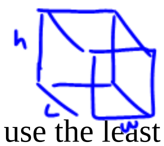
SURFACE AREA AND VOLUME RELATIONSHIPS OF RECTANGULAR PRISMS INVESTIGATION

MINIMUM SURFACE AREA

Problem 1: Teri has 64 cm^3 of sand and she wants to make a box to hold it, using as little material as possible.

2 (bottom + front + side)

$$SA=2(lw+wh+lh)$$



Complete the table to determine which of the three options will use the least amount of surface area.

$V = B \cdot h$

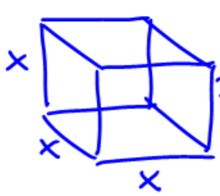
Length (m)	Width (m)	Height (m)	Surface Area (m^2)	Volume (m^3)
1	4	16	$SA=2(1\cdot4+4\cdot16+1\cdot16)=168$	64
2	4	8	$SA=2(2\cdot4+4\cdot8+2\cdot8)=112$	64
4	4	4	$SA=2(4\cdot4+4\cdot4+4\cdot4)=96$	64

The closer the box gets to being a cube, the smaller the surface area is for a given volume.

How can you predict the minimum surface area if you know the volume?

You find the side by cube rooting the volume. Then use SA formula to calculate the surface area.

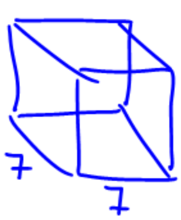
Predict the dimensions of a rectangular prism that minimizes the surface area, and has a volume of 343 mm^3.



$V = x^3$

$\sqrt[3]{343} = \sqrt[3]{x^3}$ ⇒ in calc (3 $\boxed{\sqrt[x]{\ }}$ 343)

$\boxed{7 = x}$



$SA = 6\cdot(7\cdot7)$

$= 294\,mm^2$

Problem 2: Jenny has 24 m^2 of wood to make a toy box.

Complete the table to determine how to maximize the volume of the toy box.

$SA = 2(lw + wh + lh)$

Length (m)	Width (m)	Height (m)	Surface Area (m^2)	Volume (m^3)
1	4	$24 = 2(1\cdot 4 + 4h + 1h)$ $12 = 4 + 5h$ $8 = 5h$ → $\boxed{h = 1.6}$	24	$V = 1(4)(1.6) = 6.4 m^3$
2	2	$24 = 2(2\cdot 2 + 2h + 2h)$ $12 = 4 + 4h$ → $8 = 4h$ $\boxed{2 = h}$	24	$V = (2)(2)(2) = 8 m^3$
2	3	$24 = 2(2\cdot 3 + 3h + 2h)$ $12 = 6 + 5h$ → $6 = 5h$ $\boxed{1.2 = h}$	24	$V = 2(3)(1.2) = 7.2$

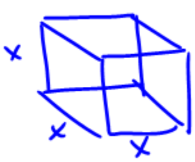
The closer the box gets to being a cube, the larger the volume is for a given surface area.

How can you predict the maximum volume if you are given the surface area?

Find the dimensions of the cube to find the volume.

Predict the dimensions of a prism that maximizes the volume and has a surface area of 54 cm^2.

Let "x" be one of the dimensions of a cube



$SA = 6(x \cdot x)$ ⟶ there're 6 same sides.

$\frac{54}{6} = \frac{6x^2}{6}$

$\sqrt{9} = \sqrt{x^2}$

$\boxed{3 = x}$

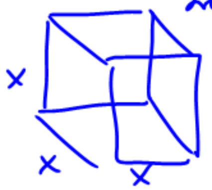
∴ The dimensions are $3 \times 3 \times 3$

Questions

1. A magician has ordered a covered water tank for his next new act. He has enough money to pay for 150 m^2 of building material. What is the largest volume of water that can be held in his water tank?

The largest volume of water can be held in a cube.

Let "x" be one of the dimensions



Step 1 $SA = 6x^2$

$\frac{150}{6} = \frac{6x^2}{6}$

$\sqrt{25} = \sqrt{x^2}$

$\boxed{x = 5}$

Step 2 $V = x^3$

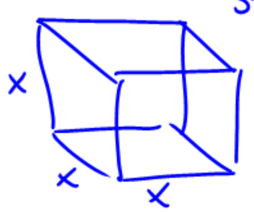
$= 5^3$

$= \underline{\underline{125\ m^3}}$

2. Dunstin and Carmila's kids need a place to put all of their toys. Dunstin decides to build them a toy box, but he only has 54 cm^2 of wood to make the box. Will he be able to make a toy box and lid that will hold all the kids toys if they have a total volume of 32 cm^3? (Do not worry about actual size of the toys, since smaller toys and fit amongst the larger ones).

Total Surface Area $= 54\ cm^2$

Let's calculate the dimensions of the shape (cube) that'll max the volume.



Step 1 $SA = 6x^2 \rightarrow$ (6 identical sides)

$\frac{54}{6} = \frac{6x^2}{6}$

$9 = x^2$

$\boxed{x = 3}$

Step 2 $V = x^3$

$= 3^3$

$= 27\ cm^3$

$\therefore$ He won't be able to fit all the toys in the box. He can donate some of the toys.

3. State the dimensions that will minimize the surface area of a shadow box that has a volume of 35,937 cm^3.

The cube will min the SA.

$V = x^3$

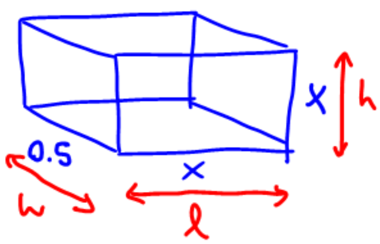
$35937 = x^3$ cube root each side $\left(3\ \boxed{\sqrt[x]{\ }}\ 35937\right)$

$\boxed{33 = x}$

∴ The dimensions are $33 \times 33 \times 33$

4. You have been asked to make a single shelf cabinet, with a volume of 4.5 m^3. However it can only be 0.5 m deep.

   a) Determine the dimensions that will minimize the surface area.
   b) Assuming that the front face of the shelf is open, what total surface area of wood is needed?



a) The cube or closer to the cube will get us the min SA

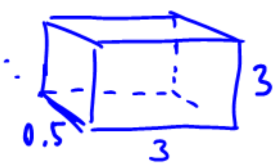
$V = l \cdot w \cdot h$

$4.5 = x \cdot 0.5 \cdot x$

$\frac{4.5}{0.5} = \frac{0.5x^2}{0.5}$

$9 = x^2$

$\boxed{x = 3}$

∴ 

b) $SA = 2(\text{bottom} + \text{side}) + \text{back}$

$= 2[3(0.5) + 3 \cdot (0.5)] + 3 \cdot 3$

$= 2[1.5 + 1.5] + 9$

$= 2[3] + 9$

$= 6 + 9$

$= 15\ m^2$

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