1. Pythagorean Theorem

2. Perimeter \& Area of Composite Figures
a) find area of given shape


| Radius $=64 / 2=32 \mathrm{~m}$ | $\begin{aligned} \text { Area of rectangle } & =1 w \\ & =100 \times 64 \end{aligned}$ |
| :---: | :---: |
| Area of circle $=\Pi r^{2}$ | $=6400 \mathrm{~m}^{2}$ |
| $\begin{aligned} & =3.14 \times 32 \times 32 \\ & =3215.36 \mathrm{~m}^{2} \end{aligned}$ |  |
| $\begin{aligned} & \text { Area of track }=\text { Area of rect. }+ \text { Area of circle } \\ &=6400+3215.36 \\ &=9615.36 \mathrm{~m}^{2} \end{aligned}$ |  |

b) find perimeter of triangle


| $a^{2}+b^{2}=c^{2}$ | $P=a+b+c$ |
| :--- | :--- |
| $3^{2}+4^{2}=c^{2}$ | $P=3+4+5$ |
| $9+16=c^{2}$ | $P=12 c m$ |
| $25=c^{2}$ |  |
| $\sqrt{25}=c$ |  |
| $5=c$ |  |

3. Volumes of Prisms, Pyramids, Cylinders, Cones, \& Spheres
a) volume of a cone


$$
\begin{aligned}
& V=\frac{\pi r^{2} h}{3} \\
& V=\frac{3.14 \times 6.2^{2} \times 9.4}{3} \\
& V=\frac{1134.6}{3} \\
& V=378.2 \mathrm{~m}^{3}
\end{aligned}
$$

b) volume of a sphere


$$
\begin{aligned}
& V=\frac{4 \pi r^{3}}{3} \\
& V=\frac{4 \times 3.14 \times 7^{3}}{3} \\
& V=\frac{4308.08}{3} \\
& V=1436.03 \mathrm{~cm}^{3}
\end{aligned}
$$

1. Maximizing the Area of a Rectangle

Rectangles with the same perimeter can have different areas. For example, all these rectangles have a perimeter of 18 cm .

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

If all 4 sides of the rectangle are enclosed, then the maximum area occurs when the length and width are closest in value. If 3 sides of the rectangle are enclosed, then the maximum occurs when the length is twice the width.

A rectangle has a perimeter of 100 cm . Complete the given table by determining the length and area of each given width.

| Width (cm) | Length (cm) | Area ( $\mathrm{cm}^{2}$ ) |
| :---: | :---: | :---: |
| 5 | $\frac{100-2(5)}{2}=45$ | $5 \times 45=225$ |
| 10 | $\frac{100-2(10)}{2}=40$ | 400 |
| 15 | 35 | 525 |
| 20 | 30 | 600 |
| 25 | 25 | 625 |

## 2. Minimizing the Perimeter of a Rectangle

Rectangles with the same area can have different perimeters.
For example, all these rectangles have an area $60 \mathrm{~m}^{2}$.


If all 4 sides of the rectangle are enclosed, then the minimum perimeter occurs when the length and width are closest in value.

1. Types of Relationships
a) weak,
positive, linear
Age and Diameter of Northern White Cedar

b) moderate strength negative linear

2. First Differences
linear relation
$\leftrightarrow \rightarrow$ graph is a straight line
$\leftrightarrow \rightarrow$ first differences are constant

| Length(cm) | Perimeter $(\mathrm{cm})$ | First <br> Differences |
| :---: | :---: | :---: |
| 4 | 14 | ---------- |
| 5 | 16 | $16-14=2$ |
| 6 | 18 | $18-16=2$ |
| 7 | 20 | $20-18=2$ |



Non-linear relation
$\leftrightarrow \rightarrow$ graph is NOT a straight line
$\leftrightarrow$ first differences are NOT constant

| Time (h) | Mass of <br> caffeine $(\mathrm{mg})$ | First Differences |
| :---: | :---: | :---: |
| 0 | 300 | ---------- |
| 6 | 150 | $150-300=-150$ |
| 12 | 75 | $75-150=-75$ |
| 18 | 37.5 | $37.5-75=-37.5$ |
| 24 | 18.75 | $18.75-37.5=-18.75$ |
| 30 | 9.375 | $0.375-18.75=-18.375$ |



1. Direct Variations

A graph that represents direct variation is a straight line that passes through the origin.

Snowboard Rental Cost


| Time <br> $\boldsymbol{t}(\mathrm{h})$ | Cost <br> $\boldsymbol{C}(\$)$ | First <br> Differences |
| :---: | :---: | :---: |
| 0 | 0 | $8-0=8$ |
| 1 | 8 | 8 |
| 2 | 16 | $16-8=8$ |
| 3 | 24 | $24-16=8$ |
| 4 | 32 | $32-24=8$ |

Rate of change $=\frac{\text { rise }}{\text { run }}=\frac{\$ 16}{2 h}=\$ 8 / \mathrm{h}$
The equation is: $C=8 t$
rate of change is $\$ 8 / \mathrm{h}$
2. Partial Variations

A graph that represents partial variation is a straight line that does not pass through the origin.


| Time <br> $t(h)$ | Cost <br> $C(\$)$ | First <br> Differences |
| :---: | :---: | :---: |
| 0 | 160 | $175-160=15$ |
| 1 | 175 |  |
| 2 | 190 | $205-190=15$ |
| 3 | 205 | $220-205=15$ |
| 4 | 220 |  |

Rate of change $=\frac{\text { rise }}{\text { run }}=\frac{\$ 30}{2 h}=\$ 15 / \mathrm{h}$
fixed cost
variable cost
The equation is: $C=160+15 t$ initial value is $\$ 160$

1. Determine the angle measure indicated by each letter. Justify your answer.
a)


$$
\begin{aligned}
& x=y \text { because the triangle is isosceles } \\
& x=\frac{180-42}{2}=69^{\circ} \\
& y=69^{\circ}
\end{aligned}
$$

b)

$b=85^{\circ}$ by the Parallel Line Theorem (corresponding angles, F - Pattern) $a=180-85=95^{\circ}$ by the Parallel Line Theorem (co-interior angles, C-Pattern)
c)

d)

$x=180-127=53^{\circ}$ because supplementary angles add up to $180^{\circ}$
$y=46^{\circ}$ because opposite angles are equal
$z=180-53-46=81^{\circ}$ because the sum of the interior angles of a triangle add to $180^{\circ}$

The octagon has 8 sides therefore the sum of the interior angles is $180 \times 6=1080^{\circ}$.

All the sides are equal so it's a regular octagon which means all the interior angles are equal to each other.
$x=\frac{1080}{8}=135^{\circ}$

1. Ratios

A ratio is a comparison of two quantities with the same units.
Two ratios are equivalent when they can be reduced to the same ratio.
For example, both $12: 16$ and $9: 12$ reduce to $3: 4$.
2. Proportions

A proportion is a statement with two ratios that are equal. For example, $2: 3=10: 15$.
To solve a proportion means to determine the value of an unknown term in a proportion.
a) Solve 2: 10 = 5: $x$

$$
\frac{2}{10}=\frac{5}{x}
$$

$$
\text { cross multiply } \rightarrow 2 x=50
$$

$$
\text { divide both sides by } 2 \rightarrow x=25
$$

3. Rates

A rate is a ratio of two terms with different units. A unit rate is a rate where the second term is 1 unit.
a) Express as 100 kilometres in 2 hours as a unit rate. $\quad \frac{100}{2}=50 \mathrm{~km} / \mathrm{h}$
4. Percents

A percent is a ratio with second term 100. A ratio can be written as a fraction, decimal, or percent.

| Ratio | Fraction | Decimal | Percent |
| :---: | :---: | :---: | :---: |
| $25: 100$ | $\frac{25}{100}$ | 0.25 | $25 \%$ |

a) A winter jacket is regularly priced at $\$ 79.99$. It is on sale for $35 \%$ off. Determine the discount and the sale price.
$\frac{100}{35}=\frac{79.99}{x}$
$100 x=2799.65$
$x=27.9965$
Therefore, the discount is $\$ 28$.
79.99-28 = 51.99 Therefore, the sale price is $\$ 51.99$.

5. Scale Diagrams

A scale is the ratio of the diagram measurement to the actual measurement. scale $=$ diagram measurement : actual measurement
a) Determine the scale used if the actual width of the design is 35 cm .
diagram : actual
$7 \mathrm{~cm}: 35 \mathrm{~cm}$
$=1: 5$ (lowest term) Therefore, the scale is $1: 5$


## MFM1P Summary Notes and Examples - Unit 7 Algebra

## 1. Polynomials

Like terms are represented by the same type of algebra tile (i.e. same variable raised to the same exponents)
$3 x^{2}$ and $-2 x^{2}$ are like terms.


The Distributive Property - Each term in the brackets is multiplied by the term outside the brackets.
a) $\left(3 x^{2}-5 x-1\right)-\left(2 x^{2}-7 x+4\right)$
$=3 x^{2}-5 x-1-2 x^{2}+7 x-4$
$=x^{2}+2 x-5$
b) $-5 x(3 x-4)+2\left(x^{2}-7 x\right)$
$=-15 x^{2}+20 x+2 x^{2}-14 x$
$=-13 x^{2}+6 x$

## 2. Solving Equations

To solve an equation means to determine the value of the variable that makes the equation true.

Solve for each unknown.
a) $2 x+10=4$

$$
2 x+10-10=4-10
$$

$$
2 x=-6
$$

$$
\frac{2 x}{2}=\frac{-6}{2}
$$

$$
x=-3
$$

b) $3 x+3=x+7$
$3 x+3-x=x+7-x$
$2 x+3=7$
$2 x+3-3=7-3$
$2 x=4$
$\frac{2 x}{2}=\frac{4}{2}$
$x=2$

d)


$$
\begin{aligned}
& x+2 x+90+90=360 \\
& 3 x+180=360 \\
& 3 x+180-180=360-180 \\
& 3 x=180 \\
& \frac{3 x}{3}=\frac{180}{3} \\
& x=60^{\circ}
\end{aligned}
$$

