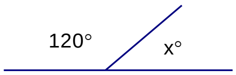
Lesson 1: Basic Angle Relationships

1. SUPPLEMENTARY ANGLES THEOREM (SAT)

Angles that make a straight line add up to 180°.

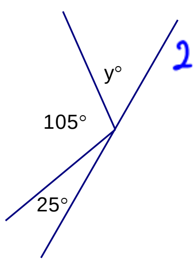
i)



$x + 120 = 180$ (-120, -120)

$\boxed{x = 60°}$

ii)



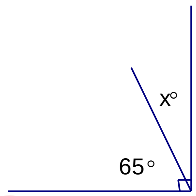
$25 + 105 + y = 180$ (-130, -130)

$\boxed{y = 50°}$

2. COMPLEMENTARY ANGLE THEOREM (CAT)

Angles that make a right angle, or add up to 90°.

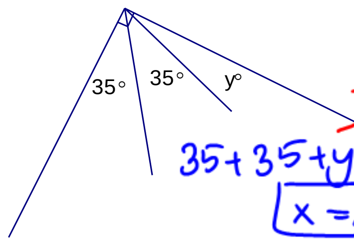
i)



$x + 65 = 90$ (-65, -65)

$\boxed{x = 25°}$

ii)



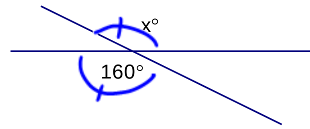
$35 + 35 + y = 90$ (-70, -70)

$\boxed{x = 20°}$

3. OPPOSITE ANGLE THEOREM (OAT)

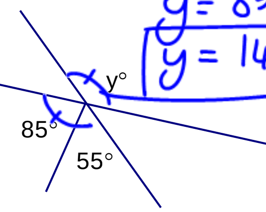
Angles that are across from each other at a point of intersection.

i)



$\boxed{x = 160°}$

ii)

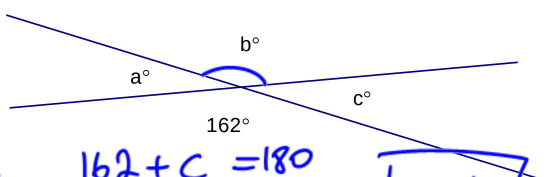


$y = 85 + 55$

$\boxed{y = 140°}$

Find each angle and state the theorem you used

.a.



$b = 162$
OAT

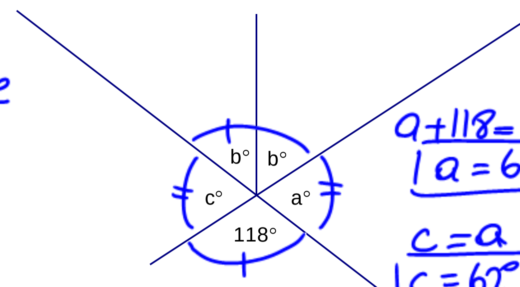
$162 + c = 180$
$c = 180 - 162$
$\boxed{c = 18°}$
SAT

$\boxed{a = 18 \quad \text{OAT}}$

b.

$\frac{118}{2} = \frac{2b}{2}$

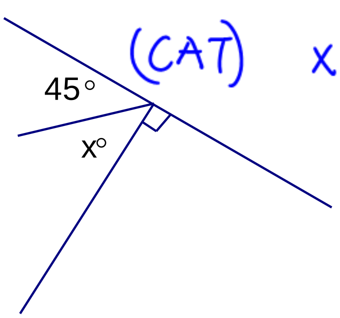
$\boxed{59° = b}$
OAT



$a + 118 = 180$ (SAT)
$\boxed{a = 62°}$

$c = a$ (OAT)
$\boxed{c = 62°}$

c.



(CAT) $x + 45 = 90°$

$\boxed{x = 45°}$

d. Determine the three angles:

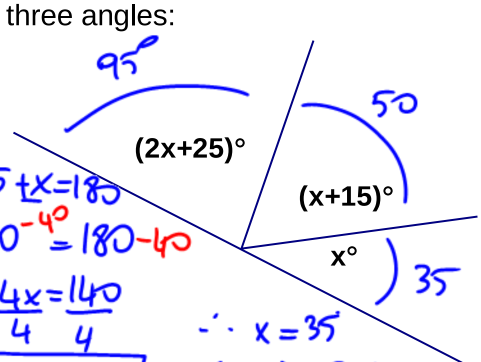
SAT

$2x + 25 + x + 15 + x = 180$

$4x + 40 - 40 = 180 - 40$

$\frac{4x}{4} = \frac{140}{4}$

$\boxed{x = 35°}$



$\therefore x = 35$
$(x+15) = 50°$
$(2x+25) = 95°$

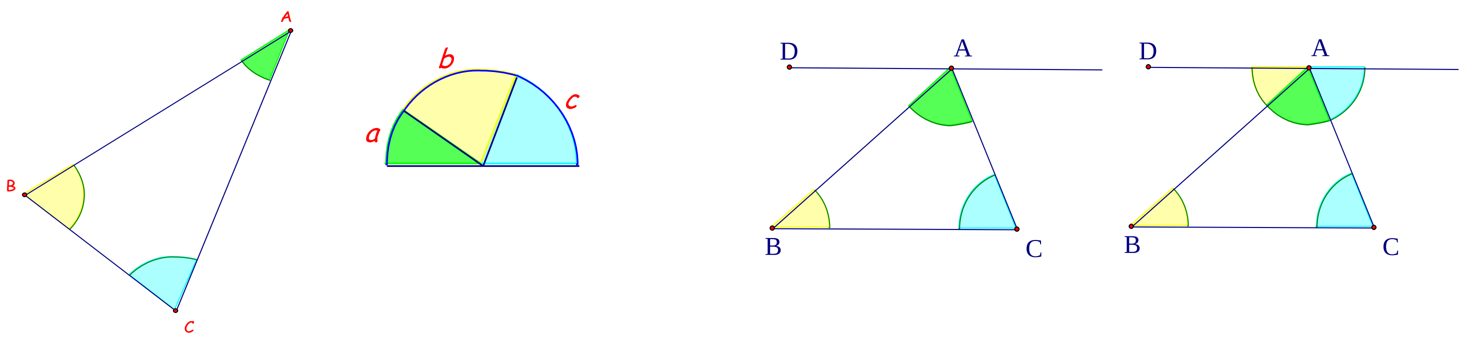
ANSWERS: a. a=18°, b=162°, c=18°, b. a=62°, b=59°, c=62°, c. x=45°, d. x =35°, x+15=50°, 2x+25=95°

Lesson 2: Interior Angles of Triangles

The sum of the interior angles in triangles is 180°.

These diagrams show how the three angles in triangles create 180° - a straight line!

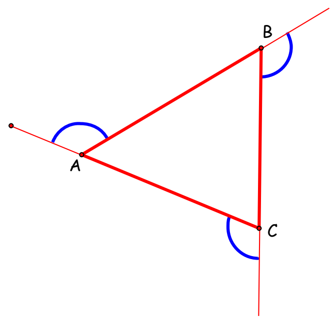
Ex 1 before	After	Ex 2 before	after



Exterior Angles of Triangles

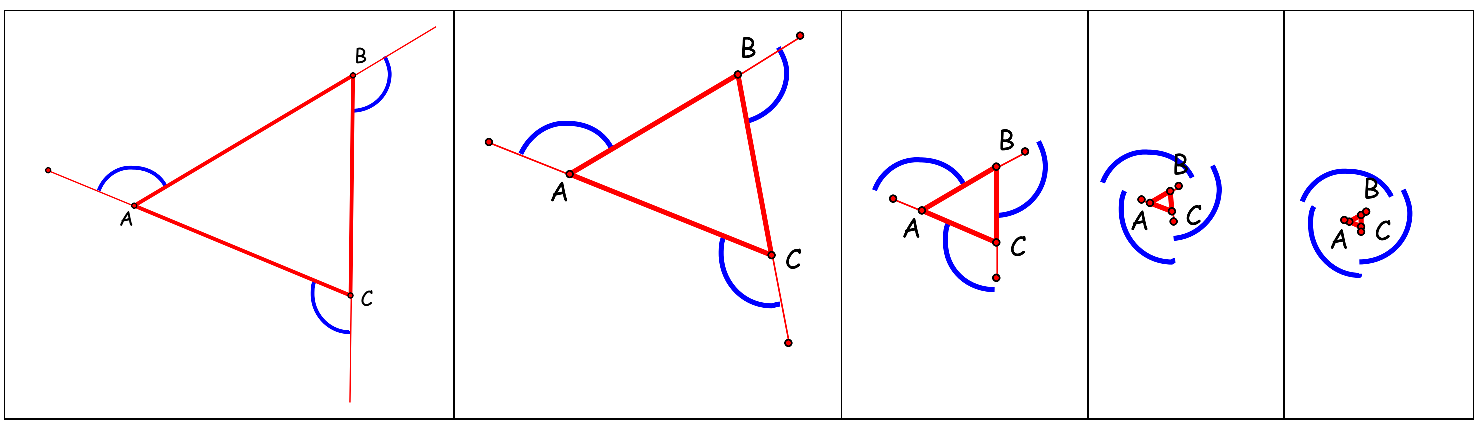
Exterior angles are angles outside of a shape. They are formed by extending the side lengths of a shape.

Example:



If you were to find the measure of the three exterior angles, you would find that their sum is 360°. Below is a diagram to show you why.

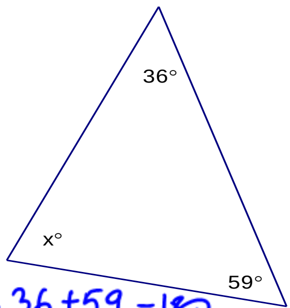
If you shrink this triangle and make it smaller and smaller by making 'similar triangles' which have the same angles, you will see how the three exterior angles come closer together and come to make a full circle. A full circle is 360°.



Practice: Interior & Exterior Angles of Triangles

Find the value of the missing angles: *** Note diagrams are not to scale so you cannot use your protractors.**

Ex1.

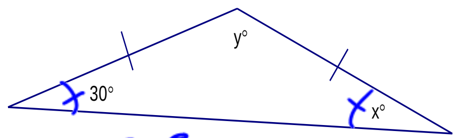


$x + 36 + 59 = 180$

$x + 95 - 95 = 180 - 95$

$\boxed{x = 85°}$

Ex2. Isosceles



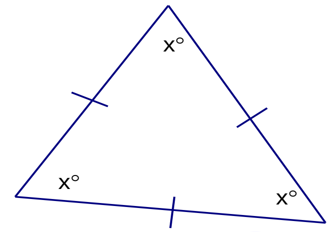
$x = 30°$

$30 + 30 + y = 180$

$y + 60 - 60 = 180 - 60$

$\boxed{y = 120°}$

Ex3. Equilateral

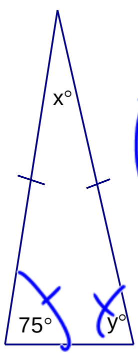


$x + x + x = 180$

$\frac{3x}{3} = \frac{180}{3}$

$\boxed{x = 60°}$

Ex4.



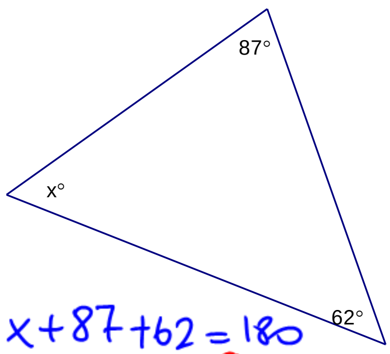
$\boxed{y = 75°}$

$x + 75 + 75 = 180$

$x + 150 - 150 = 180 - 150$

$\boxed{x = 30°}$

Ex5.

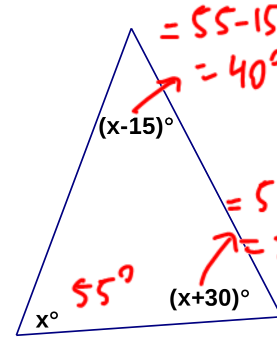


$x + 87 + 62 = 180$

$x + 149 - 149 = 180 - 149$

$\boxed{x = 31°}$

Ex6.



$= 55 - 15$

$= 40°$

$= 55 + 30$

$= 85°$

$55°$

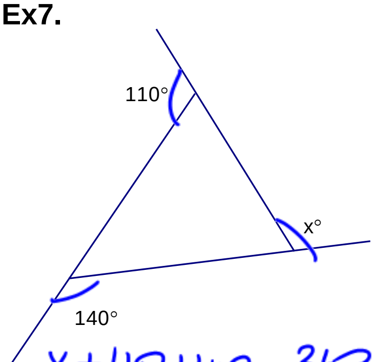
$x + (x-15) + (x+30) = 180°$

$3x + 15 - 15 = 180 - 15$

$\frac{3x}{3} = \frac{165}{3}$

$\boxed{x = 55°}$

Ex7.

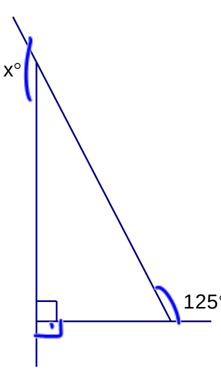


$x + 110 + 140 = 360$

$x + 250 - 250 = 360 - 250$

$\boxed{x = 110°}$

Ex8.

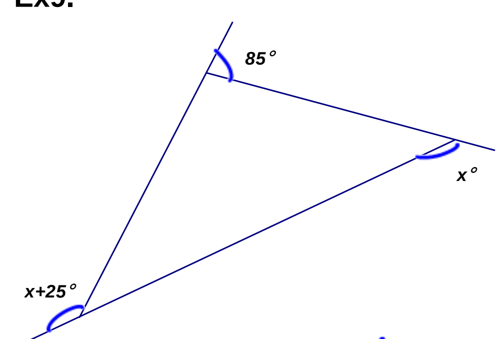


$x + 90 + 125 = 360$

$x + 215 - 215 = 360 - 215$

$\boxed{x = 145°}$

Ex9.



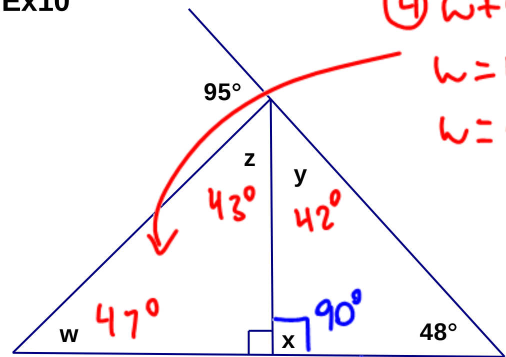
$x + 25 + 85 + x = 360$

$2x + 110 - 110 = 360 - 110$

$\frac{2x}{2} = \frac{250}{2}$

$\boxed{x = 125°}$

Ex10



① $x = 90°$

② Interior Angles of $\triangle$

$90 + 48 + y = 180$

$y = 180 - 90 - 48$

$y = 42°$

③ SAT

$y + z + 95° = 180$

$42 + z + 95 = 180$

$z = 180 - 95 - 42$

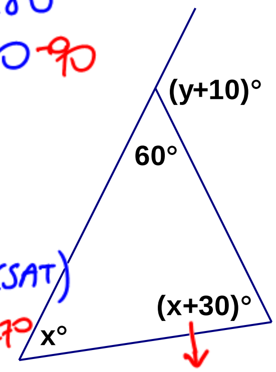
$z = 43°$

④ $w + 43 + 90 = 180$

$w = 180 - 90 - 43$

$w = 47°$

Ex11



① $x + x + 30 + 60 = 180$

Interior angles of $\triangle$

$2x + 90 - 90 = 180 - 90$

$\frac{2x}{2} = \frac{90}{2}$

$\boxed{x = 45°}$

② $60 + y + 10 = 180$ (SAT)

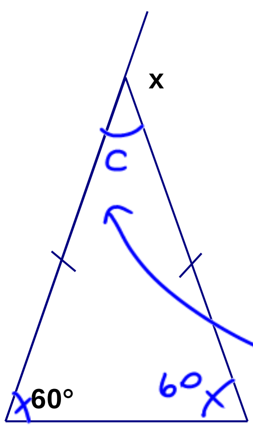
$y + 70 - 70 = 180 - 70$

$\boxed{y = 110°}$

$= 45 + 30$

$= 75°$

Ex12



Method 1

$x = 60 + 60$

$\boxed{x = 120°}$

Method 2

$c + x - x = 180 - x$

$\boxed{c = 180 - x}$

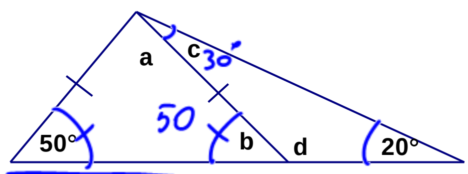
$180 - x + 60 + 60 = 180$

$300 - x - 300 = 180 - 300$

$\frac{-x}{-1} = \frac{-120}{-1}$

$\boxed{x = 120°}$

Ex13



$\boxed{b = 50°}$

$a + 50 + 50 = 180$

$a + 100 - 100 = 180 - 100$

$\boxed{a = 80°}$

$d + 50 = 180$ (SAT)

$\boxed{d = 130°}$

Interior Angles of $\triangle$

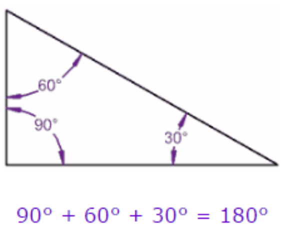
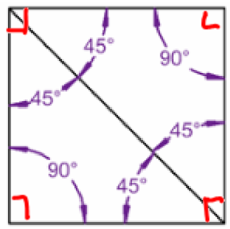
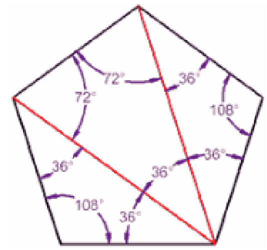
$d + 20° + c = 180$

$130 + 20 + c = 180$

$c = 180 - 130 - 20$

$\boxed{c = 30°}$

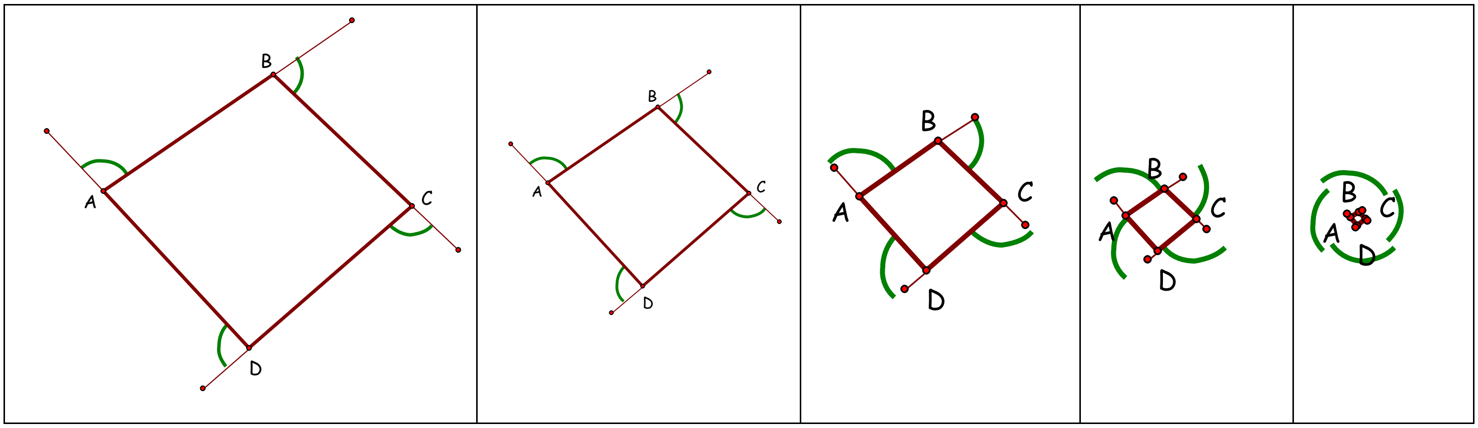
Lesson 3: Interior Angles of Quadrilaterals

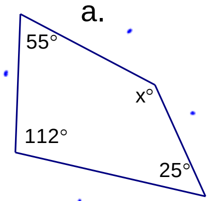
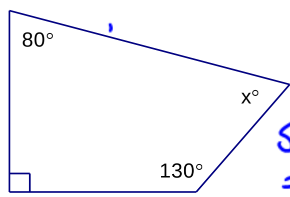
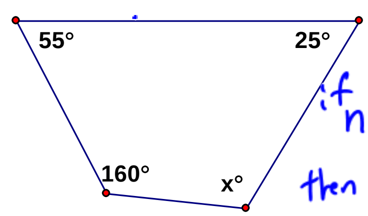
TRIANGLE	RECTANGLE	PENTAGON	ANY POLYGON
			
The interior angles in a triangle is 180^0	this square they add up to **360°**	A pentagon has 5 sides, and can be made from **three triangles**, therefore, it is **540°**	Sum of interior Angles = (**n**-2) × 180°

↑ # of sides

Exterior Angles of Quadrilaterals and Other Polygons

Check out the diagram below, showing a shrinking quadrilateral and it's exterior angles. Just as with a triangle, the sum of the exterior angles of a quadrilateral creates a circle or 360°.



a.	b.	c.
 $n=4$ $Sum=(4-2)180$ $=2\cdot180$ $=360$ $x+112+25+55=360$ $x+192-192=360-192$ $\boxed{x=168°}$	 $n=4$ $Sum=360°$ $90+80+130+x=360$ $300+x-300=360-300$ $\boxed{x=60°}$	 if $n=4$ then sum $=360$ $x+160+25+55=360$ $x+240-240=360-240$ $x=120°$
ANSWERS: a. x=168°, b. x = 60°, c. 120°		

PRACTICE:
Calculate the sum of the interior angles of a polygon with:

a. 5 sides

$Sum = (n-2)180$
$= (5-2)180$
$= 3 \cdot 180$
$= 540^\circ$

b. 10 sides

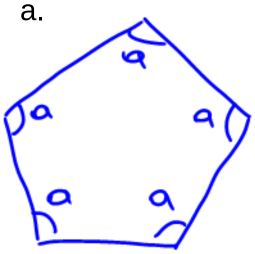
$Sum = (n-2)180$
$= (10-2)180$
$= 8 \cdot 180$
$= 1440^\circ$

c. 15 sides

$Sum = (n-2)180$
$= (15-2)180$
$= 13 \cdot 180$
$= 2340^\circ$

If each polygon above was a regular polygon (a polygon with all equal side lengths and all equal angles), determine the measure of each angle:

a.



$\frac{5a}{5} = \frac{540}{5}$
$\boxed{a = 108^\circ}$

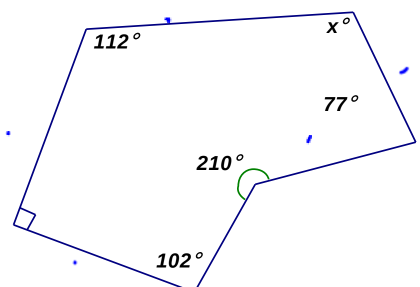
b.

$\frac{10a}{10} = \frac{1440}{10}$
$\boxed{a = 144^\circ}$

c.

$\frac{15a}{15} = \frac{2340}{15}$
$\boxed{a = 156^\circ}$

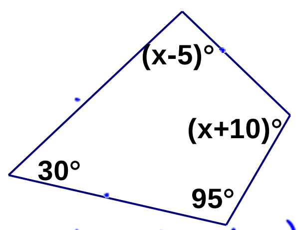
d.



$n = 6$ $Sum = (6-2)180$
$= 4 \cdot 180$
$= 720^\circ$

$x + 77 + 210 + 102 + 90 + 112 = 720$
$x + 591 - 591 = 720 - 591$
$\boxed{x = 129^\circ}$

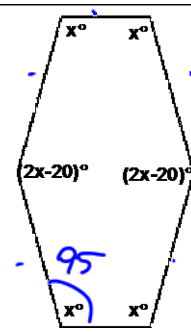
e.



$n = 4$ then $(4-2)180$
$= 360$

$30 + 95 + x + 10 + x - 5 = 360$
$130 + 2x - 130 = 360 - 130$
$\frac{2x}{2} = \frac{230}{2}$
$\boxed{x = 115^\circ}$

f.



$n = 6$ $Sum = (6-2)180$
$= 720^\circ$

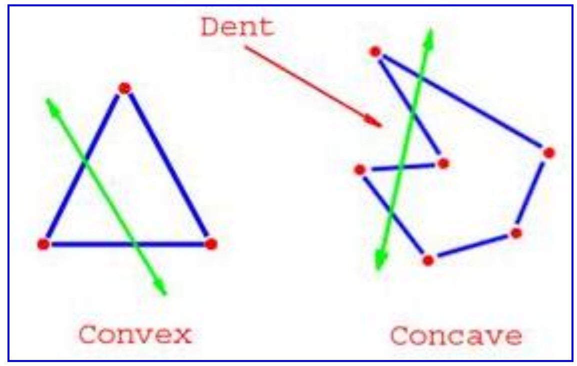
$4x + 2(2x - 20) = 720$
$4x + 4x - 40 + 40 = 720 + 40$
$\frac{8x}{8} = \frac{760}{8}$
$\boxed{x = 95^\circ}$

$= 2x - 20$
$= 2(95) - 20$
$= 170^\circ$

ANSWERS: a. x=129°, b. x=115°, x-5=110°, x+10=125°, c. x=95°, 2x-20=170°

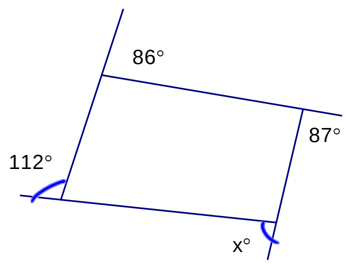
Other Polygons:

This is true for all convex polygons (a polygon where all interior angles are less than 180°). The sum of the exterior angles will always be equal to 360°.



Try some on your own. Find the missing exterior angle(s) in each polygon below:

a.

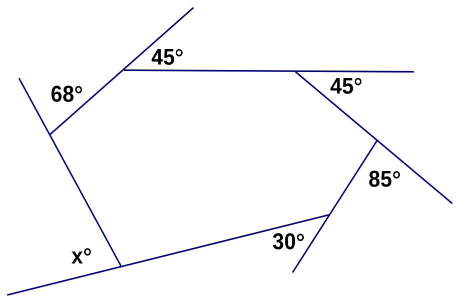


$x + 112 + 86 + 87 = 360$

$x + 285 - 285 = 360 - 285$

$\boxed{x = 75°}$

b.

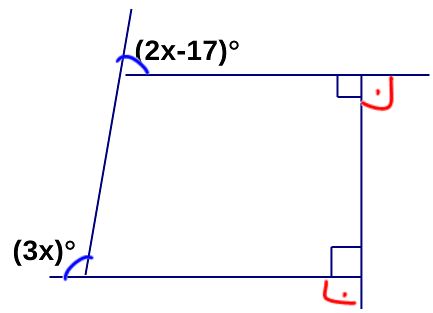


$x + 68 + 45 + 45 + 85 + 30 = 360$

$x + 273 - 273 = 360 - 273$

$\boxed{x = 87°}$

c.



$3x + 2x - 17 + 90 + 90 = 360$

$5x + 163 - 163 = 360 - 163$

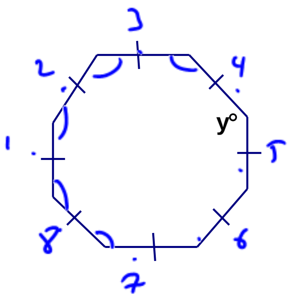
$\frac{5x}{5} = \frac{197}{5}$

$\boxed{x = 39.4}$

ANSWERS: a. x=75°, b. x=87°, c. x=39.4°, 3x=118.2°, 2x-17=61.8°

Find the measure of the missing angle(s) in each shape:

a.



$n=8$

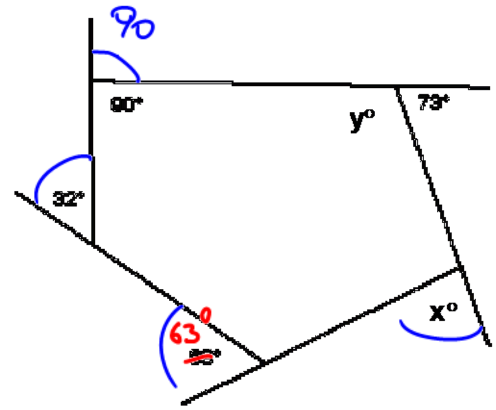
$Sum = (8-2)180$

$= 1080°$

$\frac{8y}{8} = \frac{1080}{8}$

$\boxed{y = 135°}$

b.



$x + 63 + 32 + 90 + 73 = 360$

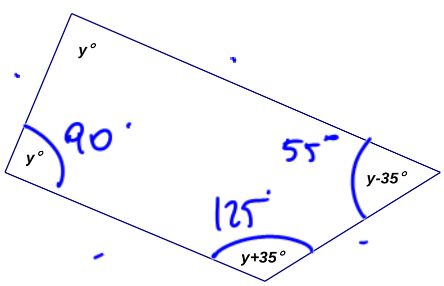
$x + 258 - 258 = 360 - 258$

$\boxed{x = 102°}$

$y + 73 = 180$ (SAT)

$\boxed{y = 107°}$

c.



$n=4$ $Sum = (4-2)180$

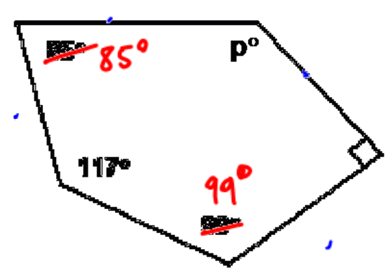
$= 360$

$y + y + y + 35 + y - 35 = 360$

$\frac{4y}{4} = \frac{360}{4}$

$\boxed{y = 90}$

d.



$n=5$

$Sum = (5-2)180$

$= 540$

$85 + 117 + 99 + 90 + p = 540$

$391 + p - 391 = 540 - 391$

$\boxed{p = 149°}$

ANSWERS: a. y=135°, b. y=107°, x=102°, c.y=90°; 90, 90, 55, & 125, d. p=149°

COMPLETE: CP page 85, 86, 87 a-i, 88 a-l