solving for Δd_x, we get

$$\Delta d_x = \Delta d_T \cos\theta$$
$$= (30.0\text{ m})(\cos 25°)$$
$$\Delta d_x = 27.19\text{ m (two extra digits carried)}$$

The *x*-component of this vector has magnitude 27.19 m (and direction [E]).

Adding the two component vectors such that $\Delta\vec{d}_x$ = 27.19 m [E] and $\Delta\vec{d}_y$ = 12.68 m [N], we get a resultant vector equal to 30.0 m [E 25° N], which was the original given vector.

Statement: The vector 30.0 m [E 25° N] has a horizontal or *x*-component of 27.2 m [E] and a vertical or *y*-component of 12.7 m [N].

Practice

1. Determine the magnitude and direction of the *x*-component and *y*-component vectors for the displacement vector $\Delta\vec{d}_T = 15\text{ m [W 35° N]}$. T/I [ans: $\Delta\vec{d}_x = 12$ m [W], $\Delta\vec{d}_y = 8.6$ m [N]]
2. Add the two component vectors from Sample Problem 1 algebraically to verify that they equal the given vector. T/I

Tutorial 3 Adding Displacement Vectors by Components

In each of the following Sample Problems we will add a pair of two-dimensional vectors together by the component method. Notice that when you draw the initial diagram in a component-method solution, you should draw all vectors, starting at the origin, on a Cartesian coordinate system. In this diagram the vectors will *not* be joined tip to tail, as we did in Section 2.1. Also notice that all *x*-components will be drawn along the *x*-axis. This will ensure that all *x*-components contain a cosine term and all *y*-components contain a sine term, minimizing your chance of making an error.

Sample Problem 1: One Vector Has a Direction Due North, South, East, or West

A cat walks 20.0 m [W] and then turns and walks a further 10.0 m [S 40° E]. What is the cat's total displacement?

Given: $\Delta\vec{d}_1 = 20.0\text{ m [W]}$; $\Delta\vec{d}_2 = 10.0\text{ m [S 40° E]}$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 20.0\text{ m [W]} + 10.0\text{ m [S 40° E]}$$

Figure 6 on the next page shows the given displacement vectors drawn on a Cartesian coordinate system, both starting at the origin. Notice that vector $\Delta\vec{d}_1$ has only one component, specifically an *x*-component. Since this vector points due west, it does not have a y-component. On the other hand, vector $\Delta\vec{d}_2$ has two components. Notice that $\Delta\vec{d}_2$ is broken down so that the x-component lies on the x-axis.

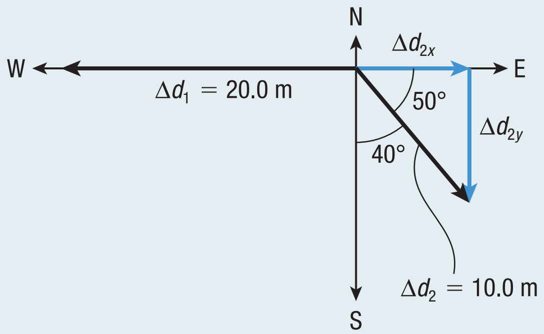


Figure 6 Start both vectors at the origin. (Diagram is not to scale.)

We begin by finding the vector sum of the x-components. In this case the vector sum of the x-components is represented by $\Delta\vec{d}_{Tx}$.

$$\Delta\vec{d}_{Tx} = \Delta\vec{d}_1 + \Delta\vec{d}_{2x}$$

From the diagram it is clear that $\Delta\vec{d}_1$ points due west, whereas $\Delta\vec{d}_{2x}$ points due east. So,

$$\begin{aligned}\Delta\vec{d}_{Tx} &= \Delta d_1\,[\mathrm{W}] + \Delta d_{2x}\,[\mathrm{E}]\\ &= \Delta d_1\,[\mathrm{W}] + \Delta d_2 \cos 50°\,[\mathrm{E}]\\ &= 20.0\text{ m }[\mathrm{W}] + (10.0\text{ m})(\cos 50°)[\mathrm{E}]\\ \Delta\vec{d}_{Tx} &= 20.0\text{ m }[\mathrm{W}] + 6.428\text{ m }[\mathrm{E}]\end{aligned}$$

We can change the direction of the smaller vector by placing a negative sign in front of the magnitude. This gives both vectors the same direction.

$$\Delta\vec{d}_{Tx} = 20.0\text{ m }[\mathrm{W}] - 6.428\text{ m }[\mathrm{W}]$$

$$\Delta\vec{d}_{Tx} = 13.572\text{ m }[\mathrm{W}]\text{ (two extra digits carried)}$$

The overall vector sum of all x-components is 13.572 m [W]. Notice that two extra significant digits have been carried here. This is to minimize rounding error. Always carry one or two extra significant digits when a calculated value will be used in subsequent calculations. You should round down to the correct number of significant digits once you have calculated the final answer to the question.

We can solve for the vector sum of all y-components in a very similar way. In this case, there is only one y-component. This is the vector $\Delta\vec{d}_{2y}$.

$$\begin{aligned}\Delta\vec{d}_{Ty} &= 0 + \Delta\vec{d}_{2y}\\ &= \Delta d_2 \sin 50°\,[\mathrm{S}]\\ &= (10.0\text{ m})(\sin 50°)[\mathrm{S}]\\ \Delta\vec{d}_{Ty} &= 7.660\text{ m }[\mathrm{S}]\end{aligned}$$

Notice that we have converted this problem into one involving two perpendicular vectors, namely $\Delta\vec{d}_{Tx}$ and $\Delta\vec{d}_{Ty}$. We can now join these two vectors tip to tail, as shown in **Figure 7**. We will use the Pythagorean theorem to determine the magnitude and the tangent function to determine the direction of the total displacement.

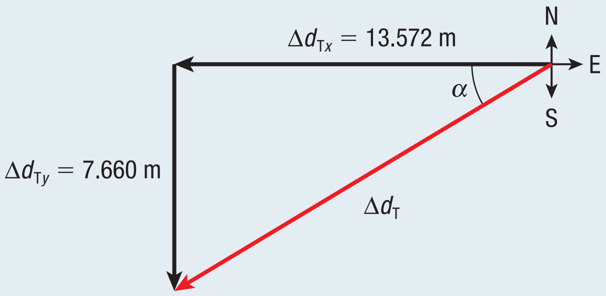


Figure 7 Use the total components to determine the total displacement.

$$\begin{aligned}\Delta d_T^2 &= (\Delta d_{Tx})^2 + (\Delta d_{Ty})^2\\ \Delta d_T &= \sqrt{(\Delta d_{Tx})^2 + (\Delta d_{Ty})^2}\\ &= \sqrt{(13.572\text{ m})^2 + (7.660\text{ m})^2}\\ \Delta d_T &= 15.6\text{ m}\end{aligned}$$

To determine the angle α (alpha) that $\Delta\vec{d}_T$ makes with the x-axis, we can use the tangent function.

$$\begin{aligned}\tan\alpha &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}}\\ \tan\alpha &= \frac{7.660\text{ m}}{13.572\text{ m}}\\ \alpha &= 29°\end{aligned}$$

Statement: The total displacement of the cat is 15.6 m [W 29° S].

Sample Problem 2: Neither Vector Has a Direction Due North, South, East, or West

A hockey puck travels a displacement of 4.2 m [S 38° W]. It is then struck by a hockey player's stick and undergoes a displacement of 2.7 m [E 25° N]. What is the puck's total displacement?

Given: $\Delta\vec{d}_1 = 4.2\text{ m }[\mathrm{S}\ 38°\ \mathrm{W}]$; $\Delta\vec{d}_2 = 2.7\text{ m }[\mathrm{E}\ 25°\ \mathrm{N}]$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 4.2\text{ m }[\mathrm{S}\ 38°\ \mathrm{W}] + 2.7\text{ m }[\mathrm{E}\ 25°\ \mathrm{N}]$$

Figure 8 shows the two displacements to be added. Both displacements start at the origin—they are not drawn tip to tail (as we did when adding vectors by scale diagram).

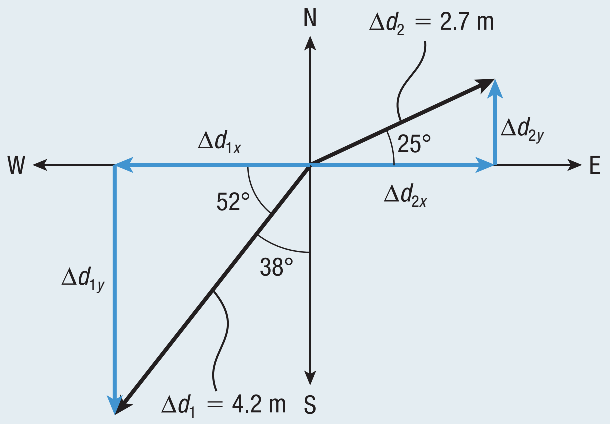


Figure 8 Draw both given vectors starting at the origin.

We begin by determining the total *x*-component and *y*-component of $\Delta\vec{d}_T$. For the *x*-component,

$$\begin{aligned}
\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x} \\
&= \Delta d_1 \cos 52° \,[W] + \Delta d_2 \cos 25° \,[E] \\
&= (4.2\text{ m})(\cos 52°)\,[W] + (2.7\text{ m})(\cos 25°)\,[E] \\
&= 2.59\text{ m }[W] + 2.45\text{ m }[E] \\
&= 2.59\text{ m }[W] - 2.45\text{ m }[W] \\
\vec{d}_{Tx} &= 0.14\text{ m }[W]
\end{aligned}$$

For the *y*-component,

$$\begin{aligned}
\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y} \\
&= \Delta d_1 \sin 52° \,[W] + \Delta d_2 \sin 25° \,[E] \\
&= (4.2\text{ m})(\sin 52°)\,[S] + (2.7\text{ m})(\sin 25°)\,[N] \\
&= 3.31\text{ m }[S] + 1.14\text{ m }[N] \\
&= 3.31\text{ m }[S] - 1.14\text{ m }[S] \\
\Delta\vec{d}_{Ty} &= 2.17\text{ m }[S]
\end{aligned}$$

We now use the total *x*-component and *y*-component to determine the magnitude of $\Delta\vec{d}_T$ (**Figure 9**).

$$\begin{aligned}
\Delta d_T^2 &= d_{Tx}^2 + d_{Ty}^2 \\
\Delta d_T &= \sqrt{d_{Tx}^2 + d_{Ty}^2} \\
&= \sqrt{(0.14\text{ m})^2 + (2.17\text{ m})^2} \\
\Delta d_T &= 2.2\text{ m}
\end{aligned}$$

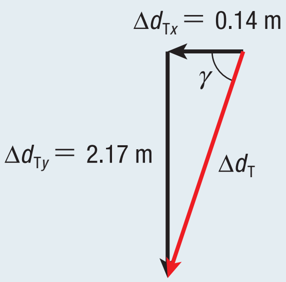


Figure 9 Determining the total displacement

We use the tangent function to determine the angle γ (gamma) that $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\begin{aligned}
\tan\gamma &= \frac{d_{Ty}}{d_{Tx}} \\
\tan\gamma &= \frac{2.17\text{ m}}{0.14\text{ m}} \\
\gamma &= 86°
\end{aligned}$$

Statement: The puck's total displacement is 2.2 m [W 86° S].

Practice

1. An ant travels 2.78 cm [W] and then turns and travels 6.25 cm [S 40° E]. What is the ant's total displacement? T/I [ans: 4.94 cm [E 76° S]]
2. A paper airplane flies 2.64 m [W 26° N] and then is caught by the wind, which causes it to travel 3.21 m [S 12° E]. What is the paper airplane's total displacement? T/I [ans: 2.62 m [W 49° S]]

Adding Velocities in Two Dimensions

What does it mean, physically, to add two velocity vectors? Imagine driving a boat across a still lake. If you know your velocity and the width of the lake, you can easily determine how long it will take you to reach the other side. If instead you are crossing a river (**Figure 10**), you have two velocities to consider: the velocity due to your boat's engine and the velocity at which the river is flowing. Does the flow of the river change your crossing time? How far downstream will you be carried as you drive across? To answer questions like this, we will use the skills of algebraic vector addition that you have already learned in Tutorial 4.

Figure 10 Two motions are involved in crossing a river—yours and the river's.

Tutorial 4 River Crossing Problems

River crossing problems are a type of two-dimensional motion problem that involve perpendicular velocity vectors. The "river crossing problem" is often first introduced in terms of boats crossing rivers, but it may also involve aircraft flying through the air, and so on. These types of problems always involve two perpendicular motions that are independent of each other.

CASE 1: DETERMINING THE TIME IT TAKES FOR A RIVER CROSSING WITHOUT TAKING CURRENT INTO ACCOUNT

Sample Problem 1

Consider the river shown in **Figure 11**. A physics student has forgotten her lunch and needs to return home to retrieve it. To do so she hops into her motorboat and steers straight across the river at a constant velocity of 12 km/h [N]. If the river is 0.30 km across and has no current, how long will it take her to cross the river?

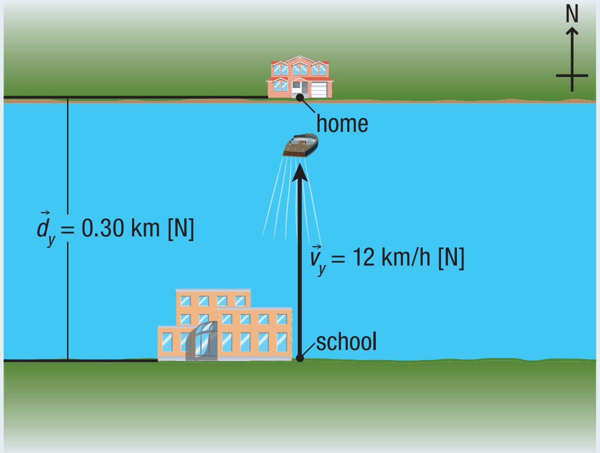


Figure 11 River crossing with no current

Let $\vec{v}_y$ represent the velocity caused by the boat's motor.

Given: $\Delta\vec{d}_y = 0.30 \text{ km [N]}$, $\vec{v}_y = 12 \text{ km/h [N]}$

Required: Δt

Analysis: Since the boat is travelling at a constant velocity, we can solve this problem using the defining equation for average velocity. Since displacement and average velocity are in the same direction, we can simply divide one magnitude by the other when we rearrange this equation.

$$\vec{v}_y = \frac{\Delta\vec{d}_y}{\Delta t}$$

$$\vec{v}_y(\Delta t) = \Delta\vec{d}_y$$

$$\Delta t = \frac{\Delta d_y}{v_y}$$

Solution: $\Delta t = \dfrac{\Delta d_y}{v_y}$

$$= \frac{0.30 \cancel{\text{km}}}{12 \dfrac{\cancel{\text{km}}}{\text{h}}}$$

$$\Delta t = 0.025 \text{ h}$$

Statement: It will take the student 0.025 h or 1.5 min to cross the river.

CASE 2: DETERMINING THE DISTANCE TRAVELLED DOWNSTREAM DUE TO A RIVER CURRENT

Sample Problem 2

The river crossing problem in Sample Problem 1 is not very realistic because a river usually has a current. So we introduce a current here, in Sample Problem 2, and see how the current affects the trip across the river. **Figure 12** shows the same boat from Sample Problem 1 going at the same velocity. Let us now assume that the gate of a reservoir has been opened upstream, and the river water now flows with a velocity of 24 km/h [E]. This current has a significant effect on the motion of the boat. The boat is now pushed due north by its motor and due east by the river's current. This causes the boat to experience two velocities at the same time, one due north and another due east. In Figure 12 these two velocity vectors are joined tip to tail to give a resultant velocity represented by $\vec{v}_T$. Notice that even though the student is steering the boat due north, the boat does not arrive at her home. Instead it lands some distance farther downstream.

(a) How long does it now take the boat to cross the river?

(b) How far downstream does the boat land?

(c) What is the boat's resultant velocity $\vec{v}_T$?

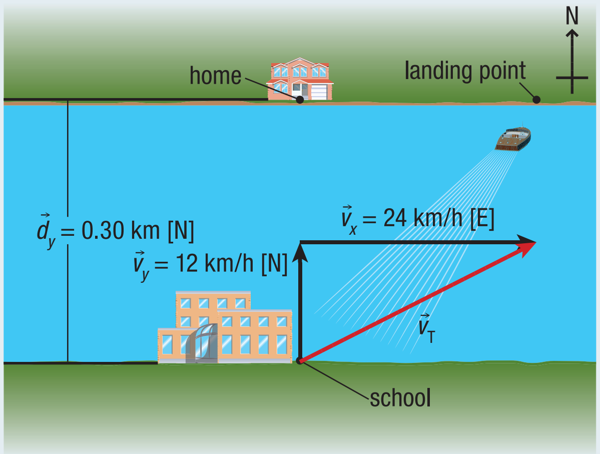


Figure 12 River crossing including a current

(a) **Given:** $\Delta\vec{d}_y = 0.30\text{ km [N]}$; $\vec{v}_y = 12\text{ km/h [N]}$;

$\vec{v}_x = 24\text{ km/h [E]}$

Required: Δt

First we will consider the motion of the boat moving across the river.

Analysis: $\vec{v}_y = \dfrac{\Delta\vec{d}_y}{\Delta t}$

$$\Delta t = \frac{\Delta d_y}{v_y}$$

Solution: $\Delta t = \dfrac{\Delta d_y}{v_y}$

$$= \frac{0.30\ \cancel{\text{km}}}{12\ \dfrac{\cancel{\text{km}}}{\text{h}}}$$

$$\Delta t = 0.025\text{ h}$$

Statement: The time it takes the boat to cross the river is still 0.025 h.

Surprisingly, the time it now takes to cross the river is precisely the same time as it took when there was no current in the river. It will still take 0.025 h. How can this be? Notice that in this Sample Problem, the current pushes the boat in a direction that is perpendicular to the direction in which the boat's motor pushed the boat. Thus, the current's velocity is perpendicular to the velocity vector caused by the boat's motor. Since the current velocity $\vec{v}_x$ does not have a component in a direction parallel to that of the velocity caused by the boat's motor, $\vec{v}_y$, it will not cause the velocity in the direction of $\vec{v}_y$ to increase or decrease.

Consider the hypothetical situation shown in **Figure 13**. This figure is almost identical to Figure 12, except that in this example the velocity due to the current $\vec{v}_c$ is not perpendicular to the velocity due to the boat's motor $\vec{v}_y$. In this case, the velocity due to the current can be broken down into perpendicular components, one moving downstream, $\vec{v}_{cx}$, and one moving across the river, $\vec{v}_{cy}$. $\vec{v}_{cy}$ is in the opposite direction to $\vec{v}_y$. This causes a reduction in the velocity of the boat across the river. If $\vec{v}_{cy}$ is greater than $\vec{v}_y$, the boat can never leave the dock. It is continuously washed back onto the shore near the school. In reality, this never happens. In real situations, the current flows parallel to the riverbanks.

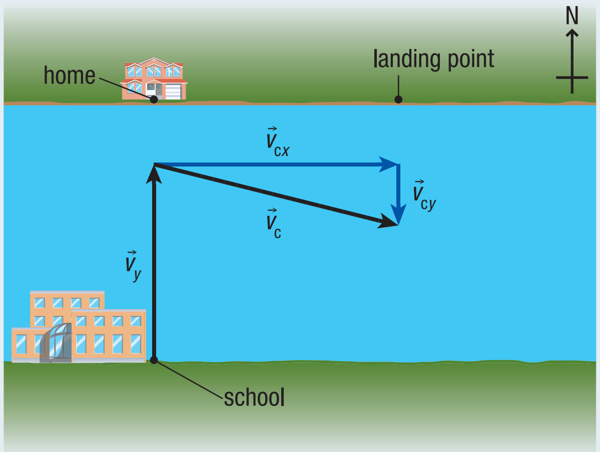


Figure 13 River crossing with an unrealistic current

(b) We now return to our Sample Problem described in Figure 12, where vector $\vec{v}_y$ is perpendicular to the vector $\vec{v}_x$. To calculate how far downstream the boat travels before it reaches the shore, we only need to consider velocities that are moving the boat downstream. In our example the only velocity moving the boat downstream is the current velocity, $\vec{v}_x$. So we can consider this to be a standard uniform velocity problem. Notice that the time that the motion downstream takes is identical to the time we calculated for the boat to move across the river in (a). In river crossing problems the time it takes to cross the river is the same as the time to move down the river. This is because the boat eventually reaches the opposite shore. We can presume that when the boat reaches the opposite shore, both the boat's motion across the river and its motion downstream will stop. As a result, in this example it takes the boat the same amount of time to cross the river as it does to travel downstream.

Given: $\vec{v}_x = 24$ km/h [E]; $\Delta t = 0.025$ h

Required: $\Delta\vec{d}_x$

Analysis: $\vec{v}_x = \dfrac{\Delta\vec{d}_x}{\Delta t}$

$$\Delta\vec{d}_x = \vec{v}_x(\Delta t)$$

Solution:
$$\begin{aligned}\Delta\vec{d}_x &= \vec{v}_x(\Delta t)\\ &= (24 \text{ km/h [E]})(0.025 \text{ h})\\ \Delta\vec{d}_x &= 0.60 \text{ km [E]}\end{aligned}$$

Statement: As a result of the current, the boat will land 0.60 km east, or downstream, of the student's home.

(c) The velocity labelled as $\vec{v}_T$ in Figure 12 is often referred to as the resultant velocity. The resultant velocity is the vector sum of the velocity due to the boat's motor and the velocity due to the current. This is the velocity that you would see the boat travelling at if you were an observer standing at the school. We can solve for $\vec{v}_T$ using the Pythagorean theorem and the tangent ratio.

Given: $\vec{v}_y = 12$ km/h [N]; $\vec{v}_x = 24$ km/h [E]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_y + \vec{v}_x$

Solution:
$$\begin{aligned}\vec{v}_T &= \vec{v}_y + \vec{v}_x\\ \vec{v}_T &= 12 \text{ km/h [N]} + 24 \text{ km/h [E]}\end{aligned}$$

Figure 14 shows the vector addition to determine the resultant velocity. This is the same technique we used in Tutorial 1.

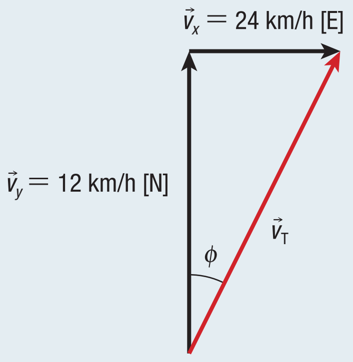


Figure 14 Determining the resultant velocity

$$\begin{aligned}v_T^2 &= v_y^2 + v_x^2\\ v_T &= \sqrt{v_y^2 + v_x^2}\\ v_T &= \sqrt{(12 \text{ km/h})^2 + (24 \text{ km/h})^2}\\ v_T &= 27 \text{ km/h}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{v_x}{v_y}\\ \tan\phi &= \frac{24 \text{ km/h}}{12 \text{ km/h}}\\ \phi &= 63^\circ\end{aligned}$$

Statement: The boat's resultant velocity is 27 km/h [N 63° E].

Practice

1. Write an email to a classmate explaining why the velocity of the current in a river has no effect on the time it takes to paddle a canoe across the river, as long as the boat is pointed perpendicular to the bank of the river. K/U C
2. A swimmer swims perpendicular to the bank of a 20.0 m wide river at a velocity of 1.3 m/s. Suppose the river has a current of 2.7 m/s [W]. T/I
   (a) How long does it take the swimmer to reach the other shore? [ans: 15 s]
   (b) How far downstream does the swimmer land from his intended location? [ans: 42 m [W]]

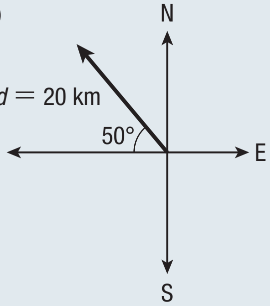
2.2 Summary

- Perpendicular vectors can be added algebraically using the Pythagorean theorem and the tangent function.
- By using the component method of vector addition, all vector addition problems can be converted into a problem involving two perpendicular vectors.
- River crossing problems involve two perpendicular, independent motions. You can solve these types of problems because the same time is taken for each motion.

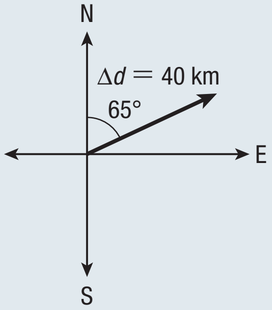
2.2 Questions

1. Break each vector down into an x-component and a y-component. T/I

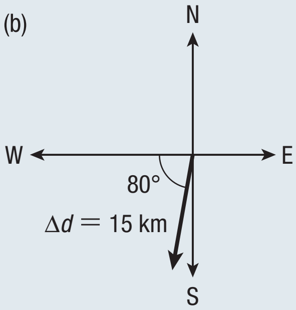
(a)



(c)

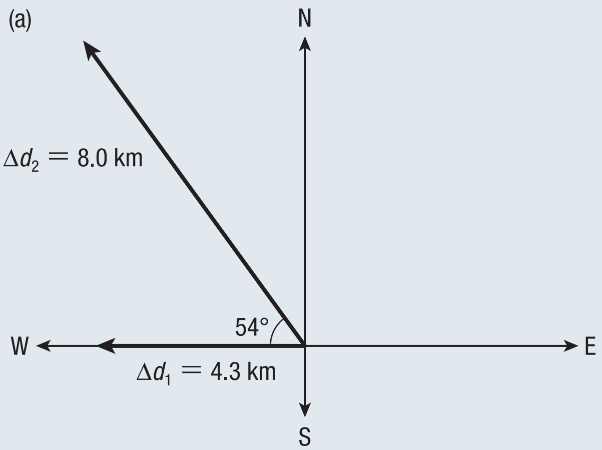


(b)

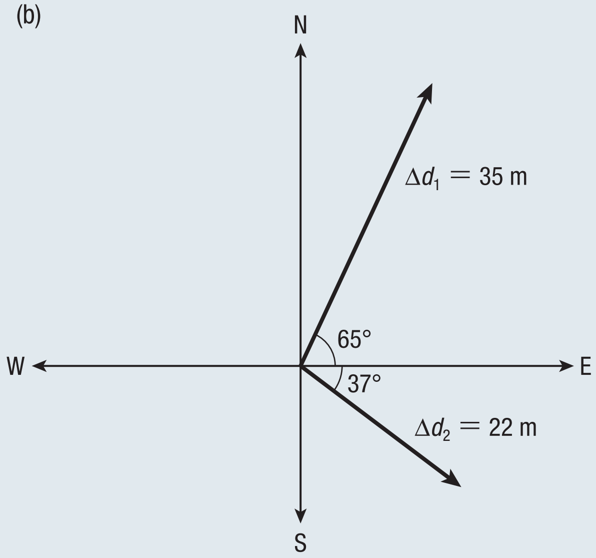


2. A motorcyclist drives 5.1 km [E] and then turns and drives 14 km [N]. What is her total displacement? T/I
3. A football player runs 11 m [N 20° E]. He then changes direction and runs 9.0 m [E]. What is his total displacement? T/I
4. What is the total displacement for a boat that sails 200.0 m [S 25° W] and then tacks (changes course) and sails 150.0 m [N 30° E]? T/I
5. Determine the total displacement of an object that travels 25 m [N 20° W] and then 35 m [S 15° E]. T/I
6. Use the component method to determine the total displacement given by the two vectors shown in each diagram.

(a)



(b)



7. Use the component method to add the following displacement vectors. T/I

$$\Delta\vec{d}_1 = 25\text{ m }[\text{N }30^\circ\text{ W}],\ \Delta\vec{d}_2 = 30.0\text{ m }[\text{N }40^\circ\text{ E}],$$
$$\Delta\vec{d}_3 = 35\text{ m }[\text{S }25^\circ\text{ W}]$$

8. A swimmer jumps into a 5.1 km wide river and swims straight for the other side at 0.87 km/h [N]. There is a current in the river of 2.0 km/h [W]. T/I
   (a) How long does it take the swimmer to reach the other side?
   (b) How far downstream has the current moved her by the time she reaches the other side?
9. A conductor in a train travelling at 4.0 m/s [N] walks across the train car at 1.2 m/s [E] to validate a ticket. If the car is 4.0 m wide, how long does it take the conductor to reach the other side? What is his velocity relative to the ground? T/I
10. Vectors can be added algebraically and by scale diagram. C
    (a) Write a letter to your teacher explaining which method you prefer and why.
    (b) Describe a situation for which the method that you do not prefer might be more suitable.