## Acceleration

Some theme parks have rides in which you are slowly carried up in a seat to the top of a tower, and then suddenly released (Figure 1). On the way down, your arms and hair may fly upward as the velocity of your seat increases. The thrill of this sudden change in motion can frighten and exhilarate you all at once. An even bigger thrill ride, however, is to be a pilot in a jet being launched from the deck of an aircraft carrier. These giant ships use catapults to move 35000 kg jets from rest ( $0 \mathrm{~km} / \mathrm{h}$ ) to $250 \mathrm{~km} / \mathrm{h}$ in just 2.5 s .

While it is true that objects sometimes move at constant velocity in everyday life, usually the velocities we observe are not constant. Objects that experience a change in velocity are said to be undergoing acceleration. Acceleration describes how quickly an object's velocity changes over time or the rate of change of velocity. We can study acceleration using a velocity-time graph, which, similar to the position-time graph, has time on the horizontal axis, but velocity rather than position on the vertical axis.

Velocity-time graphs are particularly useful when studying objects moving with uniform velocity (zero acceleration) or uniform acceleration (velocity changing, but at a constant rate). The velocity-time graphs for both uniform velocity and uniform acceleration are always straight lines. By contrast, the position-time graph of an accelerated motion is curved. Table 1 shows the position-time graphs of objects moving with accelerated motion.


Figure 1 Sudden changes in velocity are part of the thrill of midway rides.
acceleration ( $\vec{a}_{\mathrm{av}}$ ) how quickly an object's velocity changes over time (rate of change of velocity)
velocity-time graph a graph describing the motion of an object, with velocity on the vertical axis and time on the horizontal axis

Table 1 Interpreting Position-Time Graphs

| Position-time graph | Type of motion | Example |
| :---: | :---: | :---: |
| Graph A | - graph is a curve <br> - on any graph that curves, the slope or steepness of the graph changes from one point on the graph to another <br> - since the slope in graph $A$ is constantly changing, the velocity is not constant <br> - since the graph lies above the $x$-axis and its slope is increasing, the velocity of the object is also increasing (speeding up) in a positive or eastward direction |  |
| Graph B | - graph is a curve <br> - on any graph that curves, the slope or steepness of the graph changes from one point on the graph to another <br> - since the slope in graph $B$ is constantly changing, the velocity is not constant <br> - since the graph lies below the $x$-axis and its slope is negative but getting steeper, the object is speeding up as it moves in a negative or westward direction |  |

Table 1 (continued)

| Position-time graph | Type of motion | Example |
| :---: | :---: | :---: |
|  | - graph is a curve <br> - on any graph that curves, the slope changes from one point on the curve to another <br> - since the slope in graph C is constantly changing, the velocity is not constant <br> - since the graph lies above the $x$-axis and its slope is decreasing, the velocity of the object is also decreasing (slowing down) in a positive or eastward direction |  |
| Graph D <br> $t(\mathrm{~s})$ | - graph is a curve <br> - on any graph that curves, the slope or steepness of the graph changes from one point on the graph to another <br> - since the slope in graph D is constantly changing, the velocity is not constant <br> - since the graph lies below the $x$-axis and its slope is negative and getting shallower, the object is slowing down as it moves in a negative or westward direction |  |

## LEARNING TIP

## Slope and Area of

 Velocity-Time GraphsThe slope of a velocity-time graph gives the acceleration of the object. The area under a velocity-time graph gives the displacement of the object. Why is displacement related to the area under a velocity-time graph? One way to think about it is this: the greater the velocity during a given time interval, the greater the area under the graph, and the greater the displacement over that time interval.

## Determining Acceleration from a Velocity-Time Graph

Figure 2 shows a velocity-time graph for a skateboard rolling down a ramp. Notice that the line of the graph goes upward to the right and has $x$-intercept and $y$-intercept of zero. We can calculate the slope of the graph in Figure 2 using the equation

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
m & =\frac{\Delta \vec{v}(\mathrm{~m} / \mathrm{s})}{\Delta t(\mathrm{~s})}
\end{aligned}
$$

Since acceleration describes change in velocity over time, this suggests that the (average) acceleration of the skateboard is given by the equation

$$
\vec{a}_{\mathrm{av}}=\frac{\Delta \vec{v}}{\Delta t}
$$

Velocity v. Time for a Skateboard Rolling Down a Ramp


Figure 2 Velocity-time graph for a skateboard rolling down a ramp

In other words,
The slope of a velocity-time graph gives the average acceleration of an object.

Acceleration over a time interval, that is, average acceleration, is given by the equation
average acceleration $=\frac{\text { change in velocity }}{\text { change in time }}$

$$
\begin{gathered}
\overrightarrow{\mathrm{a}}_{\mathrm{av}}=\frac{\Delta \vec{v}}{\Delta t} \\
\text { or } \vec{a}_{\mathrm{av}}=\frac{\overrightarrow{\mathrm{v}}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t}
\end{gathered}
$$

Recall that the SI unit for velocity is metres per second ( $\mathrm{m} / \mathrm{s}$ ) and the SI unit for time is seconds (s). The SI unit for acceleration is a derived unit. A derived SI unit is a unit created by combining SI base units. We can derive the units for acceleration by dividing a velocity unit ( $\mathrm{m} / \mathrm{s}$ ) by a time unit ( s ), as follows:
units of acceleration = units of velocity per second

$$
\begin{aligned}
& \frac{\mathrm{m}}{\mathrm{~s}} \\
= & \frac{\mathrm{s}}{\mathrm{~m}} \\
= & \frac{\mathrm{m}}{\mathrm{~s}} \times \frac{1}{\mathrm{~s}} \\
= & \frac{\mathrm{m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

## LEARNING TIP

## Square Seconds?

What is a square second? Good question! When we write acceleration units as $\mathrm{m} / \mathrm{s}^{2}$, we are not implying that we have measured a square second. This is simply a shorter way of expressing the derived unit. You can also read the unit as "metres per second, per second"-describing how many metres per second of velocity are gained or lost during each second of acceleration.

## Tutorial 1 Calculating Acceleration

When you are given values for any three variables in the equation $\vec{a}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t}$ for acceleration, you can solve for the missing variable.

## Sample Problem 1

What is the acceleration of the skateboard in Figure 2? Consider the motion between 0 s and 10 s .
Given: $\vec{v}_{\mathrm{i}}=0 \mathrm{~m} / \mathrm{s} ; \vec{v}_{\mathrm{f}}=30 \mathrm{~m} / \mathrm{s}[\mathrm{S}] ; t_{\mathrm{i}}=0 \mathrm{~s} ; t_{\mathrm{f}}=10 \mathrm{~s}$
Required: $\vec{a}_{\mathrm{av}}$
Analysis: To calculate the average acceleration, which is the slope of the graph, we use the defining equation in the form that includes $\vec{v}_{\mathrm{i}}, \vec{v}_{\mathrm{f}}, t_{\mathrm{i}}$, and $t_{\mathrm{f}}$ :

$$
\begin{aligned}
\overrightarrow{\mathrm{a}}_{\mathrm{av}} & =\frac{\Delta \vec{v}}{\Delta t} \\
& =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t_{\mathrm{f}}-t_{\mathrm{i}}}
\end{aligned}
$$

Solution: $\vec{a}_{a v}=\frac{\vec{v}_{f}-\vec{v}_{i}}{t_{f}-t_{i}}$

$$
\begin{aligned}
& =\frac{30 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]-0 \mathrm{~m} / \mathrm{s}}{10 \mathrm{~s}-0 \mathrm{~s}} \\
\overrightarrow{\mathrm{a}}_{\mathrm{av}} & =3 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~s}]
\end{aligned}
$$

Statement: The skateboard is accelerating down the ramp at $3 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~S}]$.

What would the acceleration be if the same velocity change of $30 \mathrm{~m} / \mathrm{s}$ [S] took place over a time interval of only 5 s ?

$$
\begin{aligned}
\overrightarrow{\mathrm{a}}_{\mathrm{av}} & =\frac{\Delta \vec{v}}{\Delta t} \\
& =\frac{30 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]}{5 \mathrm{~s}} \\
\overrightarrow{\mathrm{a}}_{\mathrm{av}} & =6 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~S}]
\end{aligned}
$$

The time interval is shorter, so a more rapid acceleration occurs.

## Sample Problem 2

A bullet is found lodged deeply in a brick wall. As part of the investigation, a forensic scientist is experimenting with a rifle that was found nearby. She needs to determine the acceleration that a bullet from the rifle can achieve as a first step in linking the rifle to the bullet. During a test firing, she finds that the rifle bullet accelerates from rest to $120 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$ in $1.3 \times 10^{-2} \mathrm{~s}$ as it travels down the rifle's barrel. What is the bullet's average acceleration?
Given: $\vec{v}_{\mathrm{i}}=0 \mathrm{~m} / \mathrm{s} ; \vec{v}_{\mathrm{f}}=120 \mathrm{~m} / \mathrm{s}[\mathrm{E}] ; \Delta t=1.3 \times 10^{-2} \mathrm{~s}$
Required: $\vec{a}_{\mathrm{av}}$

## Sample Problem 3

When a hockey player hits a hockey puck with his stick, the velocity of the puck changes from $8.0 \mathrm{~m} / \mathrm{s}$ [ N ] to $10.0 \mathrm{~m} / \mathrm{s}$ [S] over a time interval of 0.050 s . What is the acceleration of the puck?
Given: $\vec{v}_{\mathrm{i}}=8.0 \mathrm{~m} / \mathrm{s}[\mathrm{N}] ; \vec{v}_{\mathrm{f}}=10.0 \mathrm{~m} / \mathrm{s}[\mathrm{S}] ; \Delta t=0.050 \mathrm{~s}$
Required: $\vec{a}_{\mathrm{av}}$
Analysis: $\vec{a}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t}$
Solution: $\vec{a}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t}$

$$
\vec{a}_{\mathrm{av}}=\frac{10.0 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]-8.0 \mathrm{~m} / \mathrm{s}[\mathrm{~N}]}{0.050 \mathrm{~s}}
$$

At this point, it would appear that we have a dilemma. In the numerator of the fraction, we must subtract a vector with a direction $[\mathrm{N}]$ from a vector with a direction $[\mathrm{S}]$.

Analysis: $\vec{a}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t}$
Solution: $\vec{a}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t}$

$$
\begin{aligned}
= & \frac{120 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]-0 \frac{\mathrm{~m}}{\mathrm{~s}}}{1.3 \times 10^{-2} \mathrm{~s}} \\
\vec{a}_{\mathrm{av}} & =9.2 \times 10^{3} \mathrm{~m} / \mathrm{s}^{2}[\mathrm{E}]
\end{aligned}
$$

Statement: The acceleration of the bullet is $9.2 \times 10^{3} \mathrm{~m} / \mathrm{s}^{2}[\mathrm{E}]$.

To solve this dilemma, we can use a technique from Section 1.2. We will change a vector subtraction problem into a vector addition problem. Recall that negative $[\mathrm{N}]$ is the same as positive $[\mathrm{S}]$.

$$
\begin{aligned}
\vec{a}_{\mathrm{av}} & =\frac{10.0 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]+8.0 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]}{0.050 \mathrm{~s}} \\
& =\frac{18.0 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]}{0.050 \mathrm{~s}} \\
\overrightarrow{\mathrm{a}}_{\mathrm{av}} & =3.6 \times 10^{2} \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~S}]
\end{aligned}
$$

Statement: The hockey puck's acceleration is $3.6 \times 10^{2} \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~S}]$.
Notice that the initial velocity of the puck is north, while its final velocity is south. The acceleration is in the opposite direction to the initial motion, so the puck slows down and comes to rest. It then continues accelerating south, increasing its velocity to $10 \mathrm{~m} / \mathrm{s}$ [S]. This is why the final velocity is due south. Sample Problem 3 can also be solved by using a vector scale diagram.

## Practice

1. A catapult accelerates a rock from rest to a velocity of $15.0 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$ over a time interval of 12.5 s . What is the rock's average acceleration? Tㅔ [ans: $1.20 \mathrm{~m} / \mathrm{s}^{2}$ [ S$]$ ]
2. As a car approaches a highway on-ramp, it increases its velocity from $17 \mathrm{~m} / \mathrm{s}[\mathrm{N}]$ to $25 \mathrm{~m} / \mathrm{s}[\mathrm{N}]$ over 12 s . What is the car's average acceleration? $T 1 /$ [ans: $\left.0.67 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~N}]\right]$
3. A squash ball with an initial velocity of $25 \mathrm{~m} / \mathrm{s}[\mathrm{W}]$ is hit by a squash racket, changing its velocity to $29 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$ in 0.25 s . What is the squash ball's average acceleration? [ans: $2.2 \times 10^{2} \mathrm{~m} / \mathrm{s}^{2}[\mathrm{E}]$ ]

You can use both the defining equation for average acceleration and velocity-time graphs to determine other information about the motion of an object, besides acceleration itself. In Tutorial 2, you will use the equation in a different form to determine a different quantity. Tutorial 3 introduces the idea of determining displacement via the area under a velocity-time graph.

## Tutorial 2 Solving the Acceleration Equation for Other Variables

In the following Sample Problem, we will explore how to solve the defining acceleration equation for other variables.

## Sample Problem 1: Solving the Acceleration Equation for Final Velocity

A racehorse takes 2.70 s to accelerate from a trot to a gallop. If the horse's initial velocity is $3.61 \mathrm{~m} / \mathrm{s}$ [W] and it experiences an acceleration of $2.77 \mathrm{~m} / \mathrm{s}^{2}$ [W], what is the racehorse's velocity when it gallops?
Given: $\Delta t=2.70 \mathrm{~s} ; \vec{v}_{\mathrm{i}}=3.61 \mathrm{~m} / \mathrm{s}[\mathrm{W}] ; \vec{a}_{\mathrm{av}}=2.77 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~W}]$
Required: $\vec{v}_{f}$
Analysis: $\vec{a}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{\Delta t}$
Solution: Rearrange the equation for acceleration to solve for the final velocity.

$$
\begin{aligned}
\vec{a}_{\mathrm{av}} \Delta t & =\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}} \\
\vec{v}_{\mathrm{f}} & =\vec{a}_{\mathrm{av}} \Delta t+\vec{v}_{\mathrm{i}} \\
& =\left(2.77 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}[\mathrm{~W}]\right)(2.70 . \mathrm{s})+3.61 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~W}] \\
& =7.48 \mathrm{~m} / \mathrm{s}[\mathrm{~W}]+3.61 \mathrm{~m} / \mathrm{s}[\mathrm{~W}] \\
\vec{v}_{\mathrm{f}} & =11.1 \mathrm{~m} / \mathrm{s}[\mathrm{~W}]
\end{aligned}
$$

Statement: When it gallops, the racehorse has a velocity of $11.1 \mathrm{~m} / \mathrm{s}$ [W].

## Practice

1. How long does it take a radio-controlled car to accelerate from $3.2 \mathrm{~m} / \mathrm{s}$ [W] to $5.8 \mathrm{~m} / \mathrm{s}$ [W] if it experiences an average acceleration of $1.23 \mathrm{~m} / \mathrm{s}^{2}$ [W]? TNI [ans: 2.1 s ]
2. A speedboat experiences an average acceleration of $2.4 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~W}]$. If the boat accelerates for 6.2 s and has a final velocity of $17 \mathrm{~m} / \mathrm{s}$ [W], what was the initial velocity of the speedboat? [TIT [ans: $2.1 \mathrm{~m} / \mathrm{s}$ [W]]

## Tutorial 3 Determining Displacement from a Velocity-Time Graph

By further analyzing the velocity-time graph shown in Figure 2 on page 22, we can determine even more information about the motion of the skateboard. Figure 3 has the same data plotted as Figure 2, except that the area under the line is shaded. The area under a velocitytime graph gives the displacement of the object. Note how the area under the straight line in Figure 3 forms a triangle. The area $(A)$ of a triangle is determined by the equation

$$
\begin{aligned}
A & =\frac{1}{2} b h \\
\Delta \vec{d} & =\frac{1}{2} b h
\end{aligned}
$$

where $b$ is the length of the base of the triangle and $h$ is the height of the triangle. We can determine the displacement of the skateboard from Figure 3 by calculating the area of this triangle.

Velocity v. Time for a Skateboard Rolling Down a Ramp


Figure 3 Velocity-time graph showing the area underneath the line

## Sample Problem 1: Determining Displacement from a Velocity-Time Graph

What is the displacement represented by the graph in Figure 3 on the previous page?
Given: $b=10.0 \mathrm{~s} ; h=30.0 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$
Required: $\Delta \vec{d}$
Analysis: $\Delta \vec{d}=\frac{1}{2} b h$

Solution: $\Delta \vec{d}=\frac{1}{2}(10.0 \mathrm{~s})\left(30.0 \frac{\mathrm{~m}}{\mathrm{~g}}[\mathrm{~S}]\right)$
$\Delta \vec{d}=150 \mathrm{~m}[\mathrm{~S}]$
Statement: The object was displaced 150 m [S] in 10 s .

## Sample Problem 2: Determining Displacement from a More Complex Velocity-Time Graph

What is the displacement represented by the graph in Figure 4 over the time interval from 0 s to 10.0 s?


Figure 4 Velocity-time graph showing more complex motion

The graph in Figure 4 is a more complex velocity-time graph than we worked with in Sample Problem 1. However, the displacement of the object can still be determined by calculating the area under the velocity-time graph.

To calculate this displacement, we will need to break the area under the line into a rectangle and a triangle, and add these two areas together. We know the area $(A)$ of a triangle is given by $A_{\text {triangle }}=\frac{1}{2} b h$. The area of a rectangle is its length (I) multiplied by its width $(w)$, or $A_{\text {rectangle }}=I w$.

Given: $b=5.0 \mathrm{~s} ; h=10.0 \mathrm{~m} / \mathrm{s}[\mathrm{S}] ; I=5.0 \mathrm{~s} ; w=10.0 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$

Required: $\Delta \vec{d}$
Analysis: $\Delta \vec{d}=A_{\text {triangle }}+A_{\text {rectangle }}$
Solution: $\Delta \vec{d}=A_{\text {triangle }}+A_{\text {rectangle }}$

$$
\begin{aligned}
& =\frac{1}{2} b h+I w \\
& =\frac{1}{2}(5.0 \mathrm{~s})\left(10.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}]\right)+(5.0 \mathrm{~s})\left(10.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}]\right) \\
& =25 \mathrm{~m}[\mathrm{~S}]+50.0 \mathrm{~m}[\mathrm{~S}] \\
\Delta \vec{d} & =75 \mathrm{~m}[\mathrm{~S}]
\end{aligned}
$$

Statement: The object has travelled $75 \mathrm{~m}[\mathrm{~S}]$ after 10.0 s .

## Practice

1. Determine the displacement represented by the graph in Figure 4 over the following time intervals:
(a) from 0 s to 4.0 s [ans: $16 \mathrm{~m}[\mathrm{~S}]$ ]
(b) from 0 s to 7.5 s [ans: 50 m [ S$]$ ]

## Mini Investigation

## Motion Simulations

Skills: Predicting, Performing, Observing, Analyzing, Communicating
In this investigation, you will use a computer simulation for four different motion scenarios and then analyze the scenarios by graphing.
Equipment and Materials: computer access; graphing paper

1. Go to the Nelson website and find the link for this Mini Investigation.
2. Go to the simulation, and take a few minutes to familiarize yourself with how it operates. You will be asked to run this simulation for each of the following scenarios:

- a positive velocity value
- a negative velocity value
- a negative initial position and a positive velocity value
- a negative initial position and a negative velocity value
A. Before you run these scenarios, write a brief statement and draw a sketch to predict how each graph will appear. Im ㄷ.
B. After you have run each scenario, sketch the resulting position-time and velocity-time graphs from the actual data you collected.
C. Compare your sketches from Question A to the graphs that were generated when you ran each scenario. Explain any discrepancies or misconceptions that you may have had.


## Instantaneous Velocity and Average Velocity

The velocity of any object that is accelerating is changing over time. In motion with uniform acceleration, the velocity of an object changes at a constant (uniform) rate. During a launch (Figure 5), a spacecraft accelerates upward at a rapid rate. NASA personnel may need to determine the velocity of the spacecraft at specific points in time. They can do this by plotting position and time data on a graph and determining the spacecraft's instantaneous velocity. Instantaneous velocity, or $\vec{v}_{\text {inst }}$ is the velocity of an object at a specific instant in time. By comparison, average velocity $\left(\vec{v}_{\mathrm{av}}\right)$ is determined over a time interval. Both types of velocity are rates of change of position, but they tell us different things about the motion of an object.

## Tutorial 4 Determining Instantaneous and Average Velocity

Figure 6 on the next page shows a position-time graph of an object that is undergoing uniform acceleration. Moving along the curve, the slope of the curve progressively increases. From this, we know that the velocity of the object is constantly increasing. To determine the instantaneous velocity of the object at a specific time, we must calculate the slope of the tangent of the line on the position-time graph at that time. A tangent is a straight line that contacts a curve at a single point and extends in the same direction as the slope of the curve at the point.

A plane mirror can be used to draw a tangent to a curved line. Place the mirror as perpendicular as possible to the line at the point desired. Adjust the angle of the mirror so that the real curve merges smoothly with its image in the mirror, which will occur when the mirror is perpendicular to the curved line at that point. Draw a line perpendicular to the mirror to obtain the tangent to the curve.


Figure 6 Position-time graph with non-constant velocity
We can use Figure 6 to determine the instantaneous velocity of the object at any specific point in time. To determine the instantaneous velocity, we must determine the slope of the tangent of the line on the position-time graph at that specific time.

## Sample Problem 1: Determining Instantaneous Velocity

Consider the point on the curve in Figure 6 at 2.0 s on the $x$-axis.
What is the instantaneous velocity of the object at this time?
Given: $t=2.0 \mathrm{~s}$; position-time graph
Required: $\vec{v}_{\text {inst }}$
Analysis: $\vec{v}_{\text {inst }}$ is equal to the slope, $m$, of the tangent to the curve at $t=2.0 \mathrm{~s}$, so $m=\frac{\Delta \vec{d}}{\Delta t}$
In Figure 6, the tangent to the point on the curve at $t=2.0 \mathrm{~s}$ has been extended until it crosses a convenient grid line. Calculate the slope of the tangent to determine the instantaneous velocity.

Solution: $m=\frac{8.0 \mathrm{~m}[\mathrm{E}]}{2.0 \mathrm{~s}}$

$$
\vec{v}_{\text {inst }}=4.0 \mathrm{~m} / \mathrm{s}[\mathrm{E}]
$$

Statement: The instantaneous velocity of the object at 2.0 s is $4.0 \mathrm{~m} / \mathrm{s}$ [ E ].

Since the object is accelerating, if we had calculated the slope of the tangent at time $t=1.0 \mathrm{~s}$, the velocity would have been smaller in magnitude, and if we had calculated the slope of the tangent at time $t=3.0 \mathrm{~s}$, the velocity would have been greater. We can also use Figure 6 to determine the average velocity of the object.

Sample Problem 2: Determining Average Velocity from a Position-Time Graph
What is the average velocity of the object in Figure 6 over the time interval from 0.0 s to 2.0 s ?
Given: $\vec{d}_{1}=0.0 \mathrm{~m} ; \vec{d}_{2}=4.0 \mathrm{~m}[\mathrm{E}] ; t_{1}=0.0 \mathrm{~s} ; t_{2}=2.0 \mathrm{~s}$
Required: $\vec{v}_{\text {av }}$
Analysis: Recall that average velocity is the total displacement of an object divided by the total time taken.

$$
\begin{aligned}
& \vec{v}_{\mathrm{av}}=\frac{\Delta \vec{d}}{\Delta t} \\
& \vec{v}_{\mathrm{av}}=\frac{\vec{d}_{2}-\vec{d}_{1}}{t_{2}-t_{1}}
\end{aligned}
$$

Solution: $\vec{v}_{\mathrm{av}}=\frac{\vec{d}_{2}-\vec{d}_{1}}{t_{2}-t_{1}}$

$$
\begin{aligned}
& =\frac{4.0 \mathrm{~m}[\mathrm{E}]-0.0 \mathrm{~m}}{2.0 \mathrm{~s}-0.0 \mathrm{~s}} \\
\overrightarrow{\mathrm{v}}_{\mathrm{av}} & =2.0 \mathrm{~m} / \mathrm{s}[\mathrm{E}]
\end{aligned}
$$

Statement: The average velocity of the object over the time interval from 0.0 s to 2.0 s is $2.0 \mathrm{~m} / \mathrm{s}$ [ E .

## Practice

1. (a) Determine the instantaneous velocity at $t=1.0 \mathrm{~s}$ for the graph shown in Figure 6 . [ans: $2.0 \mathrm{~m} / \mathrm{s}$ [ E$]$ ]
(b) Determine the instantaneous velocity at $t=3.0 \mathrm{~s}$ for the graph shown in Figure 6. [ans: $6.0 \mathrm{~m} / \mathrm{s}$ [ E$]$ ]
(c) Compare your answers to (a) and (b) with the solution to Sample Problem 1 above. Is it possible that the object is moving with constant acceleration? Explain.
2. (a) Determine the instantaneous velocity at $t=5.0 \mathrm{~s}$ for the graph shown in Figure 7. [ans: $30 \mathrm{~m} / \mathrm{s}$ [E]]
(b) Determine the average velocity from $t=0 \mathrm{~s}$ to $t=10.0 \mathrm{~s}$ for the graph Figure 7.
[ans: $30 \mathrm{~m} / \mathrm{s}$ [ E$]$ ]
(c) Notice that for the motion described in Figure $7, t=5.0 \mathrm{~s}$ is the midpoint in time. Write a statement describing the relationship that exists between the average velocity and the instantaneous velocity at the midpoint in time when an object is accelerating uniformly. Tㅔ


Figure 7

Note that in Tutorial 4, the average velocity over the first 2.0 s of the motion was less than the instantaneous velocity at 2.0 s . For motion with non-uniform velocity but uniform acceleration, average and instantaneous velocities are not necessarily equal. The only situation where the average velocity and the instantaneous velocity are the same is at the midpoint in the time interval.

### 1.3 Summary

- Acceleration describes change in velocity over time.
- The slope of a velocity-time graph gives the acceleration of the object whose motion it describes.
- The area under a velocity-time graph gives the displacement of the object whose motion it describes.
- The instantaneous velocity of an object is its velocity at a specific instant in time. It is equal to the slope of the tangent to the position-time graph at that instant in time.
- For motion with non-uniform velocity, average and instantaneous velocities are not necessarily equal.


### 1.3 Questions

1. Describe three characteristics that an accelerating object may exhibit. Give a real-world example of each characteristic. kvo co
2. Describe, in your own words, how you would determine the acceleration of an object from a velocity-time graph. K/vic
3. Describe, in your own words, how you would determine the displacement of an object from a velocity-time graph. 티니
4. Determine the average acceleration described by each of the following graphs.
(a)

(b)

Velocity v. Time for Accelerated Motion

(C)

5. One of your classmates makes the following statement: "If an object has an initial velocity of $10 \mathrm{~m} / \mathrm{s}[\mathrm{N}]$ and a final velocity of $10 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$, this object has clearly not accelerated, as it is travelling at a constant speed." Write an email to this student explaining why this statement is incorrect. 틴 C
6. (a) Describe the motion of the object in all three segments of the graph shown in Figure 8.
(b) Calculate the average acceleration of the object in all three segments of the graph in Figure 8.
(c) Calculate the total displacement of the object from 0 s to 4.0 s , from 4.0 s to 7.0 s , and from 7.0 s to 10.0 s . 제 c

Velocity v. Time for Complex Motion


Figure 8
$t(s)$
7. What is the average acceleration of a sports car that increases its velocity from $2.0 \mathrm{~m} / \mathrm{s}$ [W] to $4.5 \mathrm{~m} / \mathrm{s}$ [W] in 1.9 s?
8. If a child on a bicycle can accelerate at an average rate of $0.53 \mathrm{~m} / \mathrm{s}^{2}$, how long would it take to increase the bicycle's velocity from $0.68 \mathrm{~m} / \mathrm{s}[\mathrm{N}]$ to $0.89 \mathrm{~m} / \mathrm{s}[\mathrm{N}]$ ?
9. (a) While approaching a red light, a student driver begins to apply the brakes. If the car's brakes can cause an average acceleration of $2.90 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{~S}]$ and it takes 5.72 s for the car to come to rest, what was the car's initial velocity?
(b) What is the significance of the direction of the initial velocity and that of the acceleration?
10. What is the average acceleration of a tennis ball that has an initial velocity of $6.0 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$ and a final velocity of $7.3 \mathrm{~m} / \mathrm{s}[\mathrm{W}]$, if it is in contact with a tennis racket for 0.094 s ?
11. (a) Determine the instantaneous velocity at $t=6.0 \mathrm{~s}$ in Figure 9.
(b) Determine the average velocity of the motion depicted in Figure 9.

Position v. Time for Accelerated Motion


Figure 9

