2.1 Motion in Two Dimensions—A Scale Diagram Approach

Figure 1 The motion of these cyclists is two-dimensional in the plane of the road.

Many of the moving objects that you observe or experience every day do not travel in straight lines. Rather, their motions are best described as two-dimensional. When you pedal a bicycle around a corner on a flat stretch of road, you experience two-dimensional motion in the horizontal plane (**Figure 1**).

Think about what happens when a leaf falls from a tree. If the leaf falls on a day without any wind, it tends to fall straight down to the ground. However, if the leaf falls on a windy day, it falls down to the ground and away from the tree. In this case, the leaf experiences two different motions at the same time. The leaf falls vertically downward due to gravity and horizontally away from the tree due to the wind. We say that the leaf is moving in two dimensions: the vertical dimension and the horizontal dimension. In Chapter 1, we analyzed the motion of objects that travel in only one dimension. To fully describe the motion of a leaf falling in the wind, and other objects moving in two dimensions, we need strategies for representing motion in two dimensions.

In Chapter 1, we analyzed the motions of objects in a straight line by studying vector displacements, velocities, and accelerations. How can we extend what we have already learned about motion in one dimension to two-dimensional situations? This is the question that we will pursue throughout this chapter.

CAREER LINK

Naval officers use gyroscopic compasses and satellite navigation to navigate Canada's naval fleet. However, every Canadian MARS (Maritime Surface and Subsurface) Officer still needs to know how to navigate the old-fashioned way, using a sharp pencil, a parallel ruler, and a protractor. It is essential for Canada's naval officers to have an extensive knowledge of vectors to safely navigate Canada's coastal waters and the high seas. To learn more about becoming a naval officer,

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Direction in Two Dimensions: The Compass Rose

The compass rose, shown in **Figure 2**, has been used for centuries to describe direction. It has applications on land, on the sea, and in the air. Recall that when we draw vectors, they have two ends, the tip (with the arrowhead) and the tail. In **Figure 3**, the vector that is shown pointing east in Figure 2 is rotated by 20° toward north. We will use a standard convention for representing vectors that point in directions between the primary compass directions (north, south, east, and west) to describe the direction of this vector. Figure 3 shows how the convention can be applied to this vector.

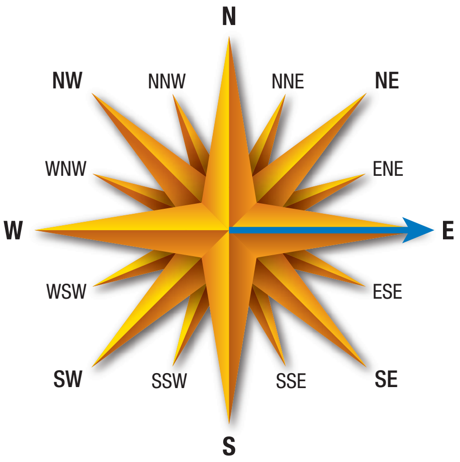

Figure 2 Compass rose, showing a vector pointing due east

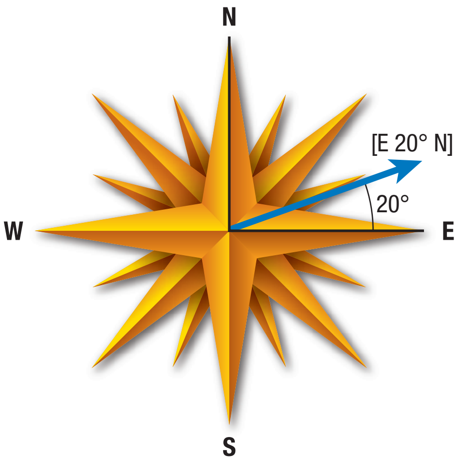

Figure 3 Convention for describing vector directions

We write the rotated vector's direction as [E 20° N]. This can be read as "point east, and then turn 20° toward north." Notice that in Figure 3 the complementary angle is 70°. Recall that complementary angles are two angles that add to 90°. So, another way of describing this vector's direction is [N 70° E], which can be read as "point north, and then turn 70° toward east." Both directions are the same, and the notation is interchangeable. The other important convention we will use is that, when using a Cartesian grid, north and east correspond to the positive y-axis and the positive x-axis, respectively.

When we are adding vectors in two dimensions, the vectors will not always point due north, south, east, or west. Similarly, the **resultant vector**—the vector that results from adding the given vectors—often points at an angle relative to these directions. So it is important to be able to use this convention to describe the direction of such vectors. In Tutorial 1, we will practise creating scale drawings of given vectors by choosing and applying an appropriate scale. In a scale such as 1 cm : 100 m, think of the ratio as "diagram measurement to real-world measurement." So a diagram measurement of 5.4 cm = 5.4 × (1 cm) represents an actual measurement of 5.4 × (100 m) = 540 m. You may find using a table like **Table 1** to be helpful.

resultant vector a vector that results from adding two or more given vectors

Table 1 Scale Conversions

	Given	Given	Required
Variable	Δd_1	Δd_2	Δd_T
before conversion (100 m)	540 m		
after conversion (1 cm)	5.4 cm		

Tutorial 1 Drawing Displacement Vectors in Two Dimensions Using Scale Diagrams

When drawing two-dimensional vectors, we must take not only the magnitude of the vector into consideration but also its direction. To draw two-dimensional vectors using a scale diagram, we need to determine a reasonable scale for the diagram. Scale diagrams should be approximately one half page to one full page in size. Generally speaking, the larger the diagram, the better your results will be.

Sample Problem 1: Draw a Displacement Vector to Scale

Draw a scale diagram of a displacement vector of 41 m [S 15° W].

Given: $\Delta\vec{d}$ = 41 m [S 15° W]

Required: Scale diagram of $\Delta\vec{d}$

Analysis: Choose a scale, and then use it to determine the length of the vector representing $\Delta\vec{d}$.

Solution: It would be reasonable to choose a scale of 1 cm : 10 m (each centimetre represents 10 m). Convert the displacement vector to the appropriate length using the following conversion method:

$$\Delta\vec{d} = (41\ \cancel{\text{m}})\left(\frac{1\ \text{cm}}{10\ \cancel{\text{m}}}\right)\ [\text{S } 15^\circ\ \text{W}]$$

$$\Delta\vec{d} = 4.1\ \text{cm [S } 15^\circ\ \text{W]}$$

In **Figure 4**, the vector is drawn with a magnitude of 4.1 cm. The direction is such that it originally pointed south and then was rotated 15° toward west.

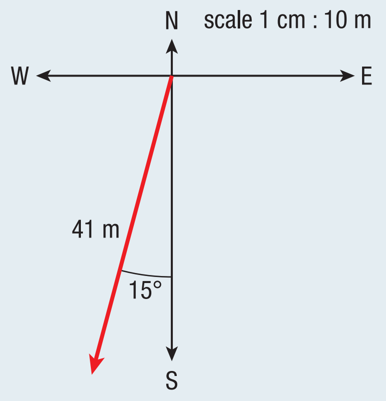

Figure 4 Scale diagram representing the displacement 41 m [S 15° W]

Statement: At a scale of 1 cm : 10 m, the given displacement vector is represented by $\Delta\vec{d}$ = 4.1 cm [S 15° W], as drawn in the diagram.

Practice

1. Choose a suitable scale to represent the vectors $\Delta\vec{d}_1$ = 350 m [E] and $\Delta\vec{d}_2$ = 410 m [E 35° N]. Use the scale to determine the lengths of the vectors representing $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$. T/I
   [ans: 1 cm : 50 m, giving 7.0 cm and 8.2 cm, or 1 cm : 100 m, giving 3.5 cm and 4.1 cm]
2. Represent the vectors in Question 1 on a scale diagram using your chosen scale. T/I C

Now that you have learned how to draw two-dimensional displacement vectors using scale diagrams, we will apply this skill to adding displacement vectors in Tutorial 2.

Tutorial 2 Adding Displacement Vectors in Two Dimensions Using Scale Diagrams

In the following Sample Problems, we will analyze three different scenarios involving displacement vectors in two dimensions. In Sample Problem 1, we will add two displacement vectors that are perpendicular to each other. In Sample Problem 2, one of the vectors to be added is pointing due north, and the other is pointing at an angle to this direction. In Sample Problem 3, we will add two vectors that do not point due north, south, east, or west.

Sample Problem 1: Adding Two Perpendicular Vectors

A cyclist rides her bicycle 50 m due east, and then turns a corner and rides 75 m due north. What is her total displacement?

Given: $\Delta\vec{d}_1 = 50\text{ m [E]}$; $\Delta\vec{d}_2 = 75\text{ m [N]}$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 50\text{ m [E]} + 75\text{ m [N]}$$

We have two perpendicular vectors that we need to add together. To add these vectors by scale diagram, we need to determine a reasonable scale for our diagram, such as 1 cm : 10 m. We can then solve the problem in four steps: draw the first vector, draw the second vector, draw the resultant vector, and determine the resultant vector's magnitude and direction.

Step 1. Draw the first vector.

Before we begin drawing our diagram, we will first draw a Cartesian coordinate system (**Figure 5**). Recall that the point where the *x*-axis and the *y*-axis of a Cartesian coordinate system cross is known as the origin. In all of our scale diagrams, the first vector will be drawn so that the tail of the vector starts at the origin. The first displacement is 50 m, or 5 × 10 m, so applying the chosen scale of 1 cm : 10 m, we draw this displacement as a 5.0 cm long vector pointing due east, starting at the origin.

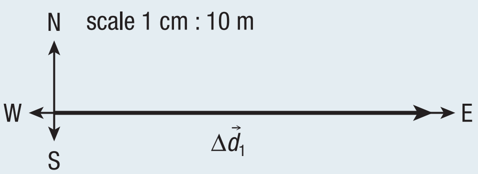

Figure 5 Vector $\Delta\vec{d}_1$, drawn to scale

Step 2. Join the second vector to the first vector tip to tail.

Figure 6 shows the second displacement vector drawn to scale represented as a vector of length 7.5 cm. Notice that the tail of this vector has been joined to the tip of the first vector. When vectors are being added, they must always be joined tip to tail.

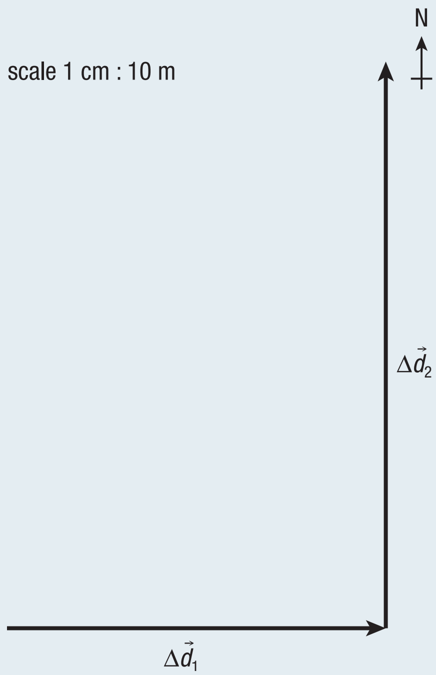

Figure 6 Add vector $\Delta\vec{d}_2$ to the scale diagram.

Step 3. Draw the resultant vector.

Figure 7 shows the resultant vector drawn from the tail of the first vector to the tip of the second vector. Resultant vectors are always drawn from the starting point in the diagram (the origin in our example) to the ending point. This diagram also indicates the angle θ (the Greek symbol theta) that the resultant vector makes with the horizontal.

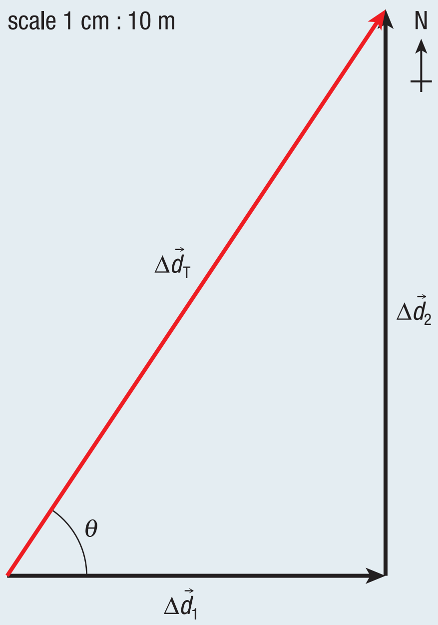

Figure 7 Draw the resultant vector.

To complete this problem, it is necessary to measure the length of the resultant vector $\Delta\vec{d}_T$ with a ruler and convert this measurement to the actual distance using the scale of the diagram. We must also measure the interior angle θ.

Step 4. Determine the magnitude and direction of the resultant vector.

As you can see from **Figure 8**, the resultant vector has length 9.0 cm. Using the scale, this vector represents a displacement of $9.0 \times (10 \text{ m}) = 90$ m. Using a protractor, the interior angle is measured to be 56° from the horizontal or [E] direction. This gives a final displacement of $\Delta\vec{d}_T = 90$ m [E 56° N].

Statement: The cyclist's total displacement is 90 m [E 56° N].

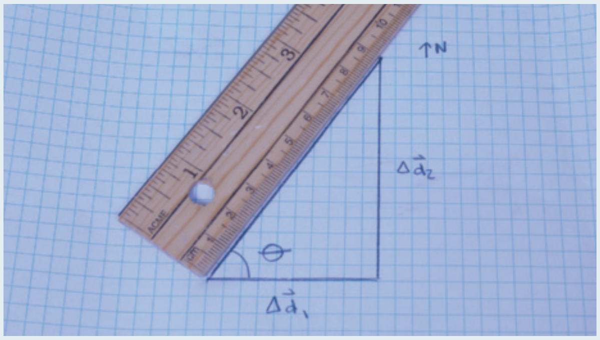

Figure 8 Using a ruler to measure the length of the resultant vector

In the next Sample Problem we will determine the total displacement of a sailboat when the direction of one of its displacements is not due north, south, east, or west.

Sample Problem 2: One Vector Is at an Angle

While in a race, a sailboat travels a displacement of 40 m [N]. The boat then changes direction and travels a displacement of 60 m [S 30° W]. What is the boat's total displacement?

Given: $\Delta\vec{d}_1 = 40 \text{ m [N]}$; $\Delta\vec{d}_2 = 60$ m [S 30° W]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 40 \text{ m [N]} + 60 \text{ m [S 30° W]}$$

At this stage, the solution looks very similar to that shown in Sample Problem 1. The scale of 1 cm : 10 m used in Sample Problem 1 can be used again here. Now we must join the two vectors tip to tail using the steps shown in Sample Problem 1.

Figure 9 shows the first displacement drawn as a vector 4.0 cm in length pointing due north. The second displacement is drawn as a vector 6.0 cm in length and is joined to the first vector tip to tail. We use a protractor to make sure the second vector points 30° west of south (not south of west!). The resultant vector is again drawn from the starting point of motion to the ending point. The resultant vector is measured using a ruler, and the displacement is calculated using our chosen scale. Notice that the displacement is in the southwest quadrant.

It is necessary to measure the angle θ with the horizontal to determine the final direction. In this case, we measure this angle from the negative horizontal or west direction, below the *x*-axis. The total displacement can be described as $\Delta\vec{d}_T = 32$ m [W 22° S].

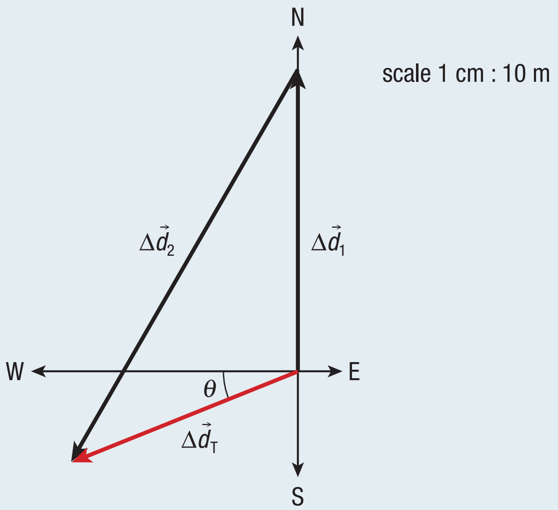

Figure 9 Adding the displacement vectors, tip to tail

Statement: The boat's total displacement is 32 m [W 22° S].

The most general vector addition problem is a situation in which neither displacement is pointing in the direction north, south, east, or west. The methods that we have used in Sample Problems 1 and 2 will also work in Sample Problem 3.

Sample Problem 3: Adding Two Non-perpendicular Vectors

A squash ball undergoes a displacement of 6.2 m [W 25° S] as it approaches a wall. It bounces off the wall and experiences a displacement of 4.8 m [W 25° N]. The whole motion takes 3.7 s. Determine the squash ball's total displacement and average velocity.

Given: $\Delta\vec{d}_1 = 6.2$ m [W 25° S]; $\Delta\vec{d}_2 = 4.8$ m [W 25° N]; $\Delta t = 3.7$ s

Required: $\Delta\vec{d}_T$; $\vec{v}_{av}$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 6.2 \text{ m [W 25° S]} + 4.8 \text{ m [W 25° N]}$$

To add these vectors, we will use a scale of 1 cm : 1 m. From **Figure 10,** we can determine the final displacement to be $\Delta\vec{d}_T = 10$ m [W 3° S].

Recall from Chapter 1 that average velocity $\vec{v}_{av}$ can be calculated algebraically as

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

We can use the value $\Delta\vec{d}_T = 10$ m [W 3° S] for the total displacement to calculate the average velocity.

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

$$= \frac{10 \text{ m [W 3° S]}}{3.7 \text{ s}}$$

$$\vec{v}_{av} = 2.7 \text{ m/s [W 3° S]}$$

Statement: The squash ball's total displacement is 10 m [W 3° S] and its average velocity is 2.7 m/s [W 3° S].

Notice that both vectors are in the same direction. This is because average velocity is calculated by dividing displacement (a vector) by time (a scalar with a positive value). Dividing a vector by a positive scalar does not affect the direction of the resultant vector (average velocity).

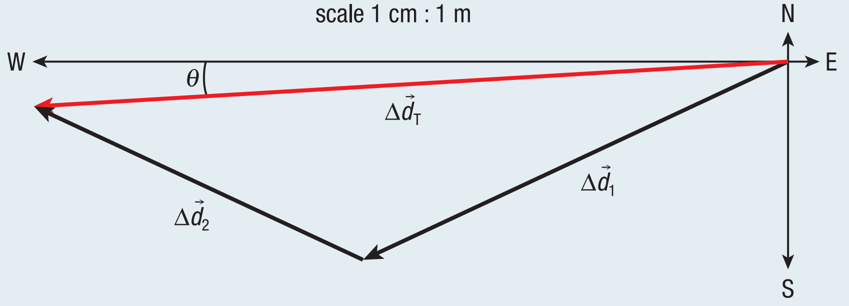

Figure 10 Adding the displacement vectors

Practice

1. Use a scale diagram to determine the sum of each pair of displacements. T/I C
   (a) $\Delta\vec{d}_1 = 72$ cm [W], $\Delta\vec{d}_2 = 46$ cm [N] [ans: 85 cm [W 33° N]]
   (b) $\Delta\vec{d}_1 = 65.3$ m [E 42° N], $\Delta\vec{d}_2 = 94.8$ m [S] [ans: 70.5 m [E 46° S]]
2. A cyclist travels 450 m [W 35° S] and then rounds a corner and travels 630 m [W 60° N]. T/I
   (a) What is the cyclist's total displacement? [ans: 740 m [W 23° N]]
   (b) If the whole motion takes 77 s, what is the cyclist's average velocity? [ans: 9.6 m/s [W 23° N]]

2.1 Summary

- Objects can move in two dimensions, such as in a horizontal plane and a vertical plane.
- The compass rose can be used to express directions in a horizontal plane, such as [N 40° W].
- To determine total displacement in two dimensions, displacement vectors can be added together using a scale diagram. To add two or more vectors together, join them tip to tail and draw the resultant vector from the tail of the first vector to the tip of the last vector.

2.1 Questions

1. Draw a Cartesian coordinate system on a sheet of paper. On this Cartesian coordinate system, draw each vector to scale, starting at the origin. T/I C
   (a) $\Delta\vec{d} = 8.0$ cm [S 15° E]
   (b) $\Delta\vec{d} = 5.7$ cm [N 35° W]
   (c) $\Delta\vec{d} = 4.2$ cm [N 18° E]
2. How could you express the direction of each vector listed in Question 1 differently so that it still describes the same vector? C
3. The scale diagram shown in **Figure 11** represents two vectors. T/I

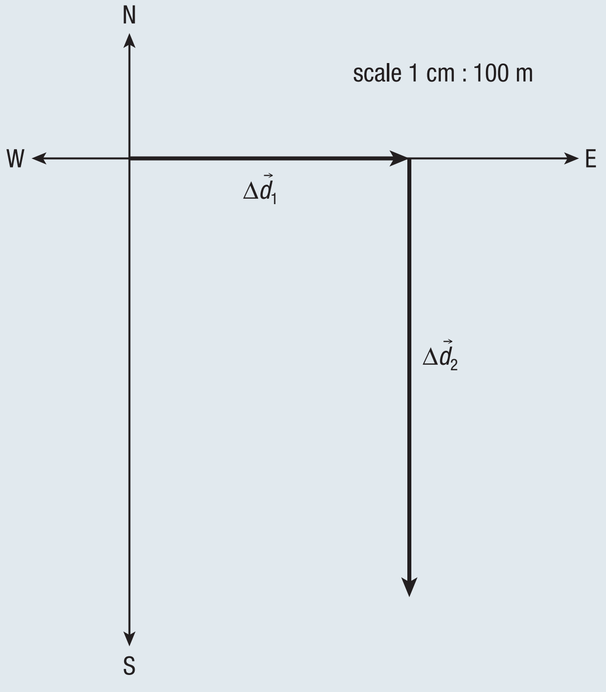

Figure 11

   (a) Use the given scale to determine the actual vectors.
   (b) Copy the scale diagram and complete it to determine the resultant vector when the two vectors are added.
4. A taxi drives 300.0 m south and then turns and drives 180.0 m east. What is the total displacement of the taxi? T/I
5. What is the total displacement of two trips, one of 10.0 km [N] and the other of 24 km [E]? T/I
6. If you added the two displacements in Question 5 in the opposite order, would you get the same answer? Explain. K/U
7. A horse runs 15 m [N 23° E] and then 32 m [S 35° E]. What is the total displacement of the horse? T/I
8. A car travels 28 m [E 35° S] and then turns and travels 45 m [S]. The whole motion takes 6.9 s. T/I
   (a) What is the car's average velocity?
   (b) What is the car's average speed?
9. An aircraft experiences a displacement of 100.0 km [N 30° E] due to its engines. The aircraft also experiences a displacement of 50.0 km [W] due to the wind. T/I
   (a) What is the total displacement of the aircraft?
   (b) If it takes 10.0 min for the motion to occur, what is the average velocity, in kilometres per hour, of the aircraft during this motion?