Circuit Analysis

Circuit analysis is a method of solving problems related to electric circuits which combines Ohm's law, Kirchhoff's laws, and the problem-solving techniques you learned for finding equivalent resistance. The Tutorials in this section will provide possible methods for solving circuit problems. There is always more than one way to solve a circuit problem, but the method you choose must follow the circuit laws.

Tutorial 1 Circuit Analysis for a Mixed Circuit

There are two general types of circuit analysis problems. In the first type, you are given only the resistance values and a source voltage. In the second type, you are given a mix of current, voltage, and resistance values. The following Sample Problems will take you through both types of problem. (Hint: When you get to a point where you have solved for two of the three variables, use Ohm's law.)

CASE 1: RESISTANCE VALUES ARE GIVEN

In this type of circuit analysis problem, you are given only the resistance values and a source voltage.

Sample Problem 1

The circuit shown in **Figure 1** has a source voltage of 12.0 V and resistance values of $R_1 = 15.0\ \Omega$, $R_2 = 25.0\ \Omega$, and $R_3 = 35.0\ \Omega$. Find values for I_{source}, I_1, I_2, I_3, V_1, V_2, V_3, and R_{total}.

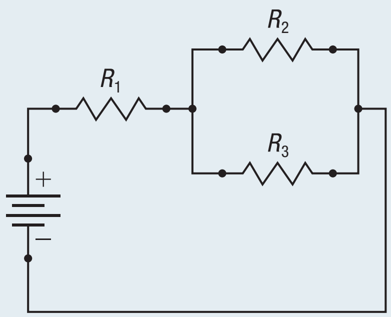


Figure 1

Step 1. Find the total resistance of the circuit. Start by finding the equivalent resistance for the parallel part of the circuit (**Figure 2**).

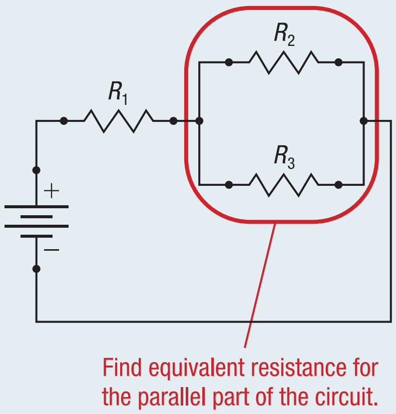


Figure 2

$$\frac{1}{R_{parallel}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{parallel}} = \frac{1}{25.0\ \Omega} + \frac{1}{35.0\ \Omega}$$

$R_{parallel} = 14.583\ \Omega$ (two extra digits carried)

Now find the total resistance.

$R_{parallel}$ is in series with R_1, so

$$\begin{aligned} R_{total} &= R_1 + R_{parallel} \\ &= 15.0\ \Omega + 14.583\ \Omega \\ R_{total} &= 29.583\ \Omega \text{ (two extra digits carried)} \end{aligned}$$

Step 2. Find I_{source} using Ohm's law written as $I = \frac{V}{R}$.

$$\begin{aligned} I_{source} &= \frac{V_{source}}{R_{total}} \\ &= \frac{12.0\ \text{V}}{29.583\ \Omega} \\ I_{source} &= 0.4056\ \text{A} \end{aligned}$$

Step 3. Apply KCL to find I_1. Note that the source is in series with I_1 and the parallel part $I_{parallel}$.

$$I_{series} = I_1 = I_2 = I_3 = \cdots$$

$I_{series} = I_{source} = I_1 = I_{parallel} = 0.4056$ A (two extra digits carried)

Step 4. Find V_1 using Ohm's law written as $V = IR$.

$$\begin{aligned} V_1 &= I_1R_1 \\ &= (0.4056\ \text{A})(15.0\ \Omega) \\ V_1 &= 6.084\ \text{V (two extra digits carried)} \end{aligned}$$

Step 5. Apply KVL to find $V_{parallel}$.

$$V_{series} = V_1 + V_2 + V_3 + \cdots$$
$$V_{source} = V_1 + V_{parallel}$$
$$V_{parallel} = V_{source} - V_1$$
$$V_{parallel} = 12.0\text{ V} - 6.084\text{ V}$$
$$V_{parallel} = 5.916\text{ V}$$

Step 6. Apply KVL to find V_2 and V_3.

$$V_{parallel} = V_1 = V_2 = V_3 = \cdots$$

$$V_{parallel} = V_2 = V_3 = 5.916\text{ V (two extra digits carried)}$$

Step 7. Find I_2 and I_3 using Ohm's law written as $I = \frac{V}{R}$.

$$I_2 = \frac{V_2}{R_2}$$
$$= \frac{5.916\text{ V}}{25.0\ \Omega}$$
$$I_2 = 0.2366\text{ A (two extra digits carried)}$$

$$I_3 = \frac{V_3}{R_3}$$
$$= \frac{5.916\text{ V}}{35.0\ \Omega}$$
$$I_3 = 0.169\text{ A}$$

Step 8. Record your final answers using the correct number of significant digits. Look back at the circuit and see if the values you have calculated coincide with Kirchhoff's laws (**Figure 3**).

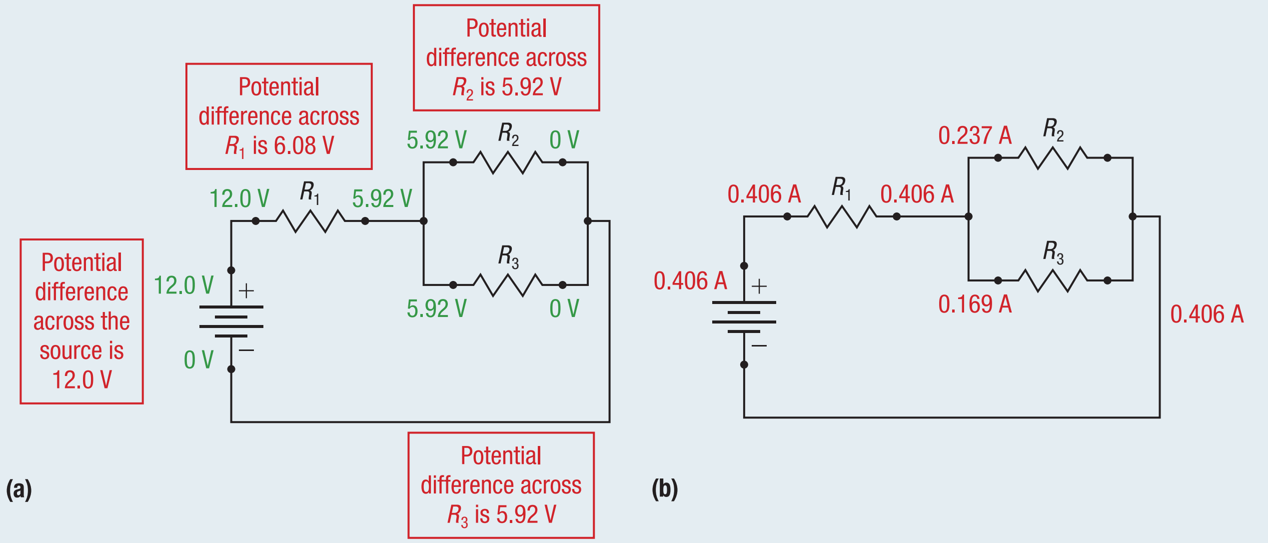


Figure 3 (a) The electric potential energies associated with the electrons are represented in green. We chose a reference point of 0 V. The red boxes represent the voltage across each point in the circuit. In each complete path, the sum of the voltage gains (12.0 V) equals the sum of the voltage drops (6.08 V + 5.92 V). Therefore, you have solved the problem correctly.
(b) The red values represent the current at various points in the circuit. The only junction is where the current splits into R_2 and R_3. The current going into the junction is 0.406 A. The current coming out is also 0.406 A. The current in each path of the parallel part of the circuit must add up to 0.406 A. A check of the values (0.237 A + 0.169 A = 0.406 A) shows that the current in the parallel part of the circuit adds up to 0.406 A.

Practice

1. Repeat Sample Problem 1 with the following values: $V_{source} = 40.0$ V, $R_1 = 25.0\ \Omega$, $R_2 = 30.0\ \Omega$, and $R_3 = 30.0\ \Omega$. T/I
   [ans: $R_{total} = 40.0\ \Omega$; $I_{source} = 1.00$ A; $I_1 = 1.00$ A; $I_2 = 0.500$ A; $I_3 = 0.500$ A; $V_1 = 25.0$ V; $V_2 = 15.0$ V; $V_3 = 15.0$ V]

CASE 2: ONLY SOME RESISTANCE VALUES ARE GIVEN

This is the type of circuit analysis problem where you are missing various values. You will use Ohm's law and Kirchhoff's laws to solve this circuit problem.

Sample Problem 2

The circuit shown in **Figure 4** has V_{source} = 12.0 V, I_1 = 0.50 A, V_3 = 2.5 V, V_4 = 5.0 V, and R_3 = 10.0 Ω. Find I_{source}, I_2, I_3, I_4, V_1, V_2, R_1, R_2, R_4, and R_{total}.

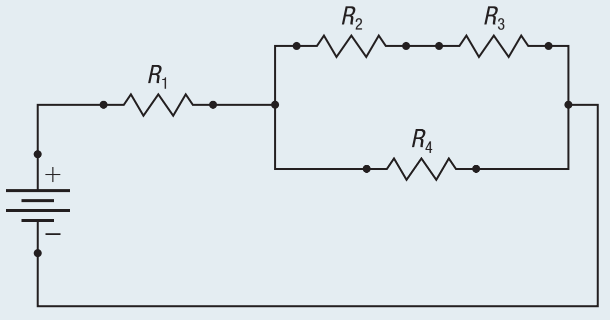


Figure 4

Step 1. Apply KVL to any *complete* pathway. In this case, one complete pathway involves the source, resistor 1, and resistor 4.

$$V_{source} = V_1 + V_4$$
$$V_1 = V_{source} - V_4$$
$$V_1 = 12.0\text{ V} - 5.0\text{ V}$$
$$V_1 = 7.0\text{ V}$$

Step 2. Apply KVL to another *complete* pathway. In this case, another complete pathway involves the source, resistor 1, resistor 2, and resistor 3.

$$V_{source} = V_1 + V_2 + V_3$$
$$V_2 = V_{source} - V_1 - V_3$$
$$V_2 = 12.0\text{ V} - 7.0\text{ V} - 2.5\text{ V}$$
$$V_2 = 2.5\text{ V}$$

Step 3. Find I_3 using Ohm's law written as $I = \frac{V}{R}$.

$$I_3 = \frac{V_3}{R_3}$$
$$= \frac{2.5\text{ V}}{10.0\ \Omega}$$
$$I_3 = 0.25\text{ A}$$

Step 4. Apply KCL to find the missing current values. Note that $I_{2,3}$ represents the current going through the path that contains I_2 and I_3.

Find I_{source}: $I_{source} = I_1$

$$= 0.50\text{ A}$$

Find I_2: $I_{series} = I_2$

$$= I_3$$
$$= 0.25\text{ A}$$
$$I_2 = 0.25\text{ A}$$

Find I_4: $I_{source} = I_{2,3} + I_4$

$$I_4 = I_{source} - I_{2,3}$$
$$I_4 = 0.50\text{ A} - 0.25\text{ A}$$
$$I_4 = 0.25\text{ A}$$

Step 5. Find all other missing values using Ohm's law.

$$R_1 = \frac{V_1}{I_1}$$
$$= \frac{7.0\text{ V}}{0.50\text{ A}}$$
$$R_1 = 14\ \Omega$$

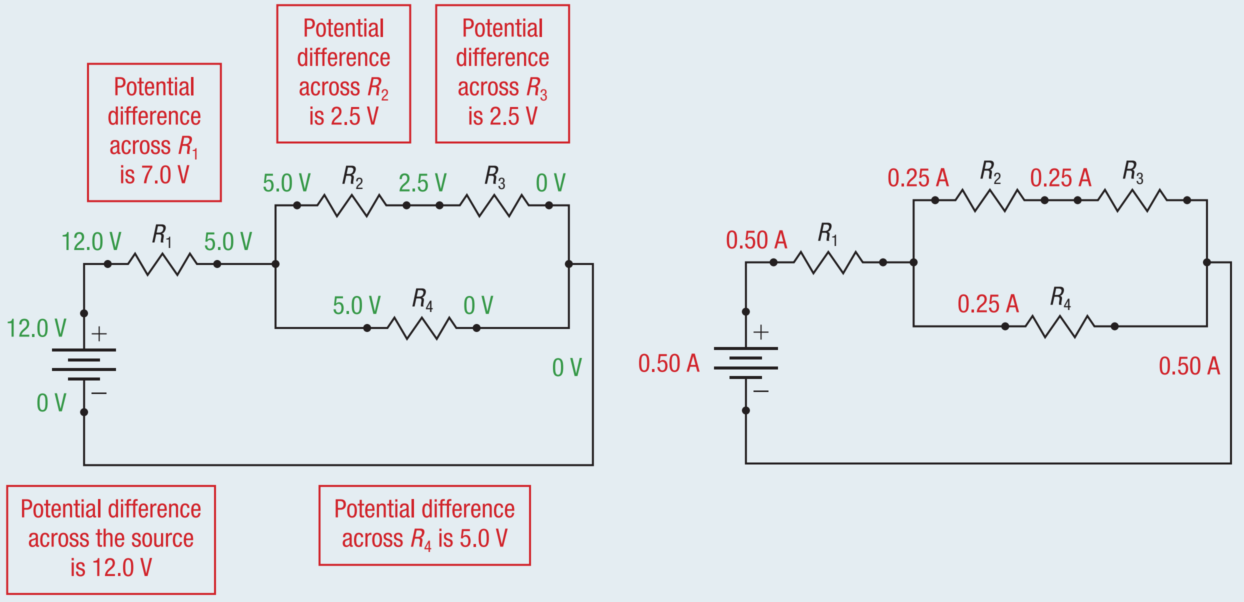
$$R_2 = \frac{V_2}{I_2}$$
$$= \frac{2.5\text{ V}}{0.25\text{ A}}$$
$$R_2 = 10\ \Omega$$

$$R_4 = \frac{V_4}{I_4}$$
$$= \frac{5.0\text{ V}}{0.25\text{ A}}$$
$$R_4 = 20\ \Omega$$

$$R_{total} = \frac{V_{source}}{I_{source}}$$
$$= \frac{12.0\text{ V}}{0.50\text{ A}}$$
$$R_{total} = 24\ \Omega$$

Step 6. Record your final answers with the correct number of significant digits.

Now that you have finished the problem, you can look back at the circuit and see if the values you have calculated coincide with Kirchhoff's laws (**Figure 5**).



(a) **(b)**

Figure 5 (a) The electric potential energies associated with the electrons are represented in green. We chose a reference point of 0 V. The red boxes represent the electric potential difference (or voltage) across the electric circuit parts. In one complete path, the sum of the voltage gains (12.0 V) equals the sum of the voltage drops (7.0 V + 2.5 V + 2.5 V). In the other complete path, the sum of the voltage gains (12.0 V) equals the sum of the voltage drops (7.0 V + 5.0 V). Therefore, you have solved the problem correctly.
(b) The red values represent the current at various points in the circuit. The only junction is at the parallel part where the current splits into $R_{2,3}$ (R_2 and R_3 together) and R_4. The current going into the junction is 0.50 A. The current coming out is also 0.50 A. The currents in both paths of the parallel part of the circuit must add up to 0.50 A. A check of the values (0.25 A + 0.25 A = 0.50 A) shows that they do. Note that the current in the two resistors connected in series (R_2 and R_3) stays constant.

Practice

1. Repeat Sample Problem 2 with the following values: $V_{source} = 42.0$ V, $I_1 = 1.75$ A, $V_2 = 8.75$ V, $V_4 = 17.5$ V, and $R_2 = 35.0\ \Omega$. Find I_{source}, I_2, I_3, I_4, V_1, V_3, R_1, R_3, R_4, and R_{total}. T/I
[ans: $I_{source} = 1.75$ A; $I_2 = 0.250$ A; $I_3 = 0.250$ A; $I_4 = 1.50$ A; $V_1 = 24.5$ V; $V_3 = 8.75$ V; $R_1 = 14.0\ \Omega$; $R_3 = 35.0\ \Omega$; $R_4 = 11.7\ \Omega$; $R_{total} = 24.0\ \Omega$]

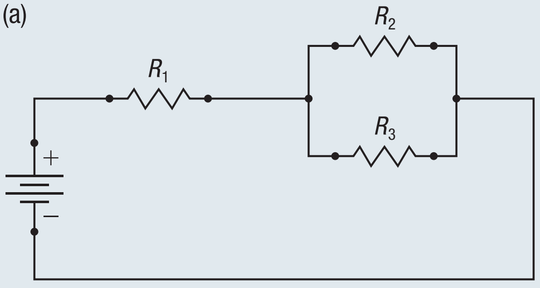
11.9 Summary

- Circuits can be analyzed using a combination of equivalent resistance, Kirchhoff's laws, and Ohm's laws.

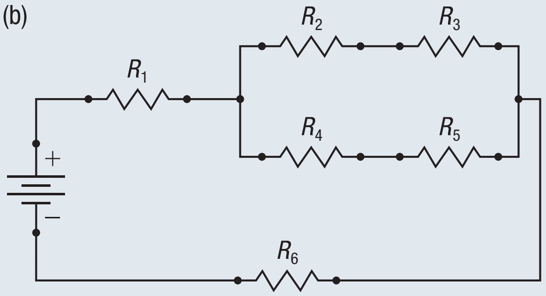
11.9 Questions

1. For each of the circuit diagrams below, the source has a voltage of 6.0 V. Each resistor has resistance 12.0 Ω. Find all the other values of current, voltage, and resistance. T/I

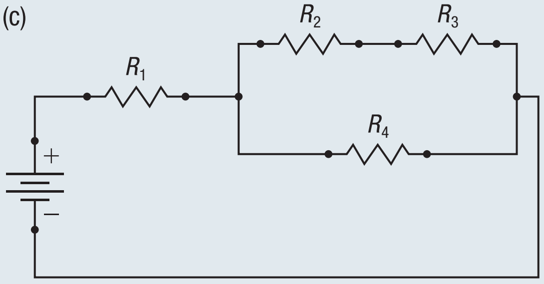
(a)



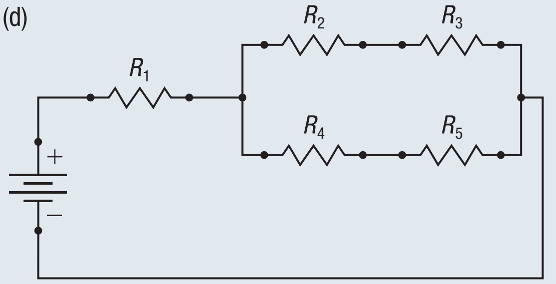
(b)



(c)



(d)



2. Draw a circuit diagram with a voltage source and three resistors connected in series. $V_{source} = 15.0$ V, $V_2 = 4.0$ V, $R_1 = 30.0\ \Omega$, and $I_2 = 0.20$ A. Find V_1, V_3, I_1, I_3, I_{source}, R_2, R_3, and R_{total}. T/I C
3. Draw a circuit diagram with a voltage source and three resistors connected in parallel. $V_{source} = 1.5$ V, $I_1 = 0.10$ A, $R_2 = 7.5\ \Omega$, and $R_3 = 5.0\ \Omega$. Find V_1, V_2, V_3, I_2, I_3, I_{source}, R_1, and R_{total}. T/I C
4. For the circuit diagram shown in **Figure 6**, $V_1 = 2.5$ V, $V_3 = 5.0$ V, $I_2 = 0.30$ A, $I_3 = 0.50$ A, $I_4 = 0.10$ A, and $R_4 = 70.0\ \Omega$. Find V_{source}, V_2, V_4, V_5, I_{source}, I_1, I_5, R_1, R_2, R_3, R_5, and R_{total}. T/I

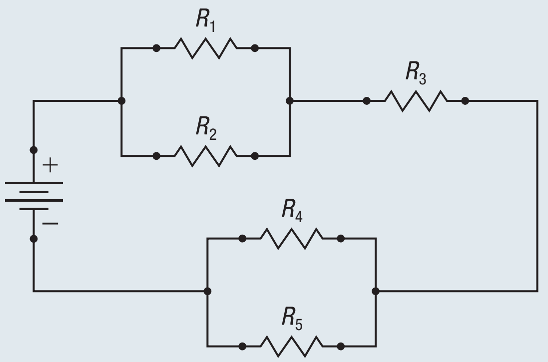


Figure 6