SOLUTIONS

Name: _____ Date: _____

Worksheets – Optimization of Cylinders (Minimize SA and Maximize Volume)

(9.5) - Key Concepts for Maximizing the volume of a cylinder

- The maximum volume for a given surface area of a cylinder occurs when its height equals its diameter. That is, $h = d$ or $h = 2r$
- The dimensions of the cylinder with maximum volume for a given surface area can be found by solving the formula: $SA = 6\pi r^2$
- and the height will be twice that value, or $2r$
  - Substitute h = 2r into the formula to solve for the SA formula above:

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi r^2 + 2\pi r(2r)$

$SA = 2\pi r^2 + 4\pi r^2$

$SA = 6\pi r^2$

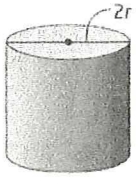
  - Rearrange the formula to solve for height:

$SA = 2\pi r^2 + 2\pi rh$

$SA - 2\pi r^2 = 2\pi rh$

$\frac{SA - 2\pi r^2}{2\pi r} = h$

1 a. Find the dimensions of a cylinder with maximum volume that can be made with 600cm^2 of aluminum. Round the dimensions to the nearest hundredth of a centimetre.



$SA = 6\pi r^2$

$600 = 6\pi r^2$

$\frac{600}{6} = \pi r^2$

$100 = \pi r^2$

$\sqrt{\frac{100}{\pi}} = r$

$5.64 = r$

∴ radius is 5.64cm, the height is 2r

$2(5.64) = 11.28$cm

b. What is the volume of this cylinder, to the nearest cubic centimetre?

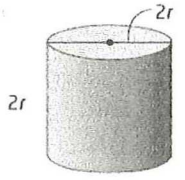
$V = \pi r^2 h$

$V = \pi (5.64)^2 (11.28)$

$V = 1127\text{cm}^3$

(9.6) - Minimize the Surface Area of a Cylinder

- The minimum surface area for a given volume of a cylinder occurs when its height equals its ___diameter___. That is, ___h=d___ or ___h=2r___
- The dimensions of the cylinder of minimum surface area for a given volume can be found by solving the formula: $V = 2\pi r3$
- and the height will be ___twice___ that value, or ___2r___
- Substitute h = 2r into the formula to solve for the V formula above:



$V = \pi r^2 h$

$V = \pi r^2 (2r)$

$V = 2\pi r^3$

2 a. Determine the least amount of aluminum required to construct a cylindrical can with a 1 litre capacity, to the nearest square centimetre.

$1L = 1000 cm^3 \rightarrow$ Volume

$\frac{1000}{2\pi} = \frac{2\pi r^3}{2\pi}$

$\frac{500}{\pi} = r^3$

$\sqrt[3]{\frac{500}{\pi}} = r$

$5.42 = r$

height is $2r$

$\therefore 10.84 cm$

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(5.42)^2 + 2\pi(5.42)(10.84)$

$SA = 554 cm^2$

↳ least amount of aluminum needed to hold 1L

b. Describe any assumptions made.

- do not take into account extra aluminum required for the seam along the lateral surface
- top / bottom edges, more likely a rim