Resistors in Circuits

You have just bought a ticket for an amusement park. There is a lineup and you do not want to waste time—you want to get on the rides. Unfortunately, there is only one turnstile open. In this case, there is a lot of resistance to the flow of people into the park. You can think of the turnstiles as resistors and the flow of people into the park as electric current.

Connecting Resistors in Series

Imagine adding another turnstile in front of the one you just passed through. Now everyone needs to pass through two turnstiles, one after the other. This slows down the flow of people into the park, which means that there is more resistance to the overall flow. The situation in an electric circuit is similar. Adding more resistors to a series circuit decreases the current in the circuit. The decreased current is a result of the increased resistance.

To find out how much the current is affected by increasing the resistance, consider the circuit shown in **Figure 1**. In this circuit, the three resistors can be reduced to a single resistor with a resistance equivalent to the total resistance of the three resistors. **Equivalent resistance** is the total resistance of a group of resistors in a circuit. You can use Kirchhoff's laws and Ohm's law to derive the equivalent resistance for resistors in a series circuit. Start with KVL for a series circuit:

$$V_{series} = V_1 + V_2 + V_3$$

Substitute Ohm's Law in the form $V = IR$:

$$I_{series}R_{series} = I_1R_1 + I_2R_2 + I_3R_3$$

In a series circuit, the current is constant and the same at all points (KCL). So the currents on the left side will cancel with the currents on the right side:

$$I_{series}R_{series} = I_1R_1 + I_2R_2 + I_3R_3$$
$$I_{series}R_{series} = I_{series}R_1 + I_{series}R_2 + I_{series}R_3$$
$$R_{series} = R_1 + R_2 + R_3$$

Therefore, in a series circuit the equivalent resistance is given by

$$R_{series} = R_1 + R_2 + R_3 + \cdots$$

So the three resistors can be reduced to a single resistor with a value equivalent to the sum of the three resistances. In the following Tutorial, you will apply the equivalent resistance equation to solve for missing values in a series circuit.
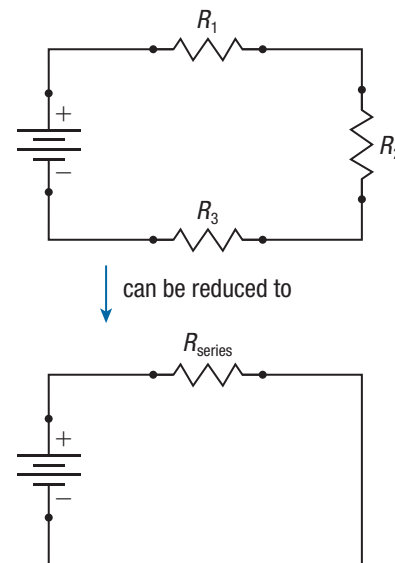
can be reduced to

Figure 1 Three resistors connected in series can be reduced to a single resistor with an equivalent value of R_{series}.

equivalent resistance the total resistance of a group of resistors connected in series or parallel

---

Tutorial 1 | Equivalent Resistance in a Series Circuit

Sample Problem 1

Four resistors are connected in series in a circuit. The resistances are as follows: $R_1 = 41\ \Omega$, $R_2 = 51.75\ \Omega$, $R_3 = 11.1\ \Omega$, and $R_4 = 102.008\ \Omega$. Calculate the equivalent resistance.

Given: $R_1 = 41\ \Omega$; $R_2 = 51.75\ \Omega$; $R_3 = 11.1\ \Omega$; $R_4 = 102.008\ \Omega$

Required: R_{series}

Analysis: $R_{series} = R_1 + R_2 + R_3 + R_4$

Solution: $R_{series} = R_1 + R_2 + R_3 + R_4$
$$= 41\ \Omega + 51.75\ \Omega + 11.1\ \Omega + 102.008\ \Omega$$
$$R_{series} = 206\ \Omega$$

Statement: The equivalent resistance is 206 Ω.

Practice

1. What is the equivalent resistance for a 25.2 Ω resistor connected in series with a 28.12 Ω resistor? ᴛ/ɪ [ans: 53.3 Ω]

2. What is the equivalent resistance for three 53.0 Ω resistors connected in series? ᴛ/ɪ [ans: 159 Ω]

Connecting Resistors in Parallel

can be reduced to
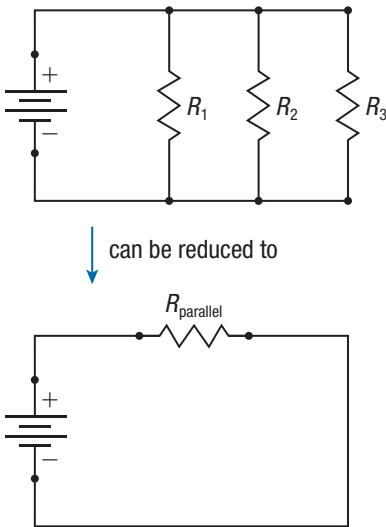
$R_{parallel}$

Figure 2 Three resistors connected in parallel can be reduced to a single resistor with an equivalent value of $R_{parallel}$.

Think about the amusement park turnstiles again. If enough turnstiles were open, each person could have their own turnstile to go through. This would greatly increase the flow of people into the park. If the flow of people into the park has increased, there must be a decrease in the resistance to overall flow. Even though each individual has experienced resistance in going through one turnstile, the overall flow of people into the amusement park has increased. A similar situation occurs when resistors are connected in parallel. The more resistors you connect in parallel, the greater the current. Therefore, the total resistance must decrease.

To understand why this occurs, consider the increase in the number of pathways when you connect resistors in parallel. With each additional parallel connection, you increase the number of pathways for the electric current. If there are more pathways, you would expect more electric current.

You can derive an equivalent resistance equation for resistors in a parallel circuit. Start with KCL for the parallel circuit shown in **Figure 2:**

$$I_{parallel} = I_1 + I_2 + I_3$$

Substitute Ohm's law in the form $I = \dfrac{V}{R}$:

$$\frac{V_{parallel}}{R_{parallel}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

In a parallel circuit, the voltage is constant and the same at all points. Using KVL, the voltages on the left side will cancel with the voltages on the right side:

$$\frac{V_{parallel}}{R_{parallel}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{\cancel{V_{parallel}}}{R_{parallel}} = \frac{\cancel{V_{parallel}}}{R_1} + \frac{\cancel{V_{parallel}}}{R_2} + \frac{\cancel{V_{parallel}}}{R_3}$$

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Therefore, in a parallel circuit the equivalent resistance is given by

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Tutorial 2 / Equivalent Resistance in a Parallel Circuit

Sample Problem 1

Three resistors are connected in parallel in a circuit. The resistances are $R_1 = 15\ \Omega$, $R_2 = 12\ \Omega$, and $R_3 = 10\ \Omega$. Calculate the equivalent resistance.

Given: $R_1 = 15\ \Omega$; $R_2 = 12\ \Omega$; $R_3 = 10\ \Omega$

Required: $R_{parallel}$

Analysis: $\dfrac{1}{R_{parallel}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

Solution: $\dfrac{1}{R_{parallel}} = \dfrac{1}{15\ \Omega} + \dfrac{1}{12\ \Omega} + \dfrac{1}{10\ \Omega}$

$\dfrac{1}{R_{parallel}} = \dfrac{4}{60\ \Omega} + \dfrac{5}{60\ \Omega} + \dfrac{6}{60\ \Omega}$ ← Find a common denominator.

$\dfrac{1}{R_{parallel}} = \dfrac{15}{60\ \Omega}$

$R_{parallel} = \dfrac{60\ \Omega}{15}$

$R_{parallel} = 4\ \Omega$ ← Find the reciprocal to solve for $R_{parallel}$.

Statement: The equivalent resistance is $4\ \Omega$.

Practice

1. What is the equivalent resistance of a 120 Ω resistor connected in parallel with a 60 Ω resistor? **T/I** [ans: 40 Ω]

2. What is the equivalent resistance of four 20 Ω resistors connected in parallel? **T/I** [ans: 5 Ω]
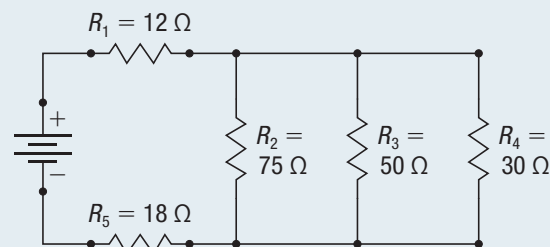
   Mixed circuits contain both series and parallel connections. As you learned in Section 11.6, to analyze a mixed circuit, you can divide the circuit into sections that are connected in parallel and sections that are connected in series. In the following Tutorial, you will use this strategy to find the equivalent resistance in a circuit with both series and parallel connections.

Tutorial 3 / Equivalent Resistance in a Mixed Circuit

Mixed circuits contain both series and parallel connections. To determine the equivalent resistance, work on each part of the circuit separately.
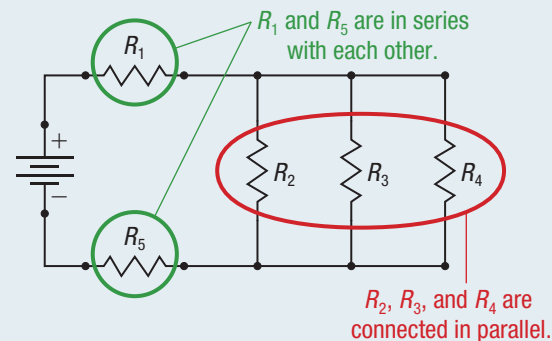
Sample Problem 1

Calculate the equivalent resistance for the circuit shown in **Figure 3**.

Figure 3

**Step 1.** Divide the circuit into series and parallel parts, as shown in **Figure 4**.

R_1 and R_5 are in series with each other.

R_2, R_3, and R_4 are connected in parallel.

Figure 4

Step 2. Find the equivalent resistance of the parallel part of the circuit.

$$\frac{1}{R_{parallel}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_{parallel}} = \frac{1}{75 \ \Omega} + \frac{1}{50 \ \Omega} + \frac{1}{30 \ \Omega}$$

$$\frac{1}{R_{parallel}} = \frac{2}{150 \ \Omega} + \frac{3}{150 \ \Omega} + \frac{5}{150 \ \Omega}$$

$$\frac{1}{R_{parallel}} = \frac{10}{150 \ \Omega}$$

$$R_{parallel} = \frac{150 \ \Omega}{10}$$

$$R_{parallel} = 15 \ \Omega$$

Step 3. Redraw the circuit using the equivalent resistance from Step 2 (**Figure 5**).
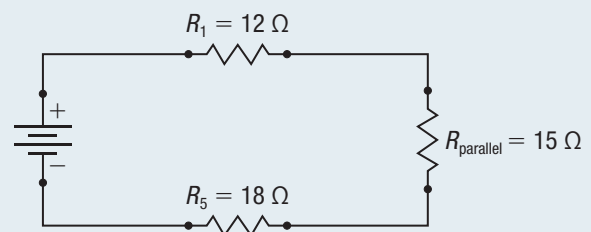
Figure 5

Step 4. Solve to determine the equivalent resistance of the remaining series circuit. Let the equivalent resistance for the complete circuit be R_{total}:

$$R_{total} = R_1 + R_{parallel} + R_5$$
$$= 12 \ \Omega + 15 \ \Omega + 18 \ \Omega$$
$$R_{total} = 45 \ \Omega$$

Statement: The equivalent resistance is 45 Ω.

Practice

1. What is the total resistance of the mixed circuits shown in **Figure 6**? Note that each resistor has resistance 5.0 Ω. T/I [ans: (a) 17.5 Ω; (b) 6.3 Ω]
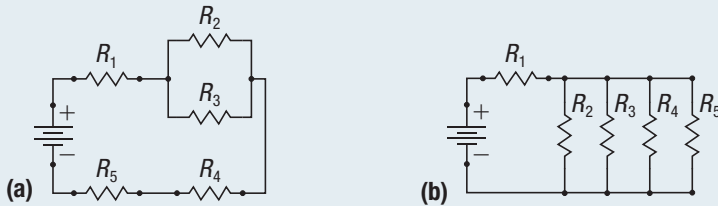

(a)   (b)

Figure 6

11.8 Summary

- Connecting resistors in series causes an increase in the total resistance and a decrease in the current.
- A group of resistors connected in series can be reduced to a single equivalent resistor with resistance $R_{series} = R_1 + R_2 + R_3 + \cdots$.
- Connecting resistors in parallel causes a decrease in the total resistance and an increase in current.
- A group of resistors connected in parallel can be reduced to a single equivalent resistor with resistance $\dfrac{1}{R_{parallel}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots$.

11.8 Questions

1. Using $R_{series} = R_1 + R_2 + R_3 + \cdots$ and Ohm's law, derive Kirchhoff's voltage law for a series circuit. T/I

2. Using $\dfrac{1}{R_{parallel}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots$ and Ohm's law, derive Kirchhoff's current law for a parallel circuit. T/I

3. Prove that the equivalent resistance of two identical resistors in parallel is equal to half the resistance of one of the resistors. T/I

4. Recall that all loads have electrical resistance. Suppose that you connect a number of loads in your home in parallel. What will happen to the amount of electric current with each load that you add? Would you be concerned about this? Explain your answer. K/U C

5. What is the equivalent resistance of the following circuits? Each resistor has resistance 12.0 Ω. K/U T/I

(a)

(b)

(c)

(d)