## LEARNING TIP

Describing Vertical Motion Directions
Vertical motion problems are vector questions that require directions to be indicated. Directions can be simplified by defining "up" as positive and "down" as negative. This means that the acceleration due to gravity should be negative in your calculations.
acceleration due to gravity (g) the acceleration that occurs when an object is allowed to fall freely; close to Earth's surface, $g$ has a value of $9.8 \mathrm{~m} / \mathrm{s}^{2}$
free fall the acceleration due to gravity of an object in the absence of air resistance

## Acceleration Near Earth's Surface

During a basketball game, the player takes the ball and shoots it toward the basket (Figure 1). The ball briefly skims the rim, then drops through the net to the floor. The player makes the basket because of her skill, with a little bit of help from the force of gravity. Gravity causes all objects to accelerate toward Earth's centre. If you have accidentally dropped an object such as a glass, you have directly experienced how significant the effect of gravity is.


Figure 1 Earth's gravity plays a key role in basketball and most other sports.

## Acceleration Due to Gravity

Acceleration due to gravity is the acceleration that occurs when an object is allowed to fall freely. The symbol for acceleration due to gravity is $g$. Physicists have determined that the average value of $g$ measured very close to Earth's surface is $9.8 \mathrm{~m} / \mathrm{s}^{2}$. Different places on Earth have different values for $g$. For example, the value of $g$ in Mexico City is slightly but measurably lower than the average of $9.8 \mathrm{~m} / \mathrm{s}^{2}$ because the city has a very high elevation and is therefore farther from Earth's centre. This had an interesting effect on the 1968 Summer Olympic Games, which were held in Mexico City. Many high jump, long jump, and pole-vaulting records were broken during these Olympic Games, attributed in part to the lower value of $g$. The value of $g$ is different on different planets and other celestial objects.

The acceleration due to gravity of an object near Earth's surface will be about $9.8 \mathrm{~m} / \mathrm{s}^{2}$ only if it is dropped in a vacuum. This type of motion is referred to as free fall, which is acceleration that occurs when there is no air resistance or other force affecting the motion of the object besides gravity.

## Uniform Vertical Acceleration

All objects that move freely in the vertical direction experience acceleration due to gravity $(g)$. In the previous section, you worked with the five key motion equations. Since we know the average value of $g$ close to Earth's surface, we can use the motion equations to explore how gravity affects objects that are moving vertically.

## Tutorial 1 Motion of an Object Falling Straight Down

The following Sample Problems will demonstrate, using the five key equations of motion, how to solve problems involving vertical motion. You might want to review the equations in Table 1 on page 37.

Sample Problem 1: Determining the Time It Takes for an Object to Fall to the Ground

A flowerpot is knocked off a window ledge and accelerates uniformly to the ground. If the window ledge is 10.0 m above the ground and there is no air resistance, how long does it take the flowerpot to reach the ground?

## Solution

We know that the motion of the flowerpot is straight down. However, we cannot describe the direction of the vector as we have previously, using $[\mathrm{E}],[\mathrm{W}],[\mathrm{N}]$, or $[\mathrm{S}]$. So we will let vectors
with directions up and to the right be indicated by positive values. Vectors with directions down and to the left will be indicated by negative values.

Since the flowerpot is not thrown upward or downward, we can assume that it was initially at rest. Therefore, the initial velocity is $0 \mathrm{~m} / \mathrm{s}$.
Given: $\Delta \vec{d}=-10.0 \mathrm{~m} ; \vec{a}=\vec{g}=-9.8 \mathrm{~m} / \mathrm{s}^{2} ; \vec{v}_{\mathrm{i}}=0 \mathrm{~m} / \mathrm{s}$
Required: $\Delta t$
Analysis: Since we are given the displacement, acceleration, and initial velocity of the flowerpot, we can use Equation 3 to solve for time.

$$
\Delta \vec{d}=\vec{v}_{\mathrm{i}} \Delta t+\frac{1}{2} \vec{a}(\Delta t)^{2}
$$

Solution: Notice that the given displacement and acceleration values are both negative, because both vectors point downward.

In general, it is not valid to divide one vector by another. However, since the motion is in a straight line, and the directions are given by the minus signs, we are able to divide the vector values. This equation can be rewritten as follows.

$$
\begin{aligned}
& \Delta \vec{d}=\frac{1}{2} \vec{a}(\Delta t)^{2} \\
& \Delta t=\sqrt{\frac{2 \Delta d}{a}} \\
& \Delta t=\sqrt{\frac{2(-10.0 \mathrm{mx})}{-9.8 \mathrm{~mm} / \mathrm{s}^{2}}} \\
& \Delta t=1.4 \mathrm{~s}
\end{aligned}
$$

We know we should take the positive root because time intervals are always positive.
Statement: The flowerpot will take 1.4 s to reach the ground.

## Sample Problem 2: Determining the Final Velocity for a Falling Object

What is the final velocity of the flowerpot in Sample Problem 1 just before it hits the ground?

## Solution

To solve this problem, we need to find the final velocity of the flowerpot just before it hits the ground. If the question asked for its velocity after it had hit the ground and came to rest, then the final velocity would be $0 \mathrm{~m} / \mathrm{s}$. Here, we are determining the velocity while the flowerpot is still in motion, just before it hits the ground.

We will again write vectors with directions that are up and to the right as having positive values and those with directions down and to the left as having negative values.
Given: $\Delta \vec{d}=-10.0 \mathrm{~m} ; \vec{a}=\vec{g}=-9.8 \mathrm{~m} / \mathrm{s}^{2} ; \vec{v}_{\mathrm{i}}=0 \mathrm{~m} / \mathrm{s}$
Required: $\vec{v}_{f}$
Analysis: We can choose any of the five key equations of motion that has $\vec{v}_{\mathrm{f}}$ as a variable. It is always wise to choose the equation
that only requires you to use given information. We will use the vector directions for the given variables but will only calculate the magnitude of the final velocity. We know that the direction of the final velocity must be downward.

$$
\begin{aligned}
v_{\mathrm{f}}^{2} & =v_{\mathrm{i}}^{2}+2 a \Delta d \\
v_{\mathrm{f}} & =\sqrt{v_{\mathrm{i}}^{2}+2 a \Delta d}
\end{aligned}
$$

Solution: Since the initial velocity is zero,

$$
\begin{aligned}
V_{\mathrm{f}} & =\sqrt{2 a \Delta d} \\
& =\sqrt{2\left(-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(-10.0 \mathrm{~m})} \\
& =14 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Statement: The flowerpot is travelling at $14 \mathrm{~m} / \mathrm{s}$ downward just before it hits the ground.

## Practice

1. A ball is dropped from the roof of a building. If it takes the ball 2.6 s to reach the ground, how tall is the building?
2. A hot air balloon is hovering at a height of 52 m above the ground. A penny is dropped from the balloon. Assume no air resistance.
(a) How long does it take the penny to hit the ground? [ans: 3.3 s ]
(b) What is the final velocity of the penny just before it hits the ground? [ans: $32 \mathrm{~m} / \mathrm{s}$ ]

## Tutorial 2 Motion of an Object Thrown Straight Up

The following Sample Problems involve analyzing the motion of an object that is first thrown straight up and then falls to Earth.

## Sample Problem 1: Determining the Height Reached by a Ball Thrown Straight Up in the Air

A tennis ball is thrown straight up in the air, leaving the person's hand with an initial velocity of $3.0 \mathrm{~m} / \mathrm{s}$, as shown
in Figure 2 on the next page. How high, from where it was thrown, does the ball go?


Figure 2 The motion of a ball thrown straight upward
Figure 2 shows the path of the tennis ball. Notice that the vectors for the initial velocity, $\vec{v}_{\mathrm{i}}$, and the acceleration, $\vec{a}$, are in opposite directions to each other. Using our previous convention, the
upward vector will be made positive and the downward vector negative. At its maximum height, the ball comes to rest momentarily, so the final velocity, $\vec{v}_{\mathrm{f}}$, is zero. Then we can use Equation 4.
Given: $\vec{v}_{\mathrm{i}}=3.0 \mathrm{~m} / \mathrm{s} ; \vec{a}=-9.8 \mathrm{~m} / \mathrm{s}^{2} ; \vec{v}_{\mathrm{f}}=0 \mathrm{~m} / \mathrm{s}$
Required: $\Delta d$
Analysis: $v_{\mathrm{f}}^{2}=v_{\mathrm{i}}^{2}+2 a \Delta d$

$$
0=v_{i}^{2}+2 a \Delta d
$$

$$
-v_{i}^{2}=2 a \Delta d
$$

$$
\Delta d=\frac{-v_{i}^{2}}{2 a}
$$

Solution: $\Delta d=\frac{-v_{i}^{2}}{2 a}$

$$
\begin{aligned}
= & \frac{-\left(3.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{2\left(-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)} \\
= & \frac{-9.0 \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}}{-19.6 \frac{\mathrm{~m}}{\frac{\mathrm{~s}^{2}}{2}}} \\
\Delta d & =0.46 \mathrm{~m}
\end{aligned}
$$

Statement: The maximum height attained by the tennis ball is 0.46 m ( 46 cm ).

## Sample Problem 2: Determining the Time for a Ball Thrown Upward to Attain Its Maximum Height

How long will it take the ball shown in Figure 2 to reach its maximum height?
Given: $\vec{v}_{\mathrm{i}}=3.0 \mathrm{~m} / \mathrm{s} ; \vec{a}=-9.8 \mathrm{~m} / \mathrm{s}^{2} ; \vec{v}_{\mathrm{f}}=0 \mathrm{~m} / \mathrm{s}$

## Required: $\Delta t$

Analysis: To determine the time that this motion takes, we will identify an equation that uses only given information.

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t} \\
\Delta t & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\overrightarrow{\mathrm{a}}}
\end{aligned}
$$

Since the final velocity is equal to zero,

$$
\Delta t=\frac{-\vec{v}_{i}}{\vec{a}}
$$

Solution: $\Delta t=\frac{-\vec{v}_{i}}{\vec{a}}$

$$
\begin{aligned}
= & \frac{-\left(3.0 \frac{\mathrm{pr}}{\mathrm{~s}}\right)}{-9.8 \frac{\mathrm{pr}}{\mathrm{~s}^{2}}} \\
\Delta t & =0.31 \mathrm{~s}
\end{aligned}
$$

Statement: It will take the tennis ball 0.31 s to reach its maximum height.

## Practice

1. A golf ball is thrown straight up in the air at a velocity of $8.3 \mathrm{~m} / \mathrm{s}$.
(a) Determine the maximum height of the golf ball. [ans: 3.5 m ]
(b) How long will it take the ball to reach its maximum height? [ans: 0.85 s ]
(c) How long will it take the ball to fall from its maximum height to the height from which it was initially launched? [ans: 0.85 s ]
2. A rock is thrown downward from a bridge that is 12 m above a small creek. The rock has an initial velocity of $3.0 \mathrm{~m} / \mathrm{s}$ downward. What is the velocity of the rock just before it hits the water? ${ }^{T I I I}$ [ans: $16 \mathrm{~m} / \mathrm{s}$ down]


Figure 3 The value of $9.8 \mathrm{~m} / \mathrm{s}^{2}$ for acceleration near Earth assumes that there are no other forces acting on an object, such as air resistance.

## Free Fall and Terminal Velocity

In real-life situations, there will always be some air resistance. Sometimes air resistance can be enough to have a significant effect on the motion of a falling object. For example, when a parachutist jumps out of an aircraft, he can control the amount of air resistance based on how he positions his body. If the parachutist dives out of the aircraft head first, he will experience very little air resistance. Most parachutists will try to fall so that as much of the surface area of their body is in contact with the air as possible. In other words, most parachutists will fall in a belly flop (Figure 3).

When the air resistance on the parachutist is equal to the force due to gravity acting on the parachutist, the parachutist will stop accelerating and stay at a constant velocity, called the terminal velocity. You will learn more about air resistance and free fall in Unit 2.

### 1.6 Summary

- The symbol $g$ is used to represent the acceleration due to gravity.
- All objects in free fall close to Earth's surface will accelerate at $9.8 \mathrm{~m} / \mathrm{s}^{2}$ toward Earth's centre.
- Air resistance can cause objects to accelerate at values less than $g$.
- When an object reaches terminal velocity, it will fall at a constant velocity.


### 1.6 Questions

1. Describe the motion of an object that is dropped close to Earth's surface.
2. A basketball player jumps up to make a basket and appears to "hang" in mid-air. Write a brief description explaining to a Grade 9 student what is occurring and why.
3. A baseball is thrown straight up in the air, reaches its maximum height, and falls back down to the height from which it was originally thrown. What is the acceleration of the ball
(a) halfway up to its maximum height?
(b) at its maximum height?
(c) halfway back down to the initial height from which it was thrown?
4. A rubber ball is dropped from a height of 1.5 m . TTII
(a) How long does it take to hit the ground?
(b) What is the velocity of the ball when it has travelled a distance halfway to the ground?
5. An arrow is shot straight up into the air at $80.0 \mathrm{~m} / \mathrm{s}$. $\mathbb{T \pi}$
(a) What is the arrow's maximum height?
(b) How long does the arrow take to reach its maximum height?
(c) Determine the total amount of time that the arrow is in the air.
6. A rock is thrown down from the top of a cliff with a velocity of $3.61 \mathrm{~m} / \mathrm{s}$ [down]. The cliff is 28.4 m above the ground. Determine the velocity of the rock just before it hits the ground.
7. Describe the motion of the object represented by the velocity-time graph in Figure 4. Give an example of an object that might undergo this type of motion.


Figure 4
8. Research and describe a real-life situation where an object or person experiences an acceleration greater than $g\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)$. A

