

**Lesson: Solving Equations - Multi-Step**

**Simplify, and then solve:**

Some equations, you may have to start by first collecting like terms to simplify the equation.

Example 1

$$3x + 10 - 6x = 8 - 4$$

$-3x + 10 = 4$  ← simplify by collecting like terms  
 ← now, solve the two-step equation

$$-3x + 10 - 10 = 4 - 10$$

or

$$-3x = 4 - 10$$

$$\frac{-3x}{-3} = \frac{-6}{-3}$$

$$\frac{-3x}{-3} = \frac{-6}{-3}$$

$$\boxed{x = 2}$$

$$\boxed{x = 2}$$

Example 2

$$7x + 3 - 4x + 5x = 3 - 5 + 9$$

$$8x + 3 = 7$$

$$8x + 3 = 7$$

$$8x + 3 - 3 = 7 - 3$$

$$8x = 7 - 3$$

$$\frac{8x}{8} = \frac{4}{8}$$

$$\frac{8x}{8} = \frac{4}{8}$$

$$\boxed{x = 0.5}$$

$$\boxed{x = 0.5}$$

**Variable on both sides:**

Other multi-step equations have the variable on both sides. These can be a little trickier. To solve, you must have all the variable terms on one side of the equation. When eliminating an entire term from an equation, we either (+) or (-) the term.

Example 3

$$8x + 8 = 2x - 4$$

$-2x$        $-2x$       ← do the same to both sides and collect your like terms

$6x + 8 = -4$       ← now, solve the two-step equation

$$6x + 8 - 8 = -4 - 8$$

$$\frac{6x}{6} = \frac{-12}{6}$$

$$\boxed{x = -2}$$

Example 4

$$5x - 23 = 3 - 8x$$

collect variables on one side, numbers the other

$$5x + 8x = 3 + 23$$

$$\frac{13x}{13} = \frac{26}{13}$$

$$\boxed{x = 2}$$

**Practice: Solving Equations - Multi-Step**

a.  $5 + 3x + 4x = 19$

$$\begin{aligned} 5 + 7x &= 19 \\ 7x &= 19 - 5 \\ 7x &= \frac{14}{7} \\ x &= 2 \end{aligned}$$

d.  $5m + 3 - 9m + 13 = 0$

$$\begin{aligned} (-4m) + 16 &= 0 \\ \frac{16}{4} &= \frac{4m}{4} \\ 4 &= m \end{aligned}$$

g.  $3b - 6 = -b - 2$

$$\begin{aligned} 8b + b &= -2 + 6 \\ \frac{4b}{4} &= \frac{4}{4} \\ b &= 1 \end{aligned}$$

j.  $5c - 3 - 4c = 2c + 2$

$$\begin{aligned} c - 3 &= 2c + 2 \\ -3 - 2 &= 2c - c \\ -5 &= c \end{aligned}$$

b.  $15y - 6 - 10y = 9$

$$\begin{aligned} 5y - 6 &= 9 \\ 5y &= 9 + 6 \\ 5y &= 15 \\ y &= 3 \end{aligned}$$

e.  $6w + 8 = 4w + 18$

$$\begin{aligned} 6w - 4w &= 18 - 8 \\ \frac{2w}{2} &= \frac{10}{2} \\ w &= 5 \end{aligned}$$

h.  $5 + 4d = -13 - 2d$

$$\begin{aligned} 2d + 4d &= -13 - 5 \\ \frac{6d}{6} &= \frac{-18}{6} \\ d &= -3 \end{aligned}$$

k.  $0 = 4x + 3 - x - 9$

$$\begin{aligned} 0 &= 3x - 6 \\ \frac{6}{3} &= \frac{3x}{3} \\ 2 &= x \end{aligned}$$

c.  $32 - 5 = -4a - 5a$

$$\begin{aligned} \frac{27}{-9} &= \frac{-9a}{-9} \\ a &= -3 \end{aligned}$$

f.  $-8k - 5 = 2k + 15$

$$\begin{aligned} -5 - 15 &= 2k + 8k \\ \frac{-20}{10} &= \frac{10k}{10} \\ -2 &= k \end{aligned}$$

i.  $7t + 8 = 3t - 12$

$$\begin{aligned} 7t - 3t &= -12 - 8 \\ \frac{4t}{4} &= \frac{-20}{4} \\ t &= -5 \end{aligned}$$

l.  $14 - n - 7 = 5n + 1$

$$\begin{aligned} 7 - n &= 5n + 1 \\ 7 - 1 &= 5n + n \\ \frac{6}{6} &= \frac{6n}{6} \\ n &= 1 \end{aligned}$$

m. Three angles are complementary (they add up to  $90^\circ$ ). For the diagram to the right, this can be expressed by the equation:  $3x + x + x - 20 = 90$ . Find the value of the three angles.

$$3x + x + x - 20 = 90$$

$$5x - 20 = 90$$

$$5x = 90 + 20$$

$$\frac{5x}{5} = \frac{110}{5}$$

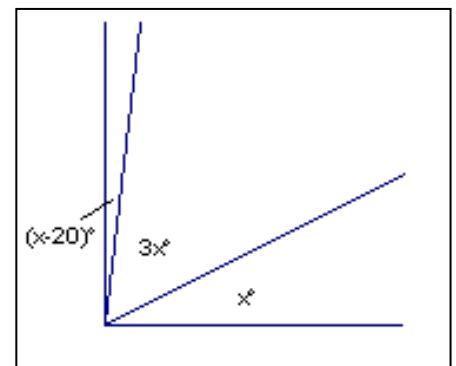
$$x = 22$$

Angles

①  $x = 22^\circ$

②  $3x = 3(22) = 66^\circ$

③  $x - 20 = 22 - 20 = 2^\circ$



ANSWERS: a.  $x=2$ , b.  $y=3$ , c.  $a=-3$ , d.  $m=4$ , e.  $w=5$ , f.  $k=-2$ , g.  $b=1$ , h.  $d=-3$ , i.  $t=-5$ , j.  $c=-5$ , k.  $x=2$ , l.  $n=1$ , m.  $2^\circ, 22^\circ, 66^\circ$

**Lesson: Solving Equations - with Brackets**

**Recap: eliminating brackets**

Simplify the following expressions:

a.  $(5x + 2) + (-x - 9)$   
 $= 5x + 2 - x - 9$   
 $= 4x - 7$

b.  $(2y + 4) - (5y - 12)$   
 $= 2y + 4 + (-5y + 12)$   
 $= 2y + 4 - 5y + 12$   
 $= -3y + 16$

c.  $3(x - 5) = 3x - 15$

Before 'solving' an equation, we sometimes will have to simplify → this may also include eliminating brackets.

**Simplify by Adding/Subtracting Polynomials**

Example 1

$(3x + 5) + (x - 1) = -2$   
 $3x + 5 + x - 1 = -2$   
 $4x + 4 = -2$   
 $4x = -2 - 4$   
 $4x = -6$   
 $\frac{4x}{4} = \frac{-6}{4}$   
 $x = -3/2$  or  $-1.5$

Example 2

$(5x - 4) - (9 - x) = -3$   
 $(5x - 4) + (-9 + x) = -3$   
 $5x - 4 - 9 + x = -3$   
 $6x - 13 = -3$   
 $6x = -3 + 13$   
 $\frac{6x}{6} = \frac{10}{6}$   
 $x = 5/3$

**Simplify using Distributive Law**

Example 3

$2(x - 4.5) + 3x = 11$   
 $2x - 9 + 3x = 11$   
 $-9 + 5x = 11$   
 $5x = 11 + 9$   
 $\frac{5x}{5} = \frac{20}{5}$   
 $x = 4$

Example 4

$3(x + 5) = 2(x - 4)$   
 $3x + 15 = 2x - 8$   
 $3x - 2x = -15 - 8$   
 $x = -23$

**Simplify**

Example 5

$5(x - 8) = (2x - 2) + (4x + 5)$   
 $5x - 40 = 2x - 2 + 4x + 5$   
 $5x - 40 = 6x + 3$   
 $-40 - 3 = 6x - 5x$   
 $-43 = x$

Example 6

$(3x + 7) - 4x = 2(9 + 4x)$   
 $3x + 7 - 4x = 18 + 8x$   
 $7 - x = 18 + 8x$   
 $7 - 18 = 8x + x$   
 $\frac{-11}{9} = \frac{9x}{9}$   
 $\frac{-11}{9} = x$

**Practice: Solving Equations – with Brackets**

a.  $5(x + 4) = 3x + 14$   
 $5x + 20 = 3x + 14$   
 $5x - 3x = 14 - 20$   
 $\frac{2x}{2} = \frac{-6}{2}$   
 $x = -3$

b.  $5q - 6 = 2(q + 3)$   
 $5q - 6 = 2q + 6$   
 $5q - 2q = 6 + 6$   
 $\frac{3q}{3} = \frac{12}{3}$   
 $q = 4$

c.  $4t + 3(2 - t) = 13$   
 $4t + 6 - 3t = 13$   
 $6 + t = 13$   
 $t = 13 - 6$   
 $t = 7$

d.  $u = 3(5 - u) + 1$   
 $u = 15 - 3u + 1$   
 $u = 16 - 3u$   
 $u + 3u = 16$   
 $\frac{4u}{4} = \frac{16}{4}$   
 $u = 4$

e.  $3(r + 4) + 2(r + 5) = 32$   
 $3r + 12 + 2r + 10 = 32$   
 $5r + 22 = 32$   
 $5r = 32 - 22$   
 $\frac{5r}{5} = \frac{10}{5}$   
 $r = 2$

f.  $5(y - 3) - 3(y - 4) = 12$   
 $5y - 15 - 3y + 12 = 12$   
 $2y - 3 = 12$   
 $2y = 12 + 3$   
 $\frac{2y}{2} = \frac{15}{2}$   
 $y = 7.5$

g.  $4(v + 3) = 2(v + 6) - 8$   
 $4v + 12 = 2v + 12 - 8$   
 $4v + 12 = 2v + 4$   
 $4v - 2v = 4 - 12$   
 $\frac{2v}{2} = \frac{-8}{2}$   
 $v = -4$

h.  $2(y - 4) = -3(y + 2) + 8$   
 $2y - 8 = -3y - 6 + 8$   
 $2y - 8 = -3y + 2$   
 $2y + 3y = 2 + 8$   
 $\frac{5y}{5} = \frac{10}{5}$   
 $y = 2$

i.  $6(3w + 4) = 10(2w - 1)$   
 $18w + 24 = 20w - 10$   
 $24 + 10 = 20w - 18w$   
 $\frac{34}{2} = \frac{2w}{2}$   
 $17 = w$

j.  $4(m + 3) + 2(m - 3) = 3(m - 2)$   
 $4m + 12 + 2m - 6 = 3m - 6$   
 $6m + 6 = 3m - 6$   
 $6m - 3m = -6 - 6$   
 $\frac{3m}{3} = \frac{-12}{3}$   
 $m = -4$

k.  $p - (4p + 3) = -3(p + 2) - (2p + 3)$   
 $p - 4p - 3 = -3p - 6 - 2p - 3$   
 $-3p - 3 = -5p - 9$   
 $-3p + 5p = -9 + 3$   
 $\frac{2p}{2} = \frac{-6}{2}$   
 $p = -3$

l. Polly solved the following equation. She is incorrect. Circle her two mistakes and explain why she is incorrect.

$3(x+5) - (x+4) = 3$   
 $3x+5 - x+4 = 3$   
 $3x - x + 5 + 4 = 3$   
 $2x + 9 = 3$   
 $+9 +9$   
 $2x = 12$   
 $\div 2 \div 2$   
 $x = 6$

$\Rightarrow 3x + 15 - x - 4 = 3$   
 $2x + 11 = 3$   
 $2x + 11 - 11 = 3 - 11$   
 $\frac{2x}{2} = \frac{-8}{2}$   
 $x = -4$

$p = -3$

ANSWERS: a)  $x = -3$ , b)  $q = 4$ , c)  $t = 7$ , d)  $u = 4$ , e)  $r = 2$ , f)  $y = 7.5$ , g)  $v = -4$ , h)  $y = 2$ , i)  $w = 17$ , j)  $m = -4$ , k)  $p = -3$ , l) 2<sup>nd</sup> line: just dropped the brackets for both polynomials. Should have  $3x + 15 - x - 4$ , AND 5<sup>th</sup> line  $+ 9$  (should have subtracted 9).