Optimization in 3D – part 1 cylinder

You have been hired by Husky Cola to design a new pop can. Husky wants the can to have a volume of 355 cm^3. Your job is to find the dimensions (radius and height) of the pop can that would minimize the amount of aluminum needed to make the can.

$$A = 2\pi rh + 2\pi r^2 \qquad V = \pi r^2 h$$

$V = 355 cm^3$
$r = 2 cm$

1. Suppose that the radius of your pop can were 2 centimetres. What would be the height?
   a. Rearrange the formula for Volume to solve it for h, the height.

   $V = \pi r^2 h$

   $\boxed{\frac{V}{\pi r^2} = h}$

   b. Substitute 355 cm^3 for V and 2 cm for r in the formula and solve for h.

   $\frac{355}{\pi (2)^2} = h$

   $\boxed{h = 28.25 cm}$

2. Using a radius of 2 cm and the height that you calculated above, calculate the surface area of the can.

   $SA = 2\pi rh + 2\pi r^2$
   $SA = 2\pi(2)(28.25) + 2\pi(2)^2$
   $\boxed{SA = 380.13 cm^2}$

3. Repeat this process (find h, then find A) to complete the table for different values of r.

Radius (cm)	Height (cm)	Surface Area (cm^2)
2.0	28.25	380.13
2.5	18.08	323.27
3.0	12.56	293.30
3.5	9.22	279.72
4.0	7.06	277.96
4.5	5.58	285.00
5.0	4.52	299.07

4. Make a prediction about how the radius should relate to the height of a cylinder in order to minimize the surface area.

   when the height is twice the radius, the SA will be minimized (the smallest)

Optimization in 3d – part 2 square-based prism

You have been hired by the Husky Juice Company to design a new container. Husky wants the container to have a volume of 355 cm^3. Your job is to find the dimensions (length and height) of a square-based prism container ($w = 1$) that would minimize the amount of cardboard used.

$$A = 21^2 + 41h \qquad V = 1^2 h$$

$V = 355 cm^3$
$l = 5$ cm

1. Suppose that the length of your juice container were 5 centimetres. What would be the height?
   a. Rearrange the formula for Volume to solve it for h, the height.

   $V = l^2 \cdot h$

   $\boxed{\frac{V}{l^2} = h}$

   b. Substitute 355 cm^3 for V and 5 cm for 1 in the formula and solve for h.

   $\frac{355}{5^2} = h$

   $\boxed{h = 14.2 \text{ cm}}$

2. Using a length of 5 cm and the height that you calculated above, calculate the surface area of the container.

   $SA = 2l^2 + 4lh$
   $SA = 2(5)^2 + 4(5)(14.2)$
   $\boxed{SA = 334 \text{ cm}^2}$

3. Repeat this process (find h, then find A) to complete the table for different values of 1.

Length (cm)	Height (cm)	Surface Area (cm^2)
5.0	14.2	334
5.5	11.73	318.56
6.0	9.86	308.64
6.5	8.40	302.9
7.0	7.24	300.72
7.5	6.31	301.8
8.0	5.54	305.28

4. Make a prediction about how the length should relate to the height of a square-based prism in order to minimize the surface area.

   when the height is equal to the length, the SA will be minimized (the smallest)